Math 1920, Prelim 2
November 8, 2016, 7:30 PM to 9:00 PM

Problem 1. (20 points) Let $f(x, y) = x^2 + 2xy - y^2$ be defined in the domain $\mathcal{D} = \{(x, y) \mid x^2 + y^2 \leq 1\}$.

(a) Find all the critical points f in the interior of $\mathcal{D}$ and determine whether they are relative maxima, minima, saddle points, or something else.

(b) In all of $\mathcal{D}$ what are the points where the maxima and minima are achieved?

Problem 2. (15 points) Let $\mathbf{F} = \langle y, z, x \rangle$ be a vector field, and $\mathbf{r}(t) = \langle \cos t, \sin t, c \rangle$, where $0 \leq t \leq 2\pi$ and c is a constant, determine a curve $\mathcal{C}$. Compute the line integral $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$. Does it depend on the value of c?

Problem 3. (15 points) Find the point on the sphere $x^2 + y^2 + z^2 = 1$ which is the closest to the plane $3x + 4y + 12z = 26$.

Problem 4. (15 points) For what value of b is the vector field $\mathbf{F} = yz\mathbf{i} + (xz + z)\mathbf{j} + (xy + by + 1)\mathbf{k}$ conservative? In this case, determine a potential function for $\mathbf{F}$.

Problem 5. (15 points) Given the vector field $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j}$, calculate the upward flux of F through the semi-circle $x^2 + y^2 = 1$, $y \geq 0$.

Problem 6. (20 points) Let $\mathcal{R}$ be the region inside the cone $z = (x^2 + y^2)^{1/2}$ and outside the sphere $x^2 + y^2 + z^2 = 1$ and bounded above by the plane $z = 2$. Set up the integral $\iiint_{\mathcal{R}} \frac{z}{(x^2+y^2+z^2)^{1/2}} dV$ in terms of spherical coordinates and cylindrical coordinates. You may leave your answer as an integral without evaluating it. But be sure to explicitly put in the limits of the integral.