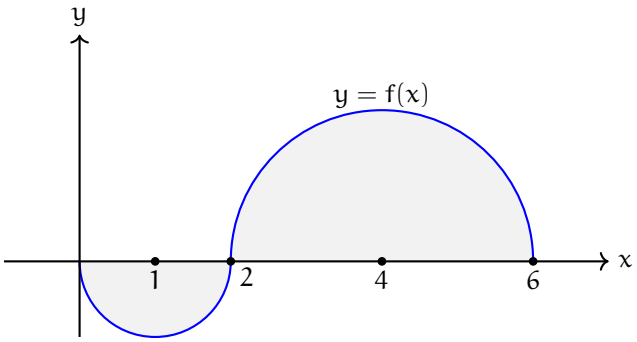
HOMEWORK 1

Math 1910, Summer 2018

NAME: _____

Due 3 July 2018

(1) Evaluate $\int_1^4 f(x)\,dx$ and $\int_1^6 |f(x)|\,dx$ using the graph below. The two parts of the graph are semicircles.



(2) Let A be the area under $f(x) = \sqrt{x}$ over the interval $[0, 1]$. Prove that $0.52 \leq A \leq 0.77$ without computing an integral. Explain your reasoning.

(3) Evaluate the indefinite integral.

(a) $\int \frac{1}{x^{4/3}}\, dx.$

(b) $\int \left(\frac{4}{x} - e^x\right)\, dx$

(c) $\int (z^5 + 4z^2)(z^3 + 1)^{12}\, dz.$

(d) $\int x^2\sqrt{x+1}\, dx$

(4) Evaluate the definite integral.

(a) $\int_1^{27} \frac{t+1}{\sqrt{t}}\, dt$

(b) $\int_0^5 |x^2 - 4x + 3|\, dx$

(c) $\int_{\pi/4}^{5\pi/8} \cos 2x\, dx$

(d) $\int_0^{\sqrt{e-1}} \frac{x^3}{x^2+1}\, dx$

(5) Show that $f(x) = \tan^2(x)$ and $g(x) = \sec^2(x)$ have the same derivative. What can you conclude about the relationship between f and g?

(6) Calculate the derivative. $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \frac{t\,dt}{t+1}$

(7) Let $N(d)$ be the number of asteroids of diameter d kilometers. Data suggest that the diameters are distributed according to a piecewise power law:

$$N'(d) = \begin{cases} 1.9 \times 10^9 d^{-2.3}, \text{ for } d < 70 \\ 2.6 \times 10^{12} d^{-4}, \text{ for } d \geq 70 \end{cases}$$

(a) Compute the number of asteroids with a diameter between 0.1 km and 100 km.

(b) Using the approximation $N(d+1) - N(d) \approx N'(d)$, estimate the number of asteroids of diameter 50km.