Selected Solutions to HW 7

15.8

PQ2)

In the first drawing ∇f is tangent to the constraint g. As no matter how you approach the point along the constraing you are moving away from the 1-level set and moving towards the 2-level set, we can assume it is a max.

In the second drawing you still have tangency, but as you approach from one side you are getting bigger, and the other side getting smaller, so it is neither a max nor a min.

18)

Let x, y, z be the lengths of the edges of the box, so we want to maximize $f(x, y, z) = xyz$ subject to $x + y + z = 300$ and $x, y, z \geq 0$. We let $G(x, y, z) = x + y + z - 300$ and using Lagrange multipliers we want to solve

$$\nabla f = (yz, xz, xy) = \lambda(1, 1, 1) = \lambda \nabla G$$

We have the equations $xy = yz = xz = \lambda$. If any of λ, x, y, or $z = 0$ then $f = 0$ which is not a maximum, so we assume that all parameters are non-zero.

With this assumption from, say, $xy = yz$ we derive $x = z$ and similarly $x = y = z$. Using this in the constraint G we find $3x = 300$ so $x = y = z = 100$ gives the maximum value for f.

26)

We want to find the maximum value of $f(x, y) = x^2y^3$ on the unit circle. We express this constraint as $G(x, y) = x^2 + y^2 - 1 = 0$. Using the multipliers we want to solve

$$(2xy^3, 3x^2y^2) = \lambda(2x, 2y)$$

or $2xy^3 = \lambda 2x, 3x^2y^2 = \lambda 2y$. If $x = 0$ or if $y = 0$ then $f(x, y) = 0$ which is clearly not a maximum. So we can assume neither x nor y, and consequently λ are all non-zero.

Using that we can rearrange the equations freely and find $y^3 = \lambda = 3x^2y/2$ and so $y^2 = 3x^2/2$. We can plug this into our constraint to find $x^2 + 3x^2/2 = 1$ or $x^2 = 2/5$. We get 4 possible points

$$\left(\sqrt{\frac{2}{5}}, \sqrt{\frac{3}{5}}\right), \left(\sqrt{\frac{2}{5}}, -\sqrt{\frac{3}{5}}\right), \left(-\sqrt{\frac{2}{5}}, \sqrt{\frac{3}{5}}\right), \left(-\sqrt{\frac{2}{5}}, -\sqrt{\frac{3}{5}}\right)$$

You can eyeball that f is biggest when $y > 0$ so $\left(\sqrt{\frac{2}{5}}, \sqrt{\frac{3}{5}}\right), \left(-\sqrt{\frac{2}{5}}, \sqrt{\frac{3}{5}}\right)$ give the maximum (they both work as the both give the same f value), which is $\frac{2}{5}\left(\frac{3}{5}\right)^{\frac{3}{2}}$