READING ASSIGNMENT 11

§8.8 (Probability), §8.9 (Numerical Integration)

NAME: SOLUTIONS

Due 20 July 2018

LEARNING OBJECTIVES

By the end of this lesson, you will be able to:

- determine whether or not a function represents a probability density function
- compute probabilities and averages of random variables, given their probability density functions,
- use the trapezoid, midpoint, and Simpson's rules to approximate integrals, and compute errors for these approximations.

REVIEW

- Review Riemann sums and sigma notation for the numerical integration section.

READING

- Read section 8.8
- Read section 8.9

QUESTIONS

(1) The function $p(x) = \cos(x)$ satisfies $\int_{-\pi/2}^{\pi} p(x)\,dx = 1$. Is p a probability density function on $[-\pi/2, \pi]$?

SOLUTION: No, because $p(x)$ is not always nonnegative.

(2) What is the graphical interpretation of Simpson's rule?

SOLUTION: Simpson's rule approximates the area under the graph of $y = f(x)$ using parabolas, instead of trapezoids or rectangles.

(3) The N-th Simpson's rule approximation can be written as $S_{2N} = \frac{2}{3}M_N + \frac{1}{3}T_N$, where M_N is the N-th midpoint rule approximation and T_N is the N-th trapezoidal approximation. Why is the midpoint-rule approximation weighted more heavily in this sum?

SOLUTION: The error bound for M_N is smaller – it is one-half that of T_N.