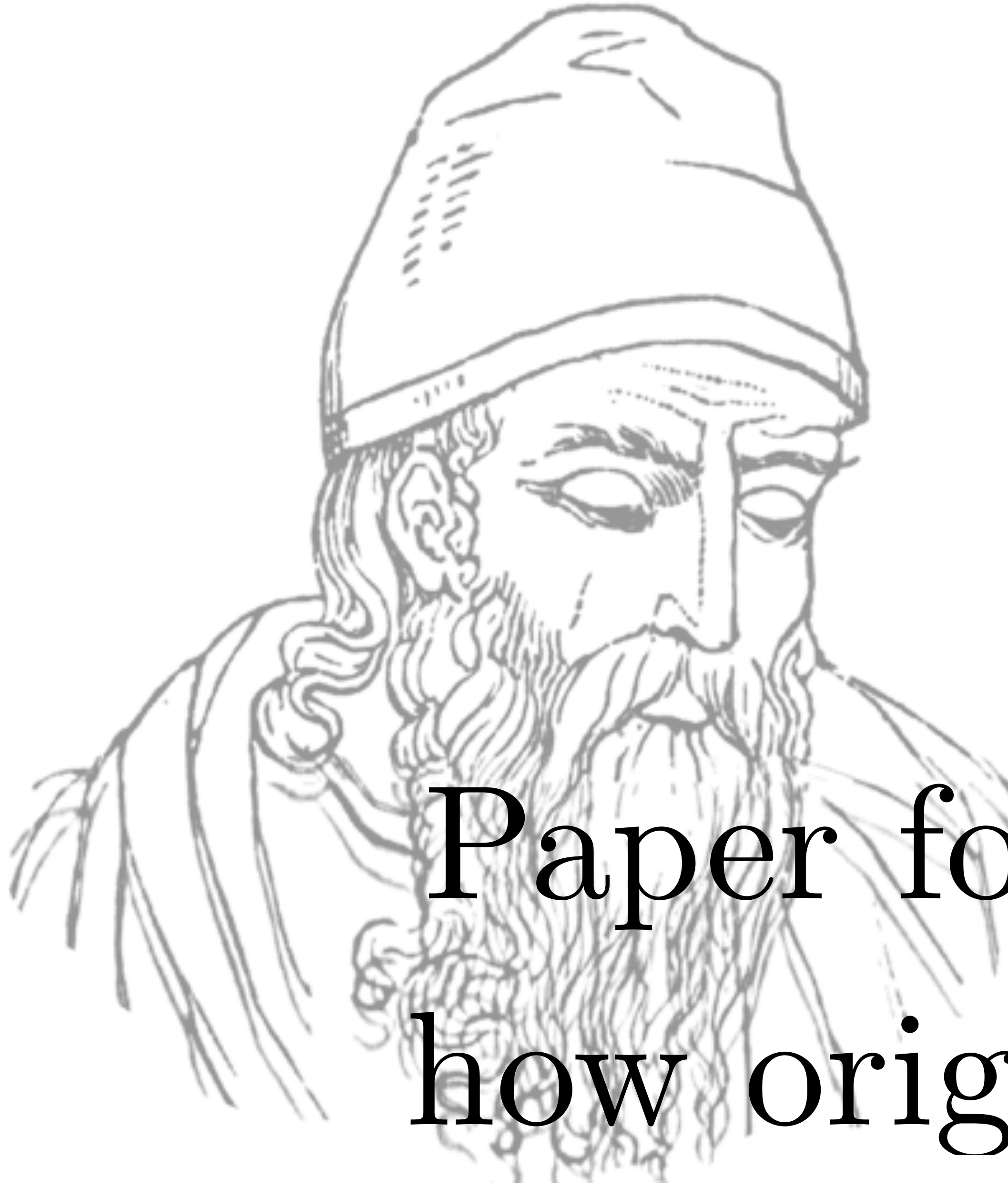

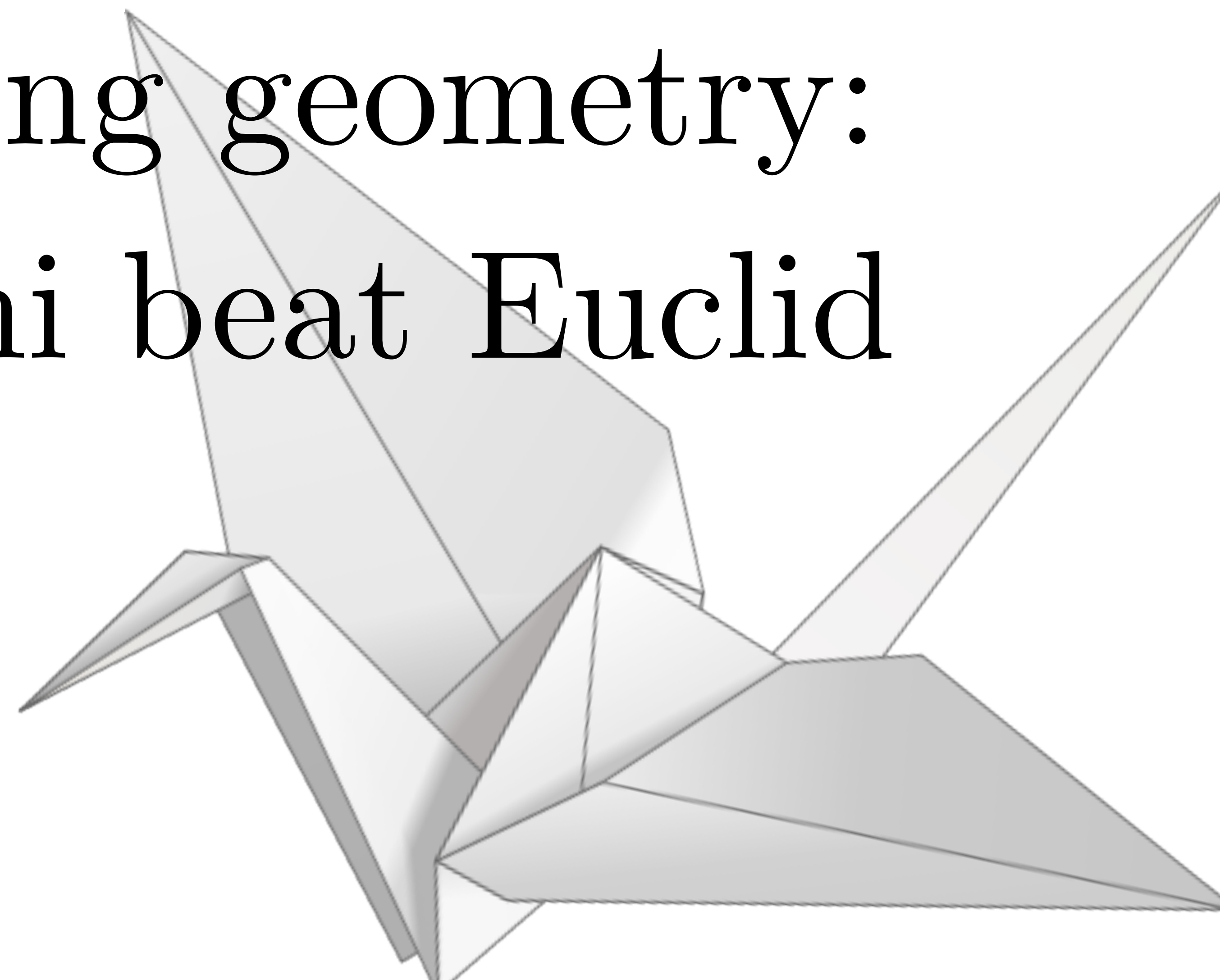

Paper folding geometry: how origami beat Euclid


MACEDONIA
THRACE
Byzantium
Pella
Therma
CHALCIDICE
Thasos
Chersonese
PHRYGIA
MT. OLYMPUS
Ilium
EPIRUS
Larissa
THESSALY
MYSIA
Corcyra
ÆGEAN SEA
Pergamum
Lesbos
LYDIA
EUBOEA
Leucas
Ithaca
Delphi
Chalcis
Sardis
Chios
Thebes
ACHAIA
Corinth
Athens
Samos
Ephesus
CARIA
Olympia
Argos
Salamis
PELOPONNESUS
Miletus
Halicarnassus
Sparta
Naxos
Melos
RHODES
Cythera
Cydonia
MT. IDA
Cnossus
CRETE

Sunday, March 3, 13


MACEDONIA
THRACE
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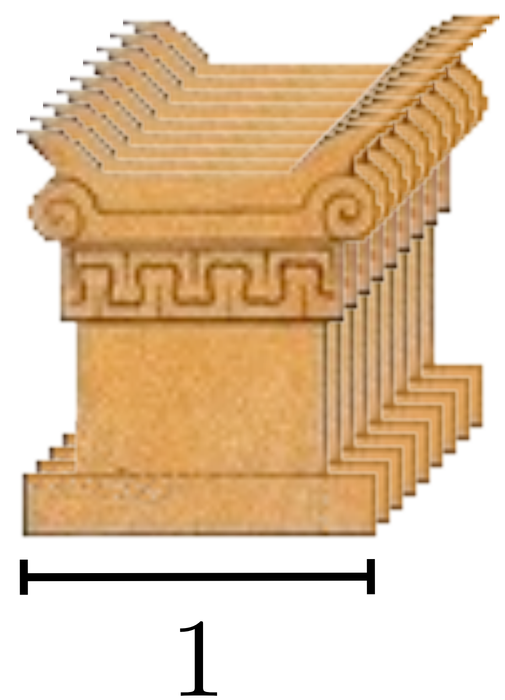
Cnossus
CRETE

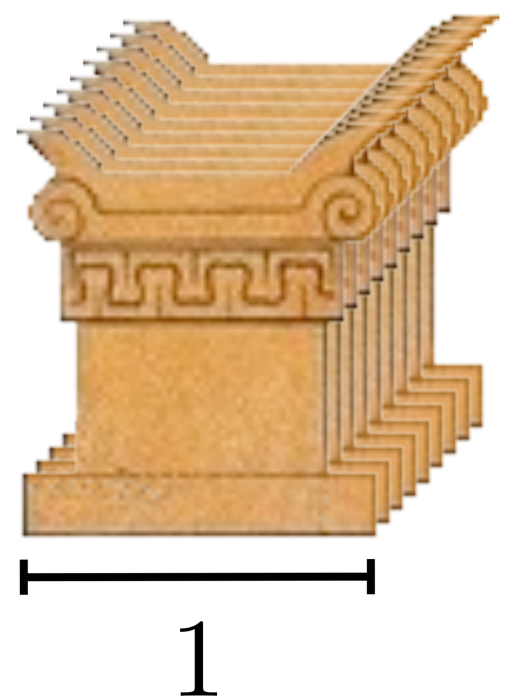

TOM. II. PL. XXXIV.

