§5.6 (Rate of Change) Name: ________________________

28 July 2018

(1) Find the displacement over the time interval $[1, 6]$ of a helicopter whose vertical velocity at time t is $v(t) = .02t^2 + t$ feet per second.

(2) A particle is moving along a straight line with velocity $v(t) = \cos t$ meters per second. Find

(a) the total displacement over the interval $[0, 4\pi]$, and

(b) the total distance travelled over the interval $[0, 4\pi]$.

(3) The velocity in feet per second of a car is recorded at half-second intervals in the table below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$v(t)$	0	12	20	29	38	44	32	35	30

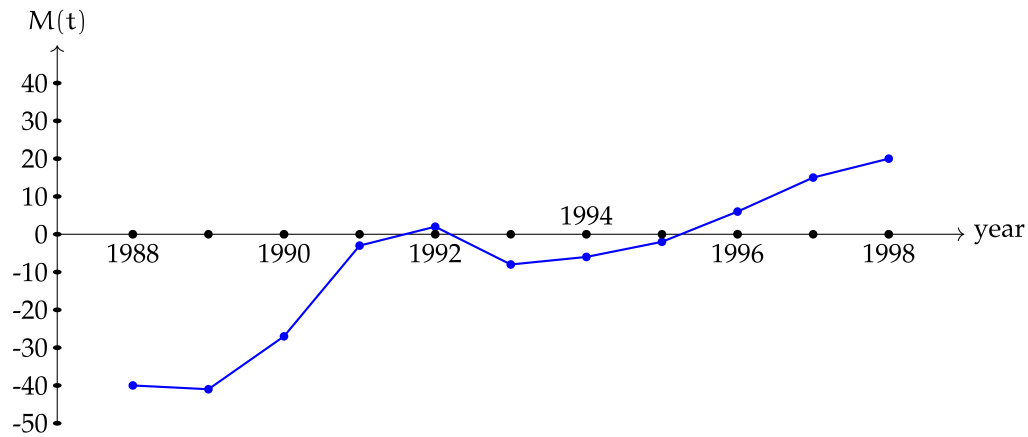
Use the average of the left-endpoint and right-endpoint approximations to estimate the total distance travelled over the time interval $[0, 4]$.

(4) The heat capacity $C(T)$ of a substance is the amount of energy (in joules) required to raise the temperature of one gram of the substance by one degree Celsius when it's temperature is T. (The heat capacity depends on the substance's current temperature.)

(a) Determine the energy required to raise the temperature of one gram from T_1 to T_2

(b) If a substance has heat capacity $C(T) = 6 + 0.2\sqrt{T}$, calculate the energy required to raise the temperature of one gram of the substance from 50° to 100° Celsius.

(5) The migration rate $M(t)$ of Ireland in the period 1988-1998 is shown in the figure below. This is the rate at which people (in thousands of people per year) move into or out of the country.



(a) Is the following integral positive or negative? What does the quantity represent? $\int_{1988}^{1998} M(t)\, dt$

(b) Did migration in the period 1988 – 1998 result in a net influx of people into Ireland or a net outflow of people from Ireland?

(c) During which two years could the Irish prime minister announce: "We're still losing population, but the trend is now improving?"

§5.7 (SUBSTITUTION) NAME:________________________

28 July 2018

The Substitution Method. To evaluate $\int f(g(x))g'(x)\,dx$:

(1) Substitute $u = g(x)$ and $du = g'(x)\,dx$ to get $\int f(u)\,du$.

(2) Integrate with respect to u.

(3) Replace u by $g(x)$.

(6) Use the substitution method to evaluate the following integrals:

(a) $\displaystyle\int_0^1 \frac{x}{(x^2+1)^3}\,dx$

(b) $\displaystyle\int_{10}^{17} (x-9)^{-2/3}\,dx$

(c) $\displaystyle\int_1^8 \sqrt{t+8}\,dt$

(d) $\displaystyle\int_1^5 \frac{e^x}{3+e^x}\,dx$

(e) $\displaystyle\int_0^{\pi/2} \sec^2(\cos\theta)\sin\theta\,d\theta$

(f) $\displaystyle\int_0^{\pi/4} \tan^3\theta\sec^2\theta\,d\theta$

(g) $\displaystyle\int \frac{dx}{(2+\sqrt{x})^3}$