Math 1920, Final Exam
December 12, 2016, 2:00 PM to 4:30 PM

You are NOT allowed to use calculators, cell phones, electronic devices, the text or any other books or notes. SHOW ALL WORK, and be sure to explain your answers! Please label the questions in your answer booklet clearly. There are ***8 questions****. Write your* *name and section number* *on each booklet you use. You may leave when you have finished, but if you have not handed in your exam booklet and left the room by 4:15 PM, please remain in your seat, so as not to disturb others who are still working.*

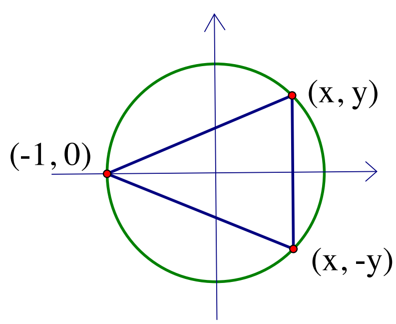
1. Consider the integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} e^{-(x^2+y^2)}dy\,dx$.

   (a) (10 points) Change the Cartesian integral into an equivalent integral with polar coordinates

   (b) (10 points) Evaluate the integral.

2. (15 points) Calculate the volume of the region inside the sphere of radius 2 and inside the cylinder of radius 1, both centered at the origin.

3. (20 points) In the figure below, a triangle is inscribed in the unit circle $x^2+y^2=1$, with one vertex at $(-1,0)$ and the opposite side vertical as shown. What is the point (x,y) that maximizes the area of the triangle? (Hint: You can use Lagrange multipliers.)



4. (20 points) Show that $\mathbf{F} = \frac{1}{x^2+y^2}\langle x, y\rangle$ is a conservative vector field, by finding a potential function $f(x,y)$ such that $\mathbf{F} = \nabla f$. Explain why $\mathbf{F}$ is incompressible as well.

5. (15 points) A vector field in 3-space is given by $\mathbf{F} = \langle x^2+e^y, y^2+\ln(z+1), z^2+(x+1)^y\rangle$. Calculate the outward flux of $\mathbf{F}$ across the boundary of the box $[0,a]\times[0,b]\times[0,c]$, where a, b, c are all positive.

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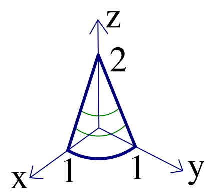
6. For the part of the curved surface of the cone $z = 2(1 - r)$, $r = \sqrt{x^2 + y^2}$, in the first octant, determine the flux away from the z-axis of the of the vector field $\mathbf{F} = \langle x, y, z \rangle$ in two ways: (a) (10 points) directly and (b) (10 points) using the the Divergence Theorem.



7. (a) (5 points) State the general form of Stokes' Theorem for a surface $\mathcal{S}$ with boundary $\partial\mathcal{S}$, and state the general form of the Divergence Theorem for a closed surface $\mathcal{S}'$.

   (b) (10 points) Let $\mathcal{S}$ be the half-ellipsoid surface defined by $x^2 + y^2 + 3z^2 = 1, z \geq 0$ and let $\mathbf{B} = \langle 2, -1, -2 \rangle$ be a vector field. We would like to compute the flux of $\mathbf{B}$ across $\mathcal{S}$, oriented with its normal vectors pointing up. We observe that there is another vector field, a vector potential, $\mathbf{A} = \langle y, -x + z, x + y \rangle$, where $\operatorname{curl} \mathbf{A} = \mathbf{B}$. Use Stokes' Theorem to calculate the flux of $\mathbf{B}$ across $\mathcal{S}$. In your answer, write the statement of Stokes' Theorem applied to $\mathcal{S}$ and the appropriate vector field.

   (c) (10 points) Let $\mathcal{D}$ be the circular disk in the xy plane whose boundary is the boundary of $\mathcal{S}$. Observe also that $\operatorname{div} \mathbf{B} = 0$ in all of 3-space, and apply the Divergence Theorem to the surface $\mathcal{S}' = \mathcal{S} \cup \mathcal{D}$, to again calculate the flux of $\mathbf{B}$ across $\mathcal{S}$. In your answer, write the statement of the Divergence Theorem applied to $\mathcal{S}'$ and the appropriate vector field.

8. (15 points) Let $\mathbf{F} = \langle x^2 - y, x - y^2 \rangle$ be a vector field in the plane. Use Green's Theorem to calculate the line integral $\oint_{\partial T} \mathbf{F} \cdot d\mathbf{r}$, where ∂T is the boundary of a trapezoid with sides of length 1 and 2 parallel to the x axis and height 1 oriented in a counter clockwise manner.