Double the altar

Double the altar



Double the cube

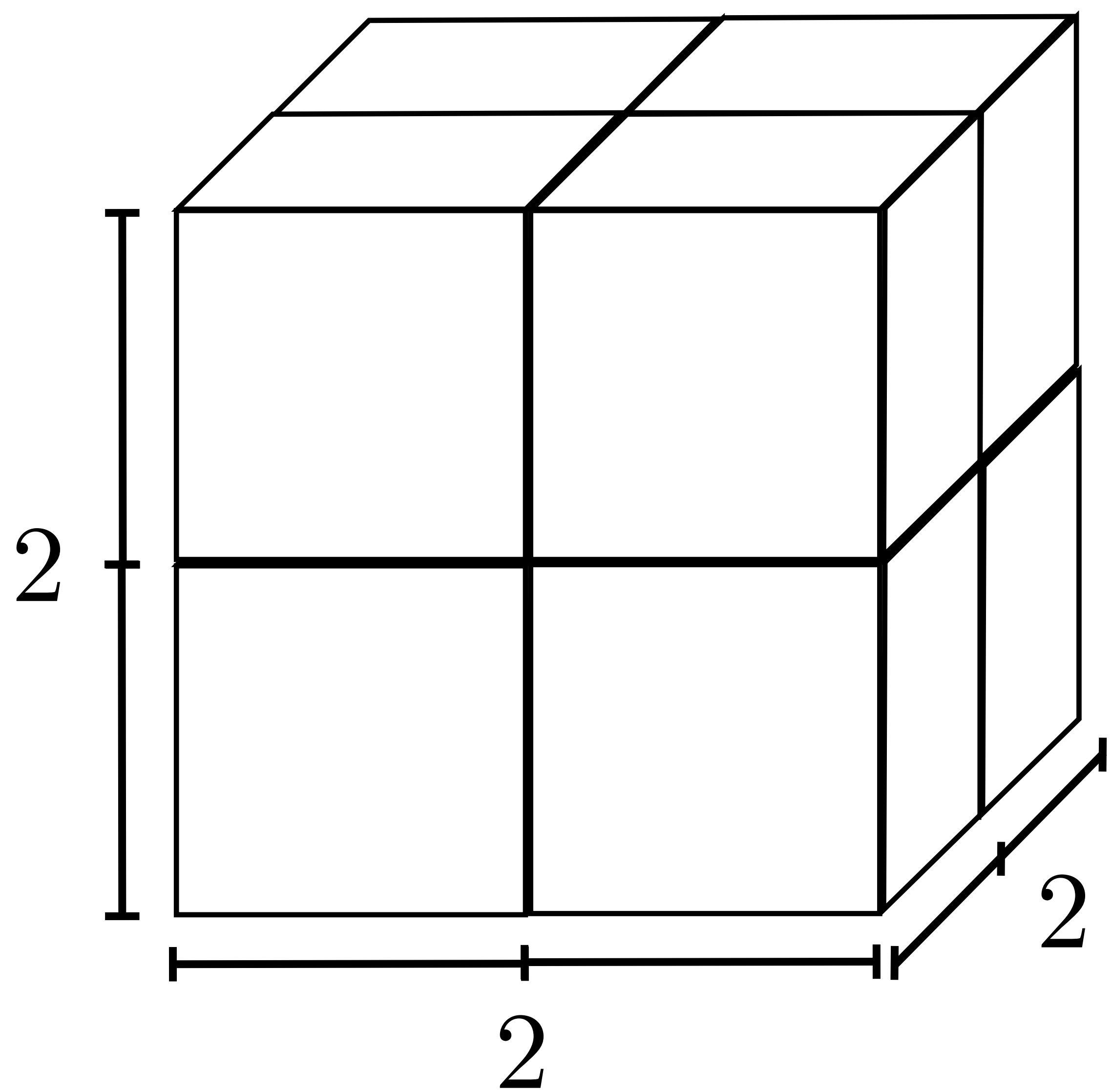


Double the cube

Double the cube

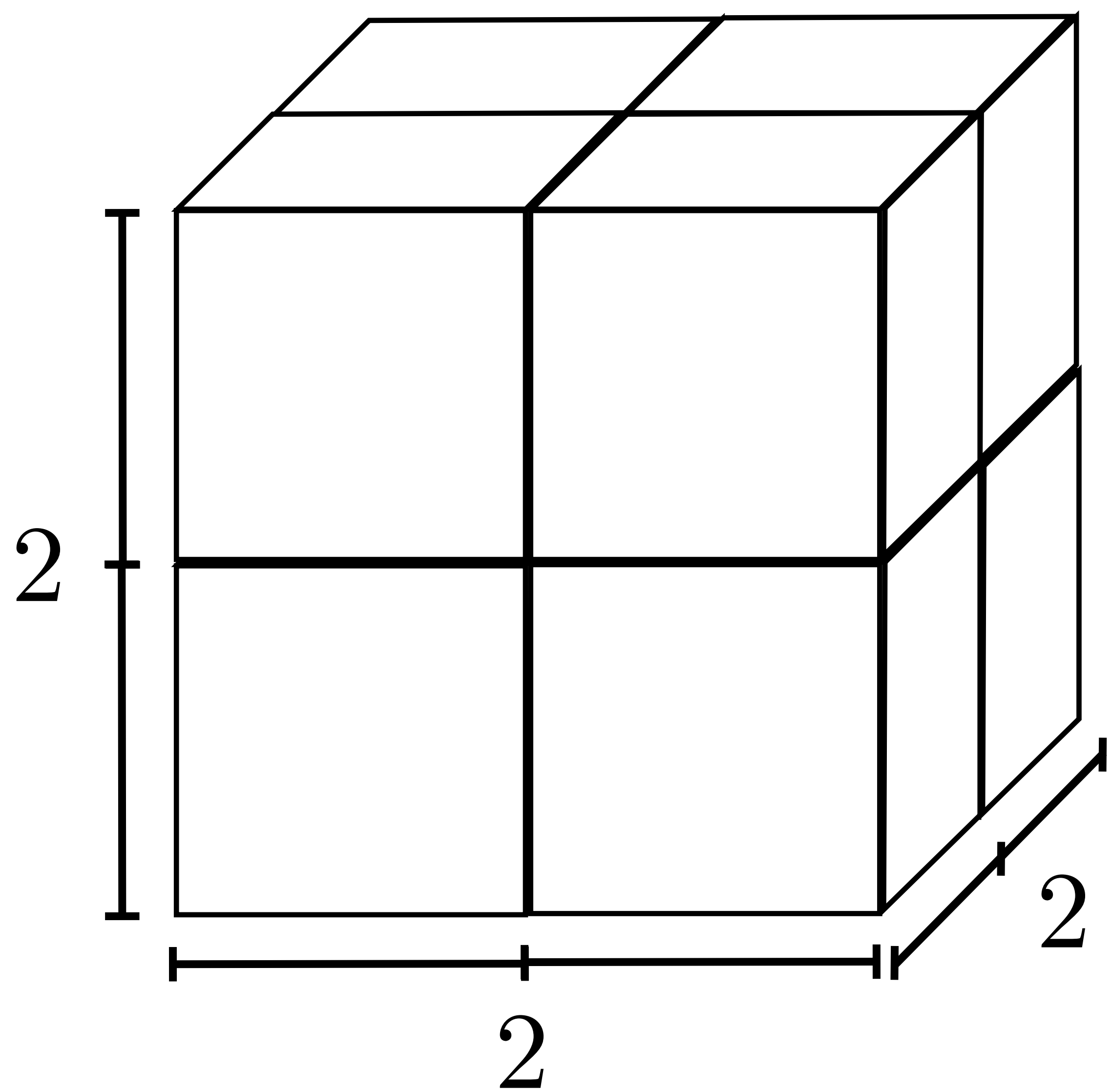
Double the cube?

Double the cube?



Double the cube?

No, octuple the cube!



Double the cube

$$\text{Volume of the cube } = (\text{edge})^3 = 2$$

$$\Rightarrow \text{edge} = \sqrt[3]{2} \simeq 1.25992105...$$

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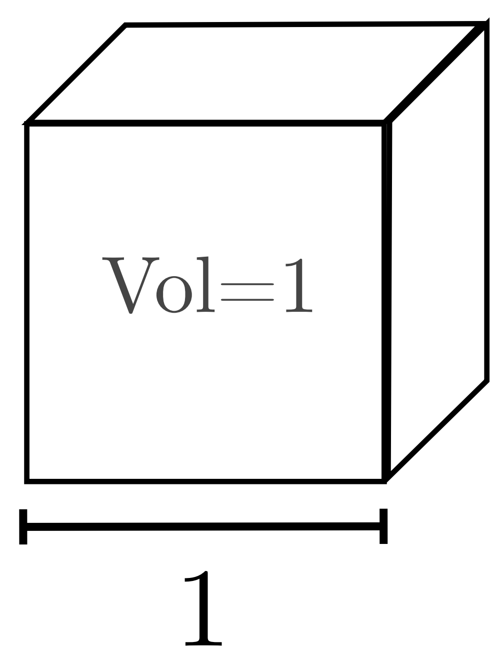

