Math 1920, Final Exam
December 13, 2013, 9:00 AM to 11:30 AM

You are NOT allowed to use calculators, cell phones, electronic devices, the text or any other book or notes (except the sheet mentioned below). SHOW ALL WORK!
Writing clearly and legibly will improve your chances of receiving the maximum credit that your solution deserves. Please label the questions in your answer booklet clearly. There are ***7 questions****. You may use one double-sided* $8\frac{1}{2}$*"-by-11" sheet in your own handwriting. Write your* *name and section number* *on each booklet you use. You may leave when you have finished, but if you have not handed in your exam booklet and left the room by 11:15 AM, please remain in your seat so as not to disturb others who are still working.*

1. (21 points) Let $f_1(x, y, z) = x^2 - y^2 + z^2$, and $f_2(x, y, z) = xy$. What is the angle between the tangent planes for the level surfaces of f_1 and f_2 at the point $(\pi, e, \sqrt{2})$?

2. (21 points) Let $\mathcal{W}$ be the region inside the unit sphere between the cones $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{3(x^2 + y^2)}$. Compute
$$\iiint_{\mathcal{W}} \frac{1}{x^2 + y^2 + z^2} dV.$$

3. (21 points) Consider the function $f(x, y) = x^3/3 - xy + y^2/2$ defined on all of space.
   (a) Find all the local maximum and minimum points, and saddle points for f.
   (b) Find the global maximum and minimum points when f is restricted to the square $0 \le x \le 1,\ 0 \le y \le 1$.

4. (24 points) Let $\mathbf{F} = \langle x + y, x, -z \rangle$, a vector field defined everywhere. Let $\mathbf{G} = \frac{1}{r^2}\mathbf{F}$, where $r^2 = x^2 + y^2 + z^2$. Note that $\mathbf{G}$ is defined everywhere except the origin.
   (a) Calculate the divergence and curl of $\mathbf{F}$.
   (b) Find a potential function for $\mathbf{F}$, if it exists.
   (c) Calculate the work done along the path $\mathbf{c}(t) = (2 \sin t \cos t, 2 \sin^2 t, 2 \cos t)$, from $t = 0$ to $t = \pi/2$ for the vector field $\mathbf{F}$.
   (d) The divergence and curl of $\mathbf{G}$ are both non-zero and are fairly complicated. Calculate the outward flux of $\mathbf{G}$ across the sphere $x^2 + y^2 + z^2 = 4$. (*Hint*: Notice how $\mathbf{F}$ and $\mathbf{G}$ are related.)
   (e) Calculate the work done along the path $\mathbf{c}(t) = (2 \sin t \cos t, 2 \sin^2 t, 2 \cos t)$, from $t = 0$ to $t = \pi/2$ for the vector field $\mathbf{G}$. (*Hint*: Notice how $\mathbf{F}$ and $\mathbf{G}$ are related.)

5. (21 points) Let $\mathcal{S}$ be the half-cylinder $x^2 + y^2 = 1$, $x \ge 0$, $0 \le z \le 1$. Assume that the z component of $\mathbf{F}$ is zero and that $\mathbf{F}(0, y, z) = \langle y^2 + z^2, 0, 0 \rangle$. Let $\mathcal{W}$ be the *solid* half-cylinder $x^2 + y^2 \le 1, x \ge 0,\ 0 \le z \le 1$, and assume that
$$\iiint_{\mathcal{W}} \operatorname{div}(\mathbf{F}) dV = 2.$$

Find the outward flux of $\mathbf{F}$ through $\mathcal{S}$.

6. (21 points) Let $\mathbf{F} = \langle z^2 - y^2, x^2 - z^2, x^2 + y^2 \rangle$. Consider the hyperboloid $g(x, y, z) = x^2 + y^2 - z^2 = 1$.

   (a) Show that at every point on the hyperboloid, $\mathrm{curl}(\mathbf{F})$ is tangent to the surface.

   (b) Let $\mathcal{S}$ be any (bounded) region on the hyperboloid, and let $\mathcal{C}$ be the boundary curve of the surface $\mathcal{S}$. Prove that the circulation $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{s} = 0$.