Math 1920

Final exam

Name ______________________________

12 December 2017 9:00–11:30 am

Place an X in the box by your discussion section number

☐ 201	Aleksandra Niepla	MW 7:30–8:20P		☐ 214	Shrinidhi Pandurangi	TR 9:05–9:55A
☐ 202	Aleksandra Niepla	MW 8:35–9:25P		☐ 215	Andres Fernandez	TR 9:05–9:55A
☐ 203	Andres Fernandez	TR 8:00–8:50A		☐ 216	Dylan Peifer	TR 9:05–9:55A
☐ 204	Dylan Peifer	TR 8:00–8:50A		☐ 217	Feng Liang	TR 12:20–1:10P
☐ 205	Thomas Reeves	TR 8:00–8:50A		☐ 218	Max Jenquin	TR 12:20–1:10P
☐ 206	Bram Wallace	TR 8:00–8:50A		☐ 219	Feng Liang	TR 1:25-2:15P
☐ 207	Shrinidhi Pandurangi	TR 8:00–8:50A		☐ 220	Ryan McDermott	TR 1:25-2:15P
☐ 209	Bill Wu	TR 8:00–8:50A		☐ 221	Itamar Oliveira	MW 7:30–8:20P
☐ 210	Thomas Reeves	TR 9:05–9:55A		☐ 222	Itamar Oliveira	MW 8:35–9:25P
☐ 211	Bram Wallace	TR 9:05–9:55A		☐ 223	Max Jenquin	TR 1:25–2:15P
☐ 213	Bill Wu	TR 9:05–9:55A		☐ 224	Ryan McDermott	TR 2:30–3:20P

Instructions—please read now

• Write your name and check the box with your discussion section number *right now*.

• There are 10 problems and this booklet has 13 sheets. You may use the back of each sheet as scratch paper.

• You have 150 minutes to complete the exam. You may leave early, but if you finish within the last 15 minutes, please remain in your seat.

• Show your work and simplify your answers. To receive full credit, your answers must be neatly written and logically organized.

• You are allowed a two-sided letter size sheet of notes. No books or electronic devices or any other resources are allowed.

• Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

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Signature of Student

Official use only

1.	______ /	10
2.	______ /	10
3.	______ /	14
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5.	______ /	12
6.	______ /	12
7.	______ /	12
8.	______ /	14
9.	______ /	24
10.	______ /	22
Total	______ /	140

1 (10 points)**.** Suppose a function $g(u,v)$ satisfies $g(1,0) = 2$ and $\nabla g(1,0) = \langle 2,-1\rangle$. Define a function $f(x,y,z)$ by $f(x,y,z) = g(e^{xz} + y, \sin(x - y + 2z))$.

(a) Find $\nabla f(0,0,0)$.

(b) Find the equation of the tangent plane at $(0,0,0)$ to the surface $f(x,y,z) = 2$.

2 (10 points)**.** Evaluate the double integral $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 x e^{y^5}\, dy\, dx$.

3 (14 points)**.** Find the volume and the centroid of the solid region between the square $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ in the xy-plane and the surface given by $z = xy + 1$. (Recall that the centroid of a solid is the center of mass with respect to a constant mass density.)

4 (10 points). Determine the local minima, local maxima and saddle points of the function $f(x, y) = x^3 + y^2 - xy$.

5 (12 points). Let a, b and c be positive constants. Let $\mathscr{D}$ be the surface in $\mathbf{R}^3$ given by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

and the inequalities $x \geq 0$, $y \geq 0$, $z \geq 0$. What is the maximum of the function $f(x, y, z) = xyz$ over the region $\mathscr{D}$? At what point of $\mathscr{D}$ is this maximum attained? (The answers will depend on a, b and c.)

6 (12 points). True or false? If true, justify. If false, explain why or give a counterexample.

(a) If $f(x,y)$ is a continuous function, then $\displaystyle\int_0^1\int_0^x f(x,y)\,dy\,dx = \int_0^1\int_0^y f(x,y)\,dx\,dy$.