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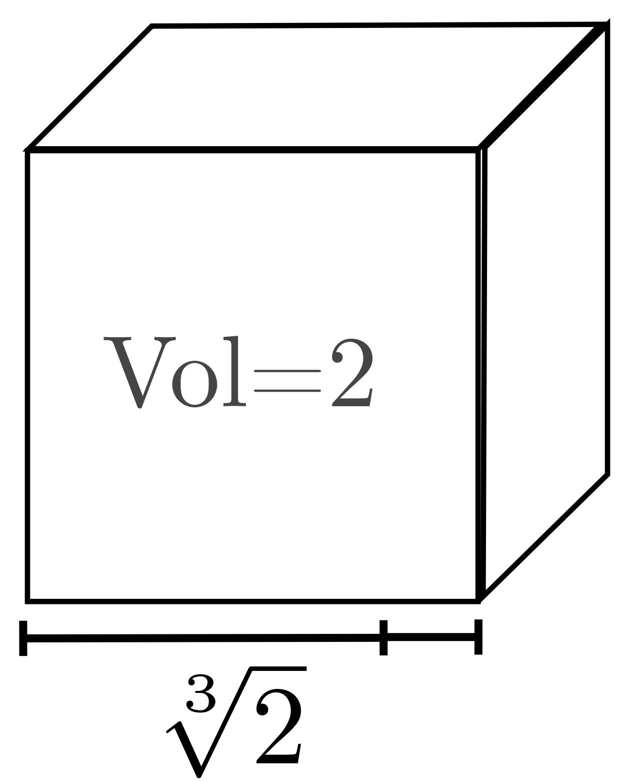
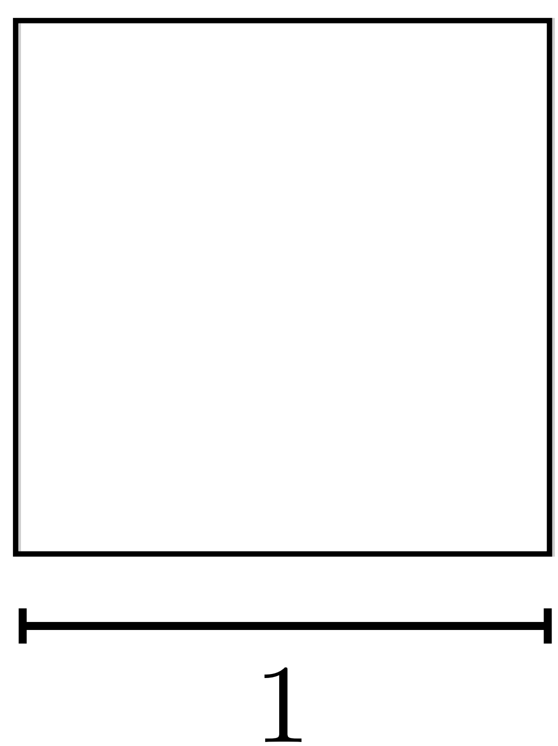
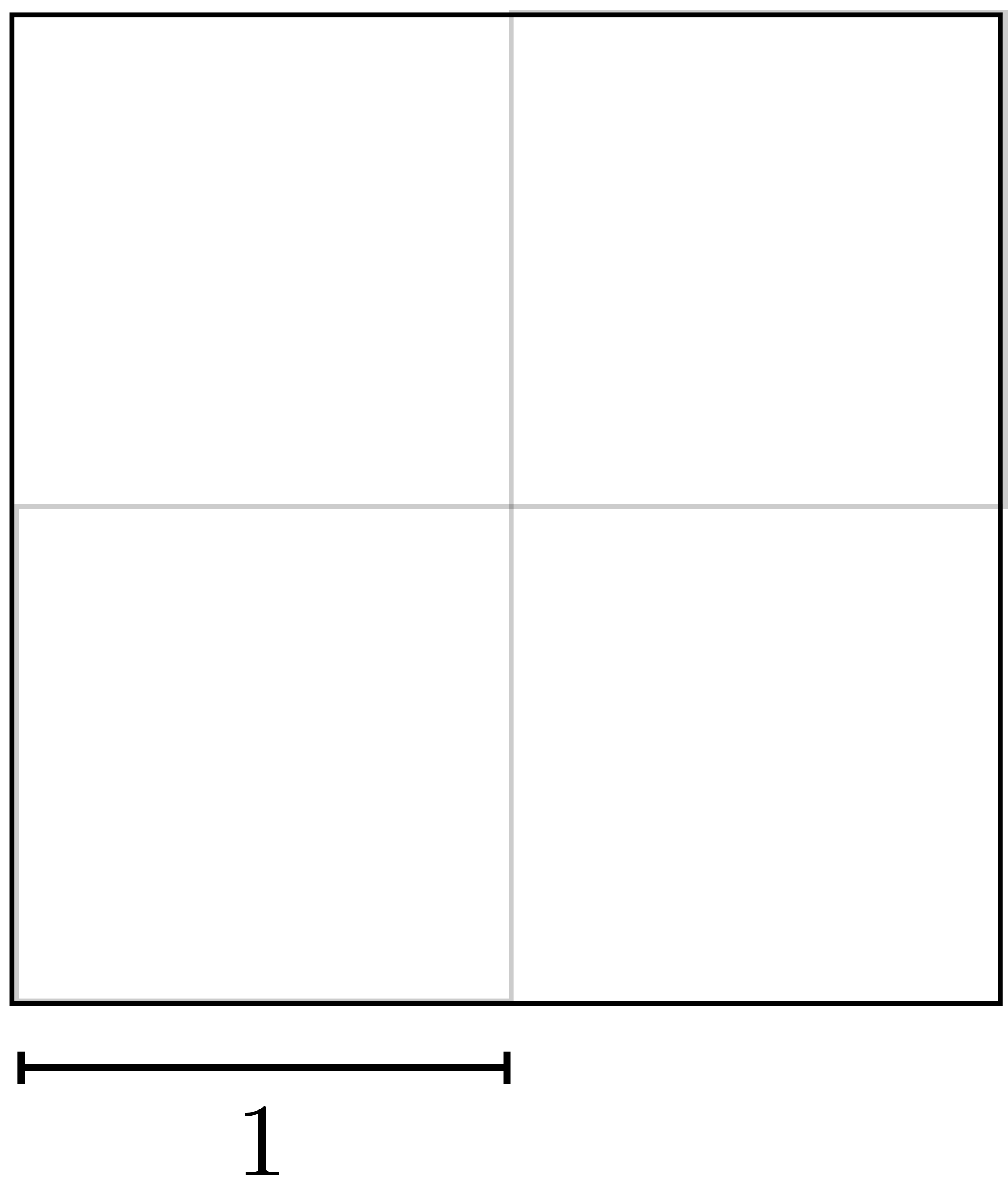


Double the square

Double the square



Double the square



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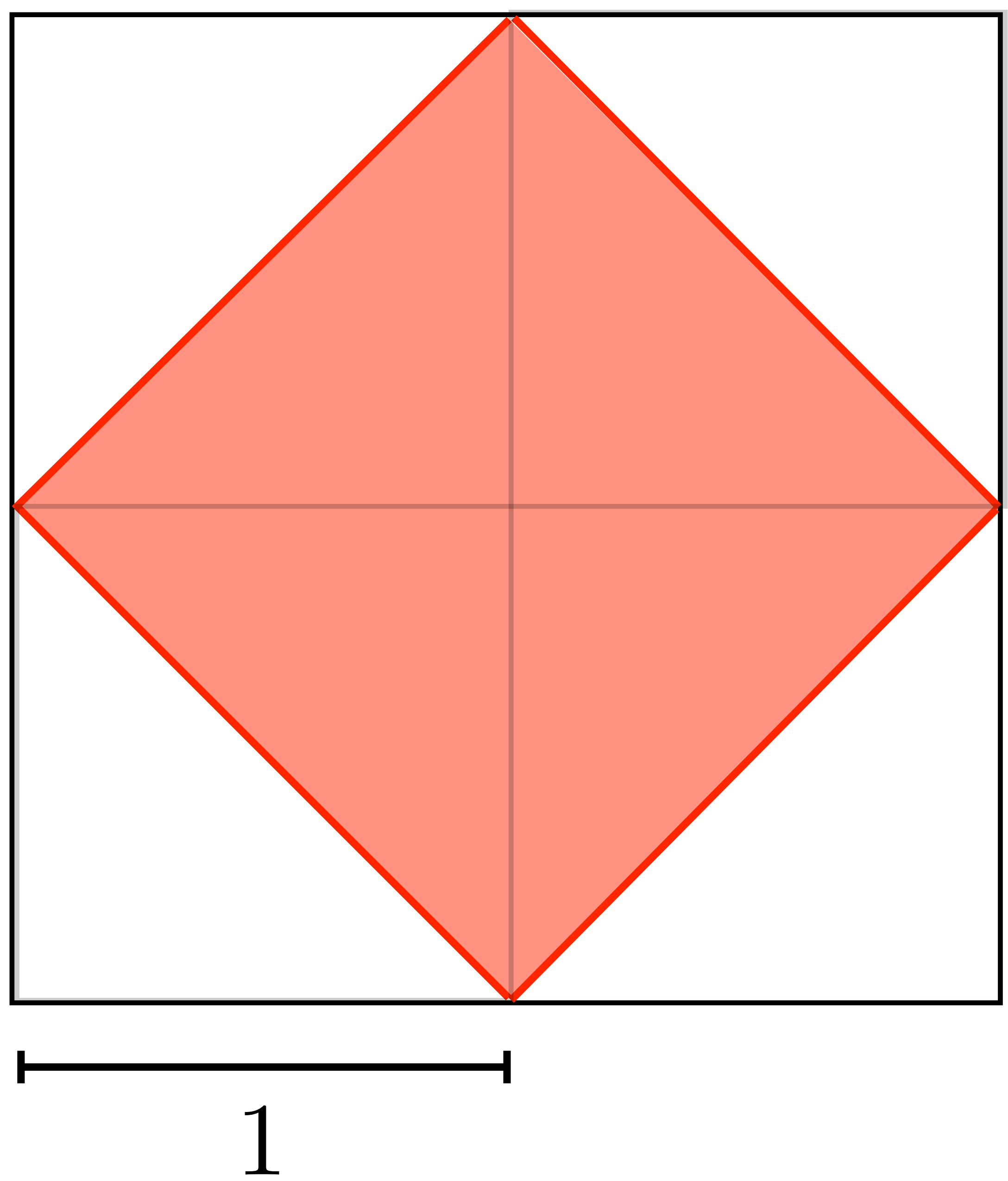

