(1) Determine the limit of the sequence $x_n = \dfrac{e^n + (-3)^n}{5^n}$ or show that it diverges.

(2) Give an example of a divergent sequence $\{a_n\}$ such that $\lim\limits_{n \to \infty} |a_n|$ converges.

(3) Find the sum: $\dfrac{1}{1 \cdot 3} + \dfrac{1}{3 \cdot 5} + \dfrac{1}{5 \cdot 7} + \cdots$

(4) Express the integral $\displaystyle\int_0^1 \arctan(x^2)\,dx$ as an infinite series and find its value to within 10^{-4}.

(5) Determine convergence or divergence of the series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n + n}{e^{2n} - n^2}$

(c) $\displaystyle\sum_{n=4}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

Question (5), continued.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + \sin n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

(6) Find the Taylor series centered at 0 and the interval on which the expansion is valid.

(a) $x^4 + 3x - 1$

(b) $(x^2 + 2x)e^x$

(c) $\dfrac{1}{3x - 2}$

Question (6), continued.

(a) $\cos^2(x)$

(b) $\displaystyle\int_0^x e^{t^2}\, dt$