(b) Let $f(x,y)$ be a function which is defined for all x and y. Let $g(x,y) = (x^2+y^2)^{2/3} f(x,y)$. If $f(x,y)$ is continuous at the origin, then $\dfrac{\partial g}{\partial x}(0,0)$ is well-defined and equal to 0.

7 (12 points)**.** True or false? If true, justify. If false, explain why or give a counterexample.

(a) Let $f(x, y) = y - x$ and let $\mathscr{C}$ be the unit circle $x^2 + y^2 = 1$, oriented in the counterclockwise direction. Then $\int_{\mathscr{C}} \nabla f \cdot d\mathbf{r} = 2\pi$, twice the area enclosed by $\mathscr{C}$.

(b) Suppose a vector field $\mathbf{F}$ defined on $\mathbf{R}^3$ is tangent to a closed surface $\mathscr{S}$, which is the boundary of a solid region $\mathscr{W}$ in $\mathbf{R}^3$. Then $\iiint_{\mathscr{W}} \operatorname{div}(\mathbf{F})\, dV = 0$.

8 (14 points)**.** (a) Let $\mathscr{C}$ be the circle in the xz-plane given by $x^2 + z^2 = 1$, $y = 0$, oriented counterclockwise when viewed from along the positive y-axis, and let

$$\mathbf{F}(x, y, z) = \langle 3x^2 + 2xyz^2, 3y^2 + x^2z^2, 3z^2 + 2x^2yz \rangle.$$

Show that $\int_{\mathscr{C}} \mathbf{F} \cdot d\mathbf{r} = 0$.

(b) Let

$$\mathbf{F}(x, y, z) = \langle x\cos(xy) + x\sin(xz), xye^{xyz} - y\cos(xy), -z\sin(xz) - xze^{xyz} \rangle.$$

Show that $\iint_{\mathscr{S}} \mathbf{F} \cdot d\mathbf{S} = 0$ for every closed oriented surface $\mathscr{S}$ in $\mathbf{R}^3$.

9 (24 points)**.** Let $\mathscr{S}$ be the surface in $\mathbf{R}^3$ defined by $x^2 + y^2 + z^2 = 1$ and $0 \le z \le h$, where h is a constant satisfying $0 \le h \le 1$. Let $\mathscr{S}$ be oriented by the upward pointing normal vector.

(a) Sketch the surface $\mathscr{S}$. Include adequate detail such as coordinate axes and an indication of scale. In your sketch indicate the orientation of the boundary of $\mathscr{S}$.

(b) Parametrize the surface $\mathscr{S}$. Specify the domain $\mathscr{D}$ of the parametrization.

(c) Calculate the normal vector field $\mathbf{N}$ to the surface.

(d) Verify Stokes' Theorem for the surface $\mathscr{S}$ and the vector field

$$\mathbf{F}(x, y, z) = \langle z - y, 0, y \rangle.$$

(The answers will depend on h.)

(You may continue your solution on the next sheet.)

Use this space to continue your solution of Problem 9.

10 (22 points)**.** Let $\mathbf{A}$ be the vector field in space defined by $\mathbf{A} = \langle -yr^a, xr^a, 0\rangle$, where a is a constant and $r = \sqrt{x^2 + y^2 + z^2}$. Let $\mathbf{F} = \mathrm{curl}(\mathbf{A})$. (Note that for $a < 0$ the vector fields $\mathbf{A}$ and $\mathbf{F}$ are not defined at the origin.)

(a) Show that $\mathbf{F} = (a+2)r^a\mathbf{k} - azr^{a-1}\mathbf{e}_r$, where $\mathbf{e}_r = \dfrac{1}{r}\langle x, y, z\rangle$ is the unit radial vector field.

(Continued on next page.)

10 (continued). (b) Show that for $a = -3$ the vector field $\mathbf{F}$ is conservative.