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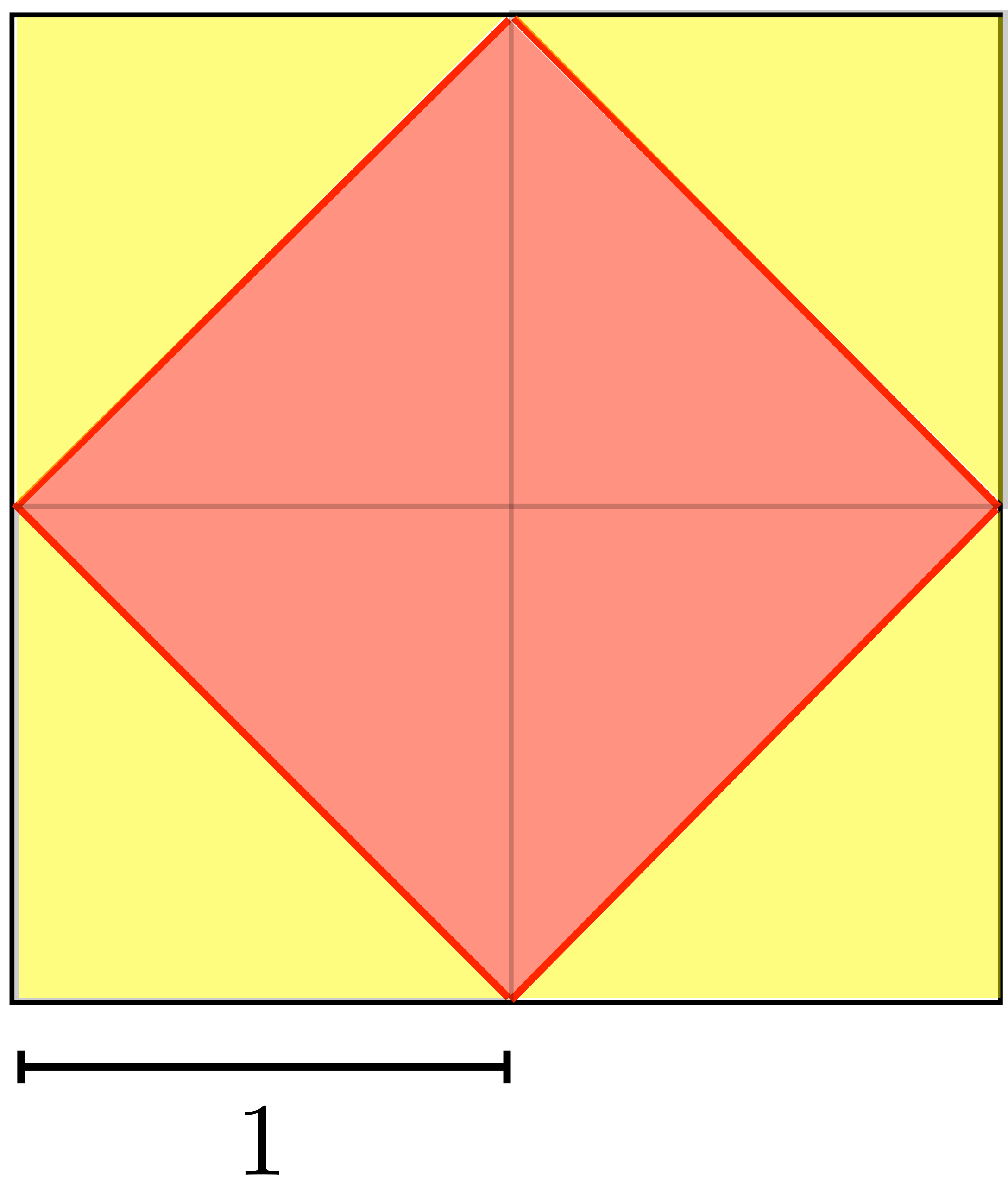
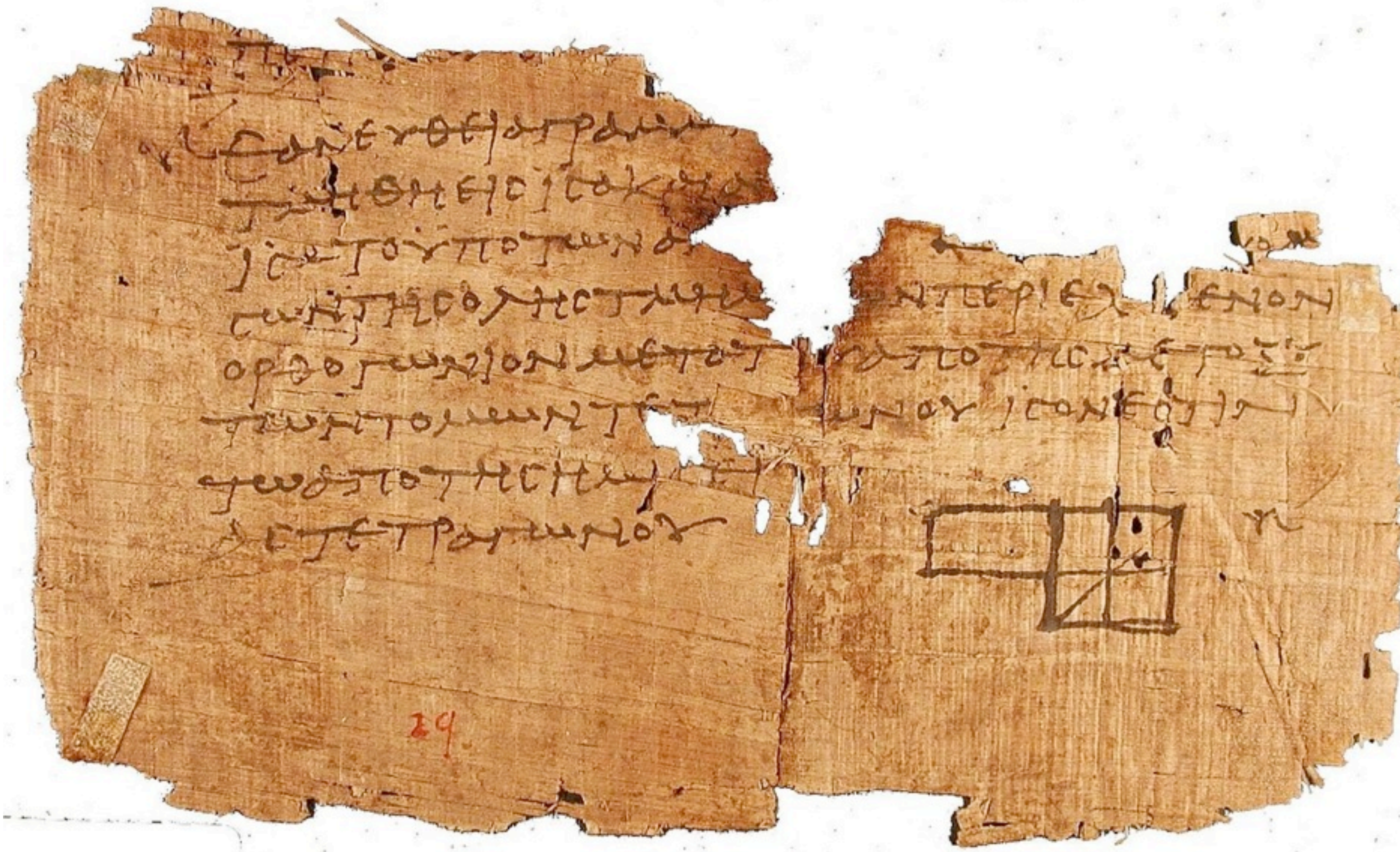
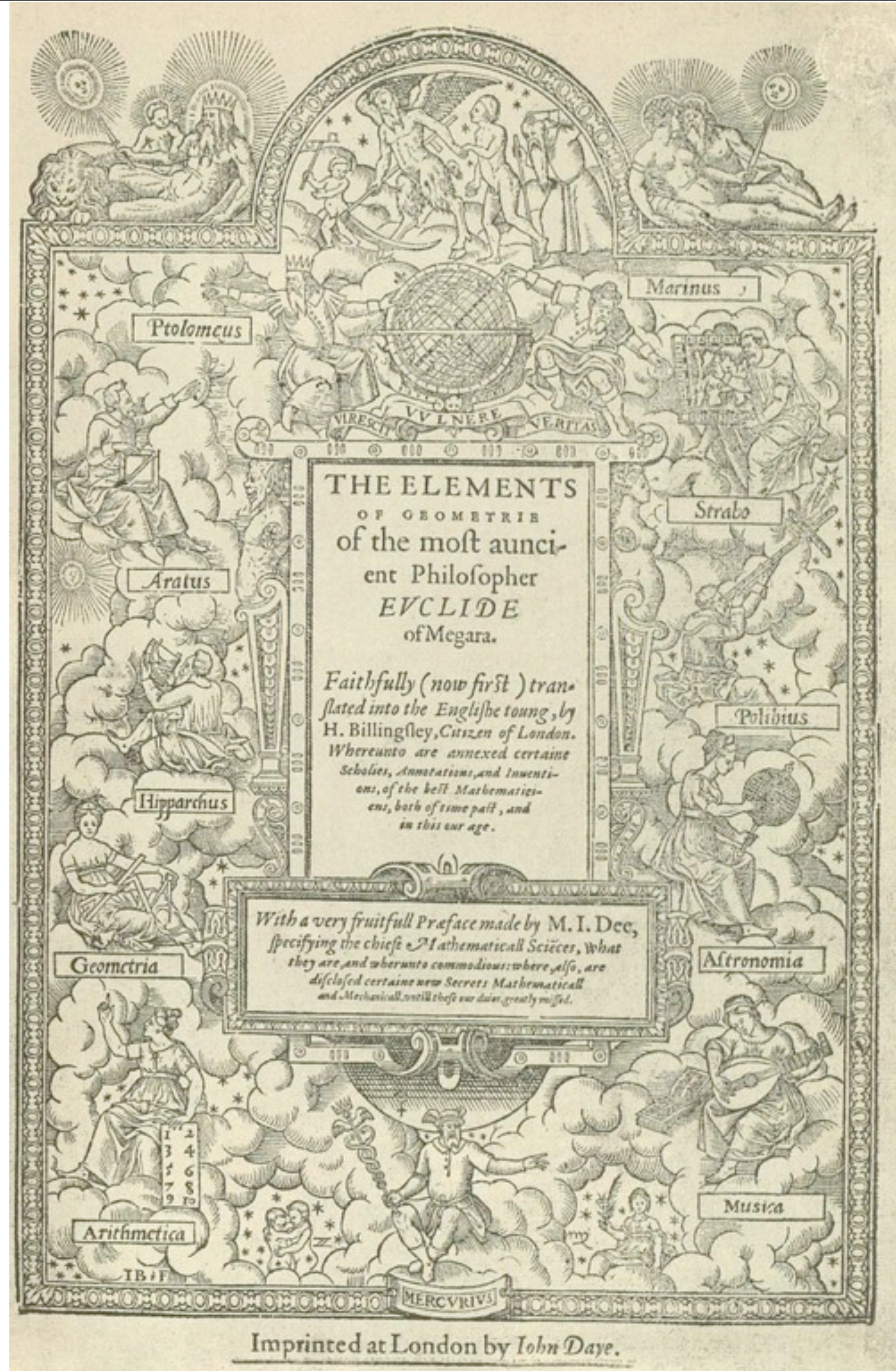






THE ELEMENTS
OF GEOMETRIE
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.

Faithfully (now first) translated into the Englishe toung, by H. Billingsley, Citizen of London. Whereunto are annexed certaine Scholies, Annotations, and Inventions, of the best Mathematiciens, both of time past, and in this our age.

With a very fruitfull Præface made by M. I. Dee, specifying the chiefe Mathematicall Scieces, what they are, and wherunto commodious: where, also, are disclosed certaine new Secrets Mathematicall and Mechanicall, untill these our daies, greatly missed.

Imprinted at London by Iohn Daye.

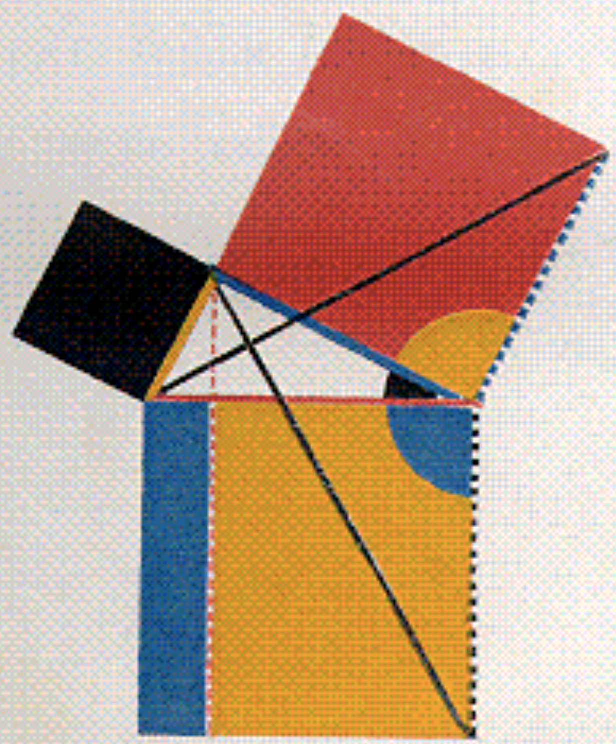
THE FIRST SIX BOOKS OF

THE ELEMENTS OF EUCLID

IN WHICH COLOURED DIAGRAMS AND SYMBOLS
ARE USED INSTEAD OF LETTERS FOR THE
GREATER EASE OF LEARNERS

BY OLIVER BYRNE

SURVEYOR OF HER MAJESTY'S SETTLEMENTS IN THE FALKLAND ISLANDS
AND AUTHOR OF NUMEROUS MATHEMATICAL WORKS



LONDON
WILLIAM PICKERING
1847

THE ELEMENTS OF EUCLID.

BOOK I.

DEFINITIONS.

I.

A *point* is that which has no parts.

II.

A *line* is length without breadth.

III.

The extremities of a line are points.

IV.

A ſtraight or right line is that which lies evenly between its extremities.

V.

A ſurface is that which has length and breadth cnly.

VI.

The extremities of a ſurface are lines.

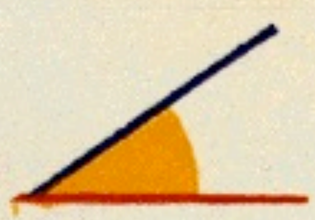
VII.

A plane ſurface is that which lies evenly between its extremities.

VIII.

A plane angle is the inclination of two lines to one another, in a plane, which meet together, but are not in the ſame direction.

IX.



A plane rectilinear angle is the inclination of two ſtraight lines to one another, which meet together, but are not in the ſame ſtraight line.

Sunday, March 3, 13

Postulates:

1. Let it be granted that a straight line may be drawn from any one point to any other point.

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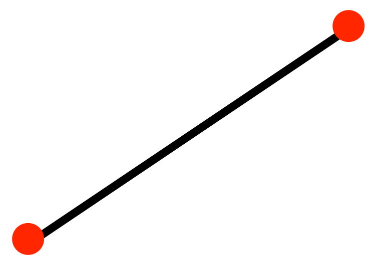

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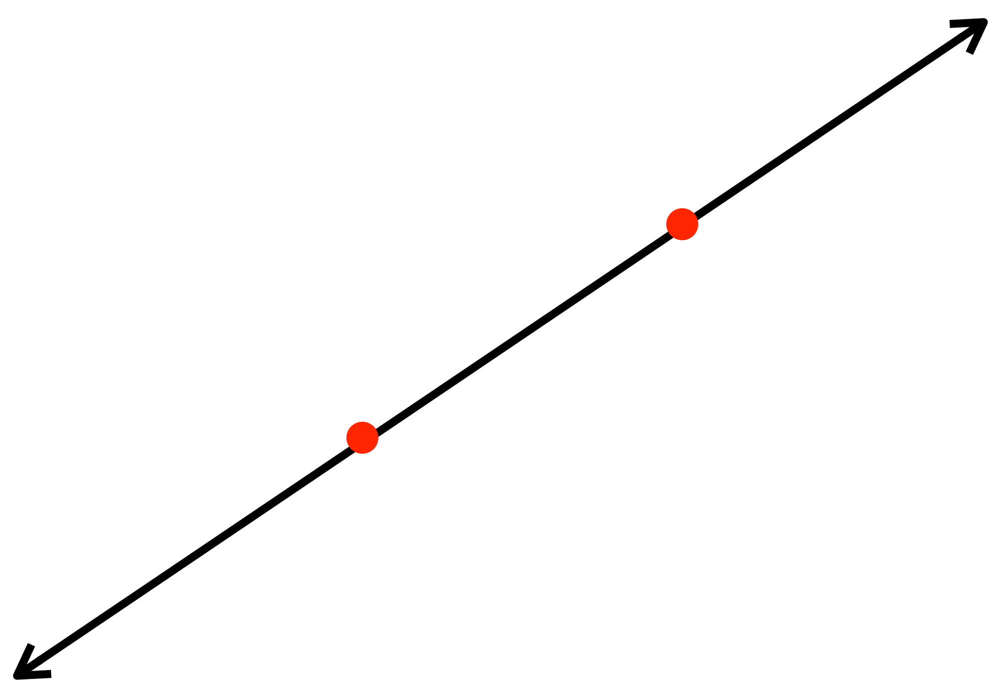
2. Let it be granted that a finite straight line may be produced to any length in a straight line.



Postulates:

1. Let it be granted that a straight line may be drawn from any one point to any other point.

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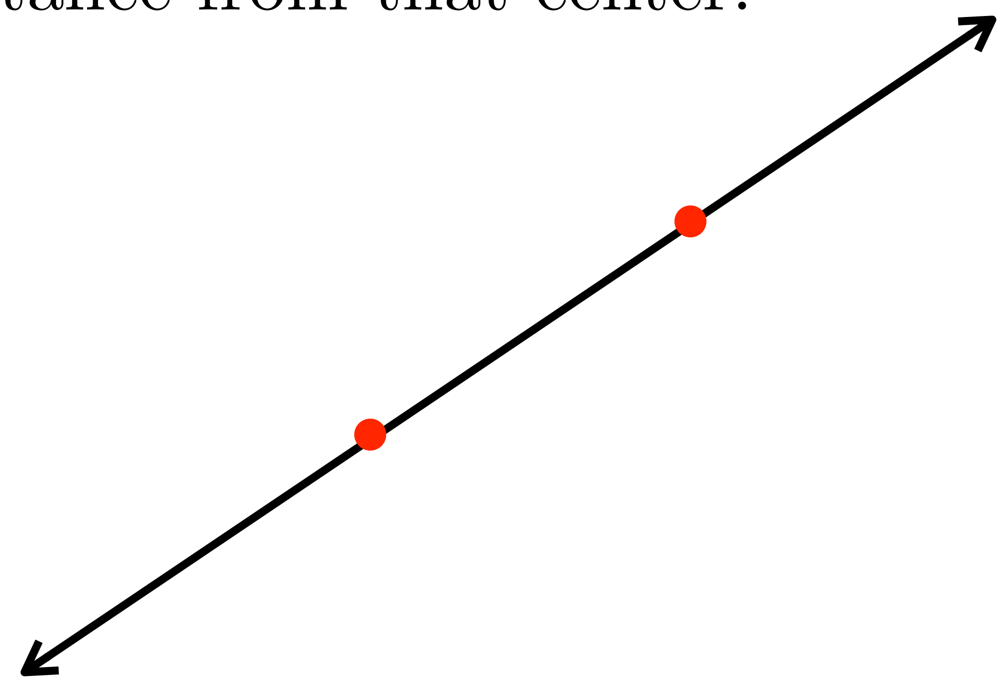
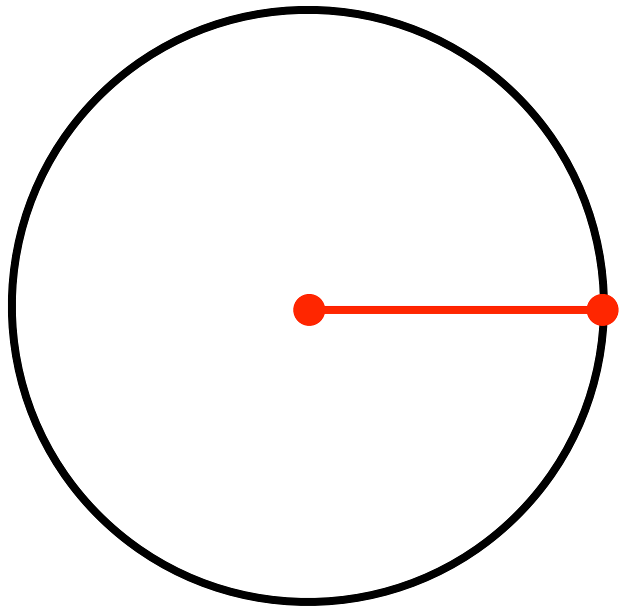
3. Let it be granted that a circle may be described with any center at any distance from that center.





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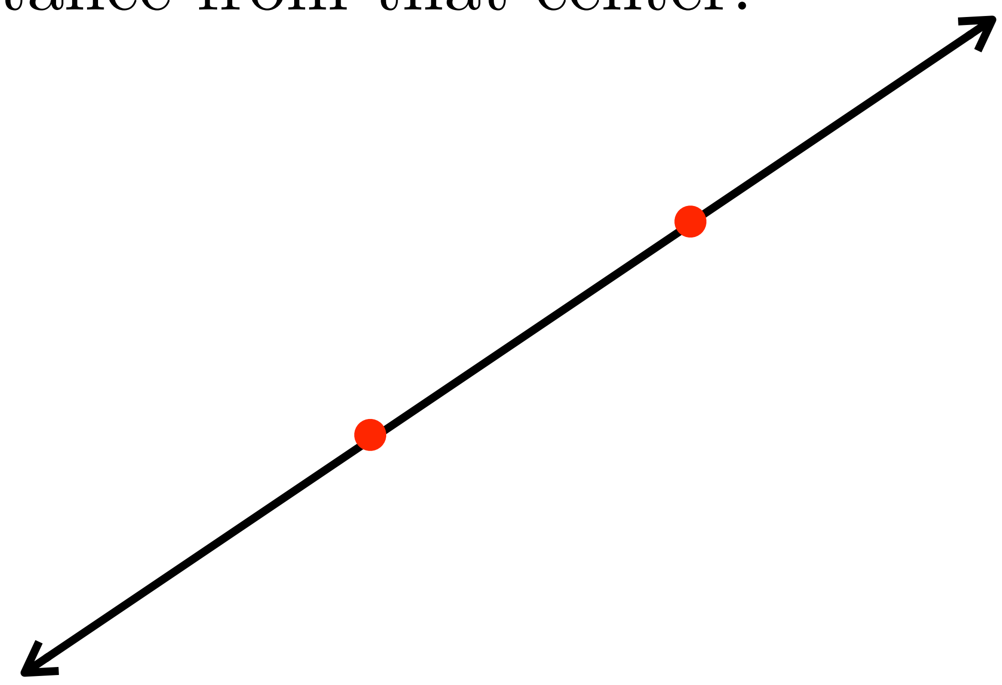

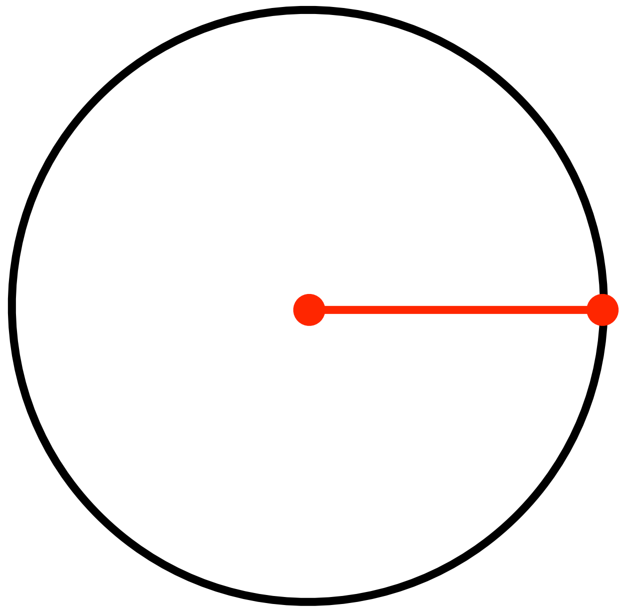

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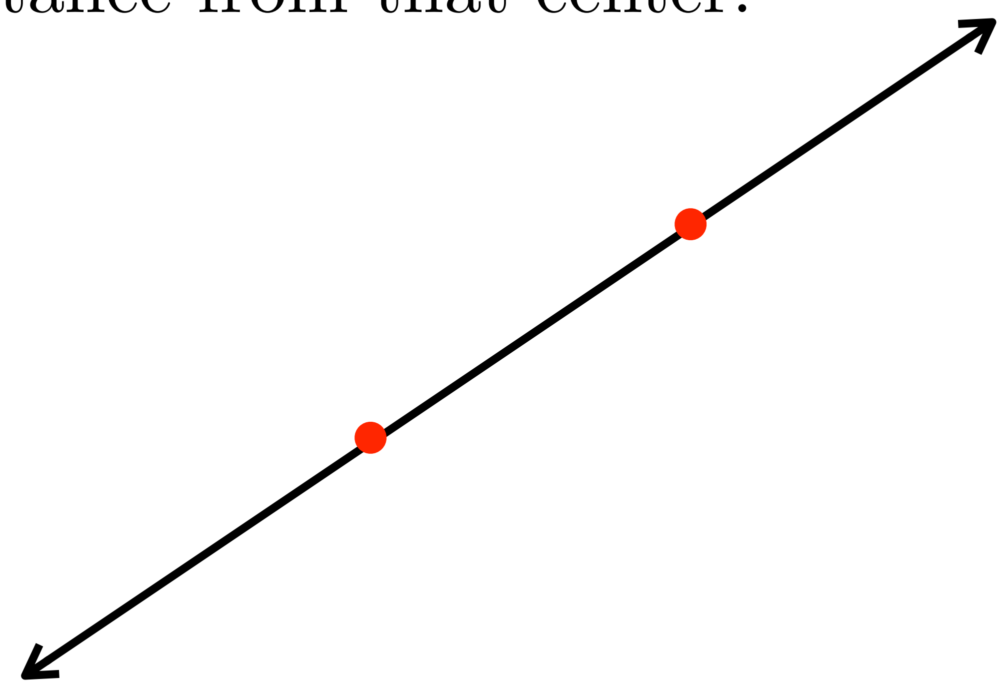

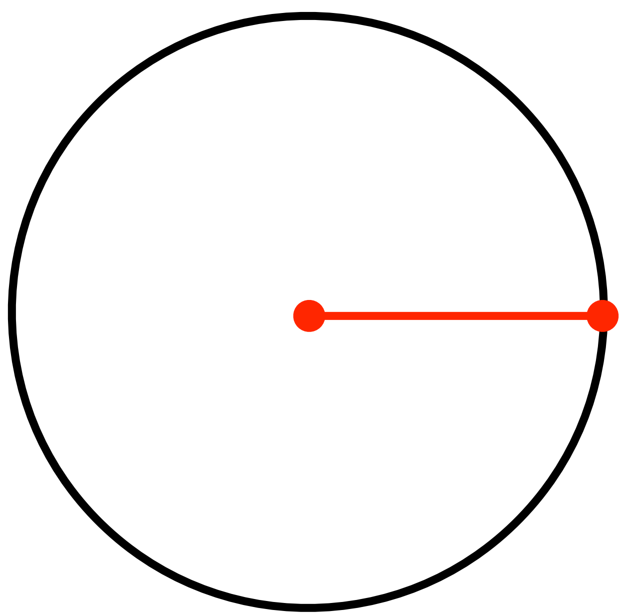

4. All right angles are equal.

5. If two straight lines meet a third straight line so as to make the two interior angles on the same side less than two right angles, then those two lines will meet if extended indefinitely on the side on which the angles are less than two right angles.

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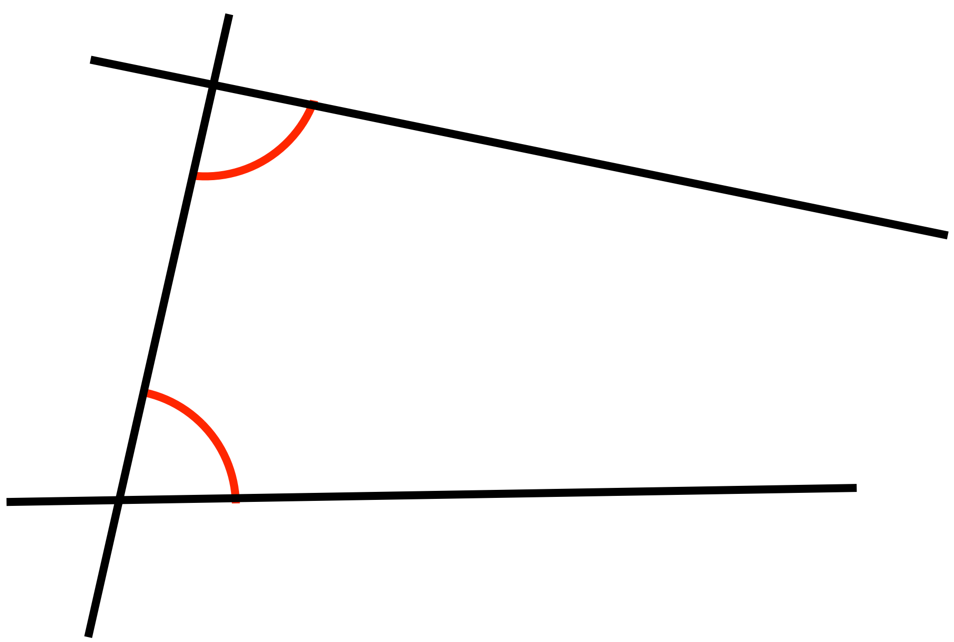

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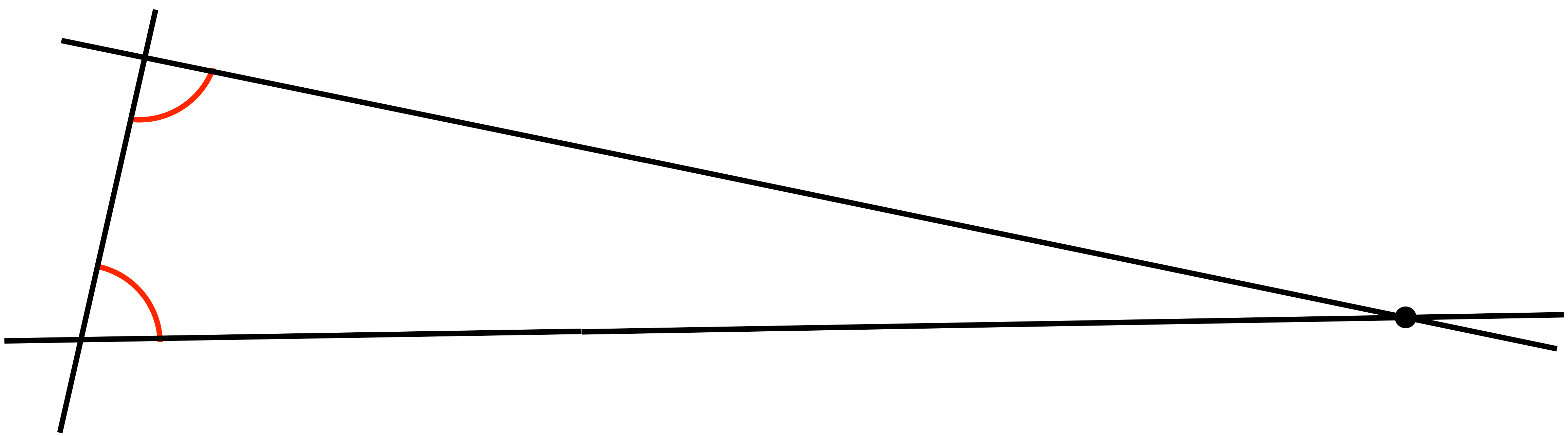

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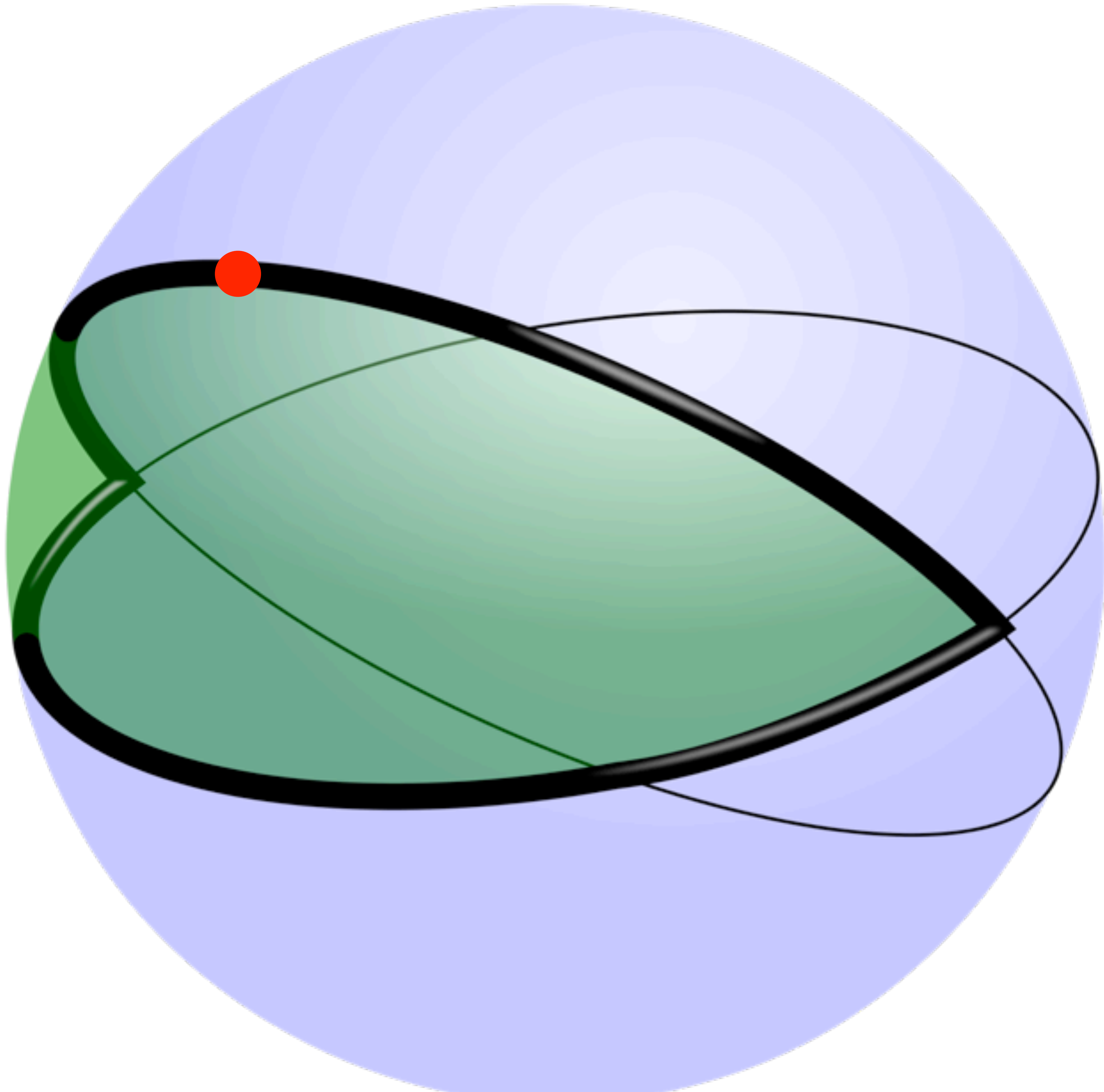

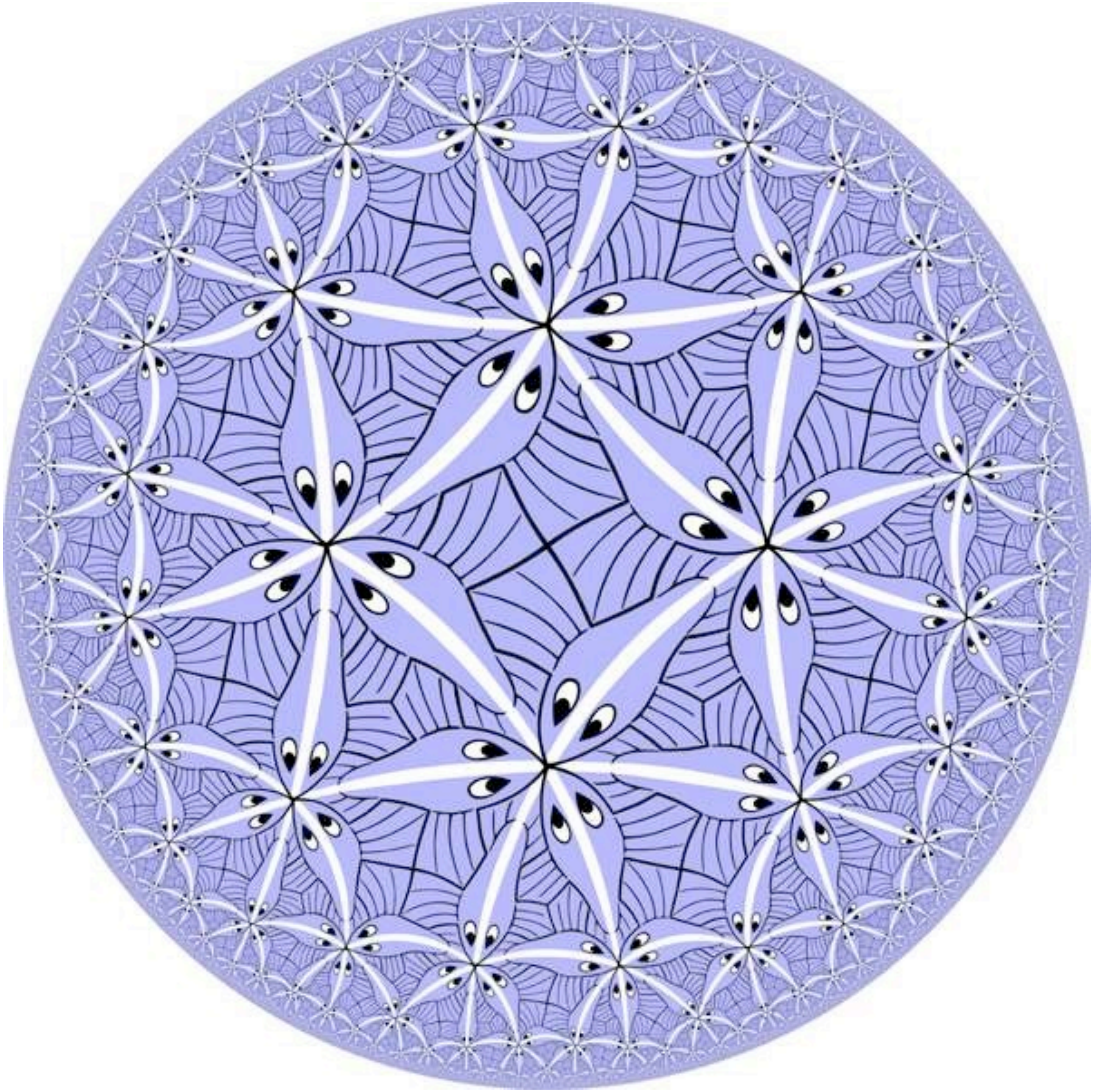

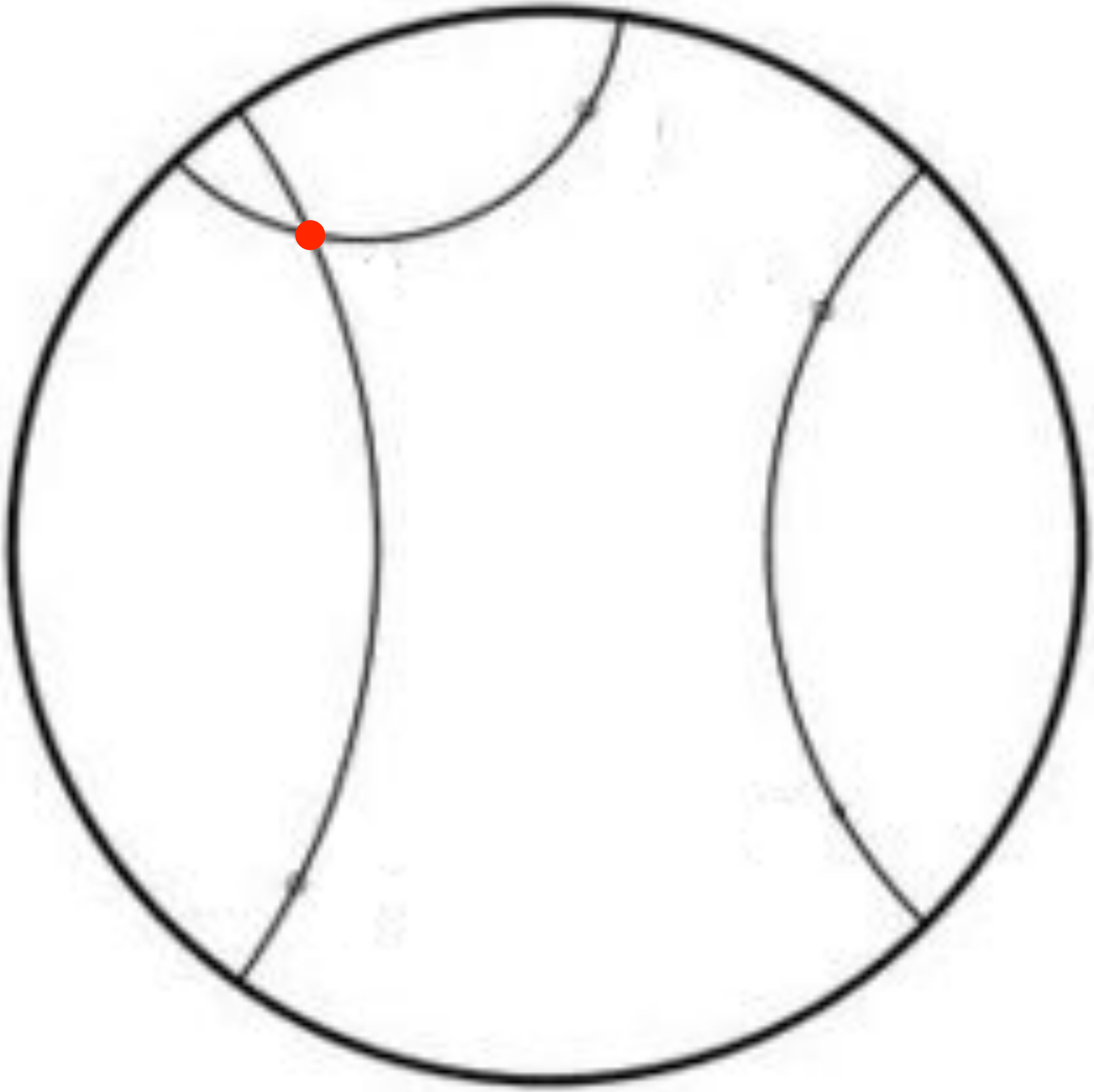

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To construct an equilateral triangle on a given finite straight line.

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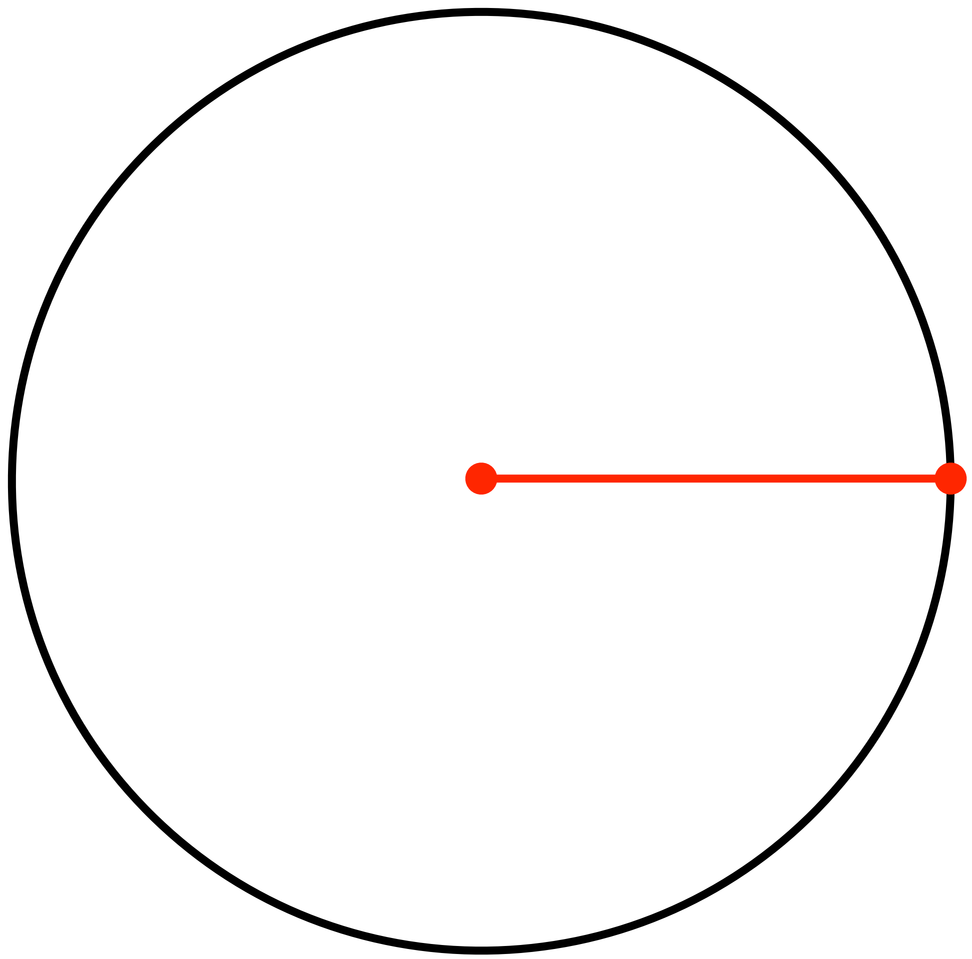

