4

Math 1920 Homework ~~3.5~~ Selected Solutions

14.3

12)

$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ and $t = 1$. Then $\mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle$ and so

$$\|\mathbf{r}'(t)\| \,|_{t=1} = \sqrt{1 + 2^2 + 3^2} = \sqrt{14}$$

is the speed at $t = 1$.

15.1

20)

(a) and (c) have linear level sets, the ones for (c) can be rewritten $4x - c = 3y$ and so match with (C), and, consequently, (a) with (B). (d) is quadratic so (D), and this leaves (b) with (A).

44)

$\rho(B) = 1.0265$ and $\rho(A) = 1.0240$ so the net change from B to A is -0.0025. The change in T is 15 so the average change is

$$-\frac{25}{10000} \cdot \frac{1}{15}$$

$\cancel{3}$ 2

46)

If you fix a salinity level (i.e., an x-coordinate) and then walk in increasing temperature (positive y-direction) then you observe the level sets are decreasing in value. Hence the funciton is decreasing in T for fixed S.

Math 1920 Homework 4 Selected Solutions

15.2

PQ2)

By the continuity of $f(x,y)$ at $(2,3)$,

$$\lim_{(x,y)\to(2,3)} f(x,y) = f(2,3).$$

In particular,

$$f(2,3) = \lim_{y\to 3} f(2,y) = \lim_{y\to 3} y^3 = 3^3 = 27.$$

14)

Let $f(x,y) = xy/(x^2+y^2)$. Along the x-axis $y=0$ and along the y-axis $x=0$, so we compute

$$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} 0 = 0, \quad \lim_{y\to 0} f(0,y) = \lim_{y\to 0} 0 = 0.$$

Now we compute the limit as we approach the origin along the line $x=y$:

$$\lim_{x\to 0} f(x,x) = \lim_{x\to 0} \frac{x^2}{x^2+x^2} = \frac{1}{2} \neq 0.$$

Hence, the limit as $(x,y)\to(0,0)$ of f does not exist as there are two different paths with different values of the limit along those paths.