Math 2940 HW 4: Required additional problems

These problems are about the relationship between *implicit* and *explicit* characterizations of subspaces of $\mathbf{R}^n$. The overarching point of view is that if a subspace is defined implicitly, then it is "naturally" the null space of a matrix. If a subspace is defined explicitly, then it is "naturally" the column space of a matrix.

1. Let

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ -2 & 3 & 2 & -5 \end{bmatrix}.$$

Find a matrix B such that $\text{Nul}(A) = \text{Col}(B)$.

Hint: $\text{Nul}(A)$ is defined implicitly. The usual procedure to find a basis for $W = \text{Nul}(A)$ yields an explicit characterization of W. Once you have found the basis, you should be able to "read off" a matrix B whose column space is W.

2. Let

$$A = \begin{bmatrix} 2 & -6 \\ -1 & 3 \\ -4 & 12 \\ 3 & -9 \end{bmatrix}.$$

Find a matrix B such that $\text{Col}(A) = \text{Nul}(B)$.

Hint: $\text{Col}(A)$ is defined explicitly. The task is to figure out a list of conditions that a vector $\mathbf{b}$ must satisfy to be in $\text{Col}(A)$, that is, find an implicit characterization of $W = \text{Col}(A)$. To do this, row-reduce the augmented matrix

$$\left[\begin{array}{cc|c} 2 & -6 & b_1 \\ -1 & 3 & b_2 \\ -4 & 12 & b_3 \\ 3 & -9 & b_4 \end{array}\right]$$

to determine conditions on $b_1, \ldots, b_4$ that must be satisfied in order for the equation $A\mathbf{x} = \mathbf{b}$ to have a solution. Once you have the list of conditions, you have found an implicit characterization of W, which means you should be able to "read off" a matrix B whose null space is W.