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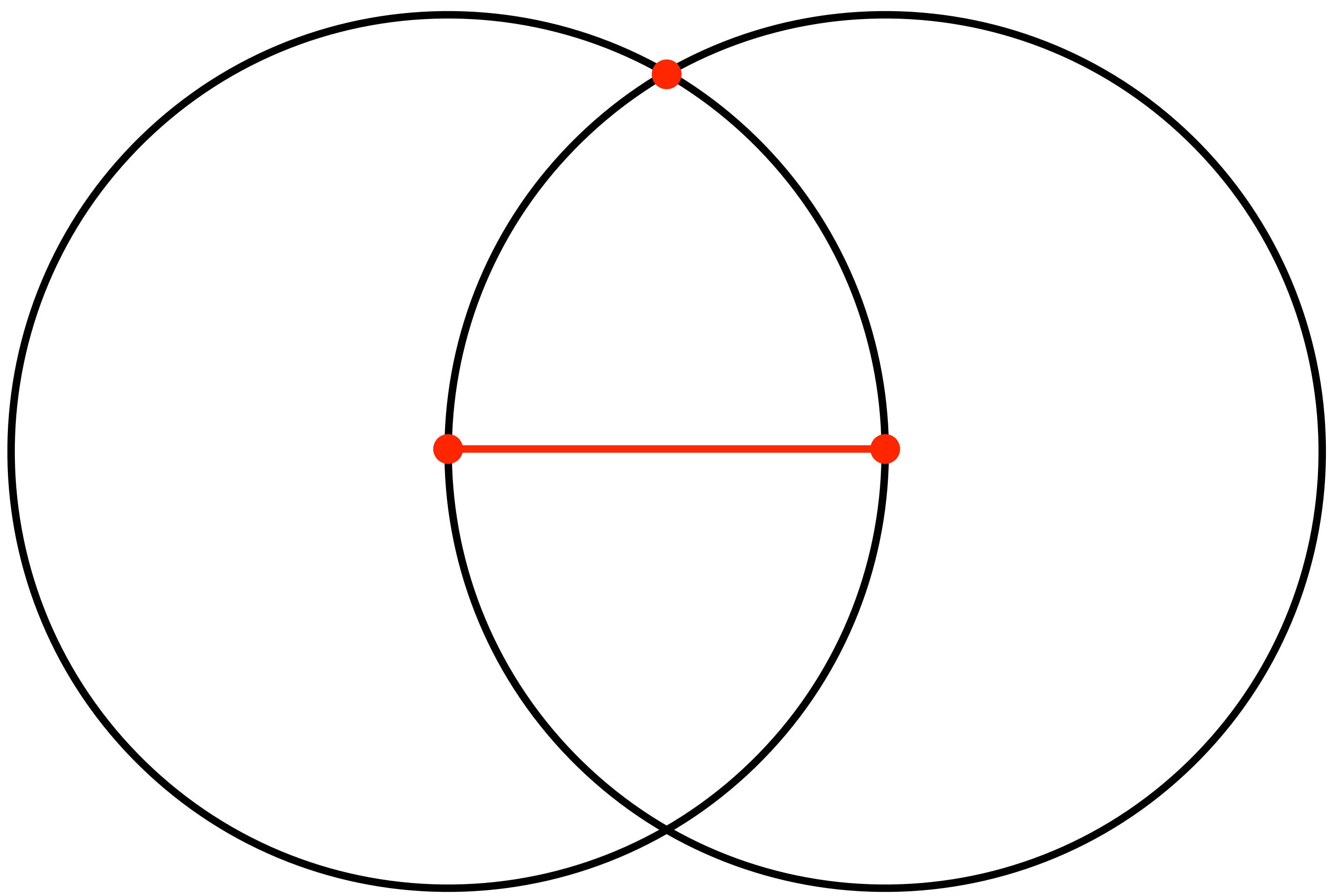

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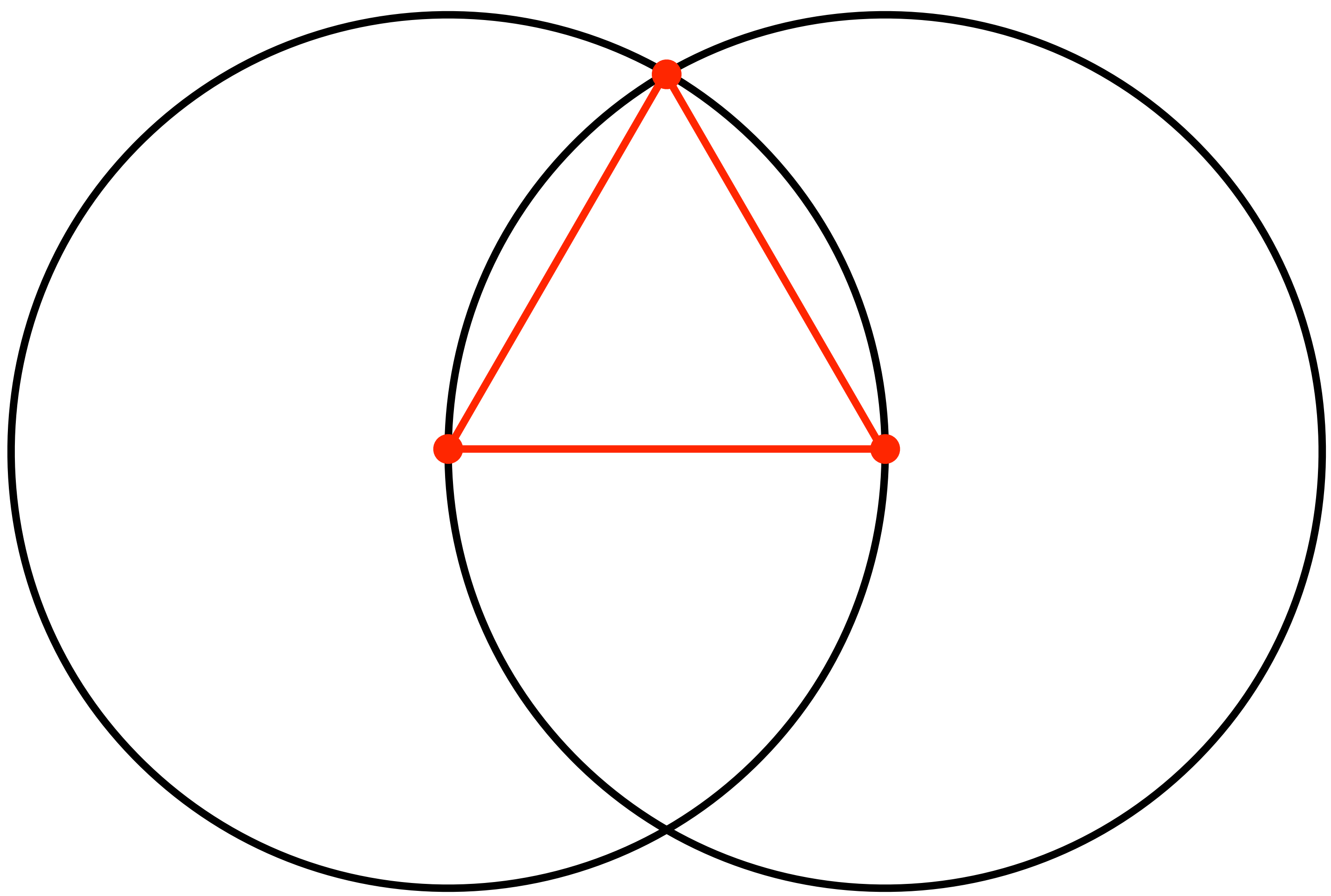
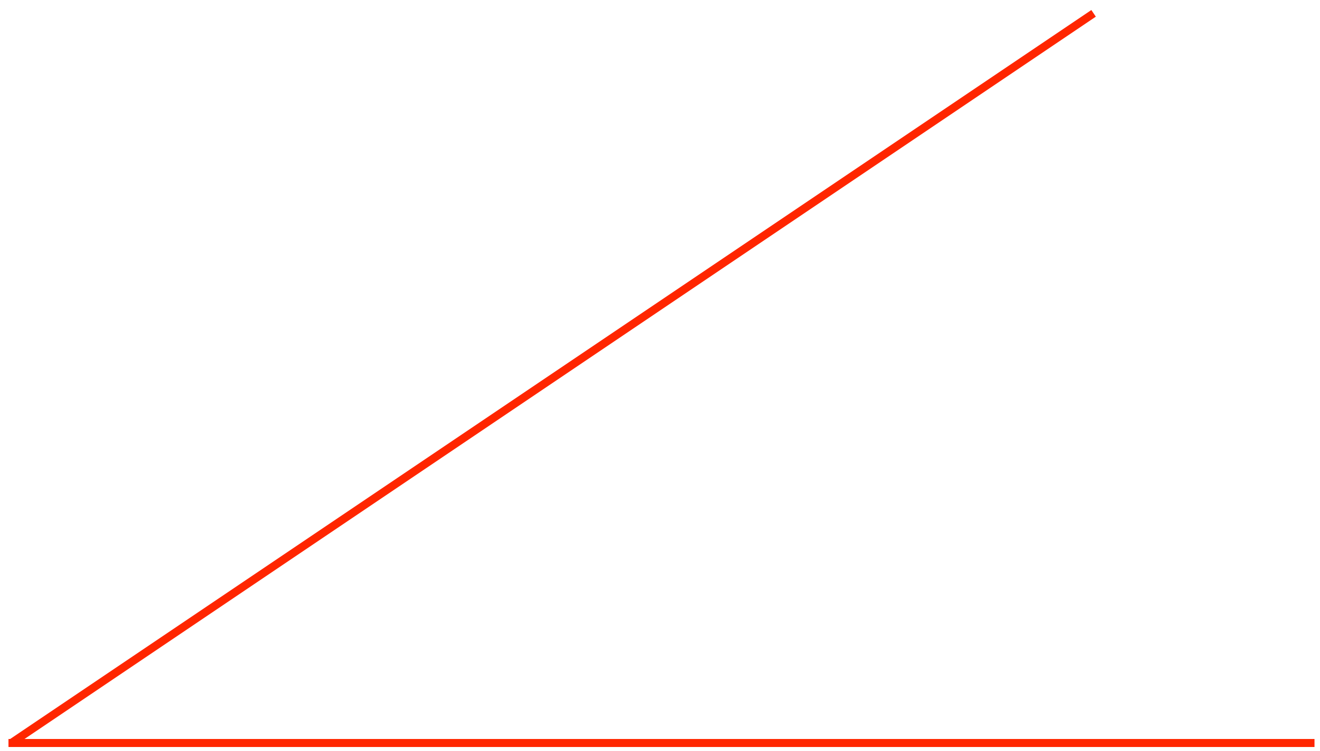


Proposition 9:

To bisect a given rectilinear angle.



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