HOMEWORK 3

NAME: ____________________________

Math 1910, Summer 2018

Due 24 July 2018

(1) Evaluate the following integrals, or state that they diverge.

(a) $\displaystyle\int_0^\infty e^{-x}\cos(x)\,dx$

(b) $\displaystyle\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$

(c) $\displaystyle\int_4^\infty \frac{1}{(x-2)(x-3)}\,dx$

(d) $\displaystyle\int_0^1 \frac{1}{x^{1/3}+x^{2/3}}\,dx$

(2) Find a constant C such that $p(x)$ is a probability density function on the given interval, and compute the probability indicated.

(a) $p(x) = \dfrac{C}{(x+1)^3}$ on $[0, \infty)$; $\qquad P(0 \leq X \leq 1)$.

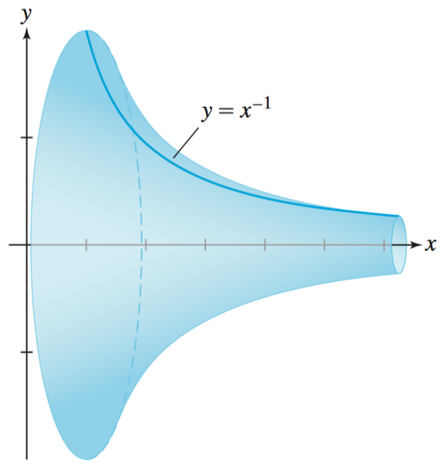
(b) $p(x) = \dfrac{Ce^{-x}}{1+e^{-2x}}$ on $(-\infty, \infty)$; $\qquad P(X \leq -4)$.

(3) The distance r between the electron and the nucleus in a hydrogen atom is a random variable with probability density $p(r) = 4a_0^{-3}r^2e^{-2r/a_0}$ for $r \geq 0$, where a_0 is the Bohr radius, $a_0 \approx 5.29 \times 10^{-11}$ m.

(a) Calculate the probability P that the electron is within one Bohr radius of the nucleus.

(b) Calculate the average distance between the electron and the nucleus.

(4) The solid S obtained by rotating the region below the graph of $y = x^{-1}$ around the x axis for $1 \leq x < \infty$ is called *Gabriel's Horn*.



(a) Compute the volume of S.

Question (4), continued.

(b) Compute the surface area of S.

(c) What is surprising about this? Would you rather use one of these as a cup or cut it up and use it the pieces as paper?

(5) Find the surface area of the torus obtained by rotating the circle $x^2 + (y - b)^2 = r^2$ around the x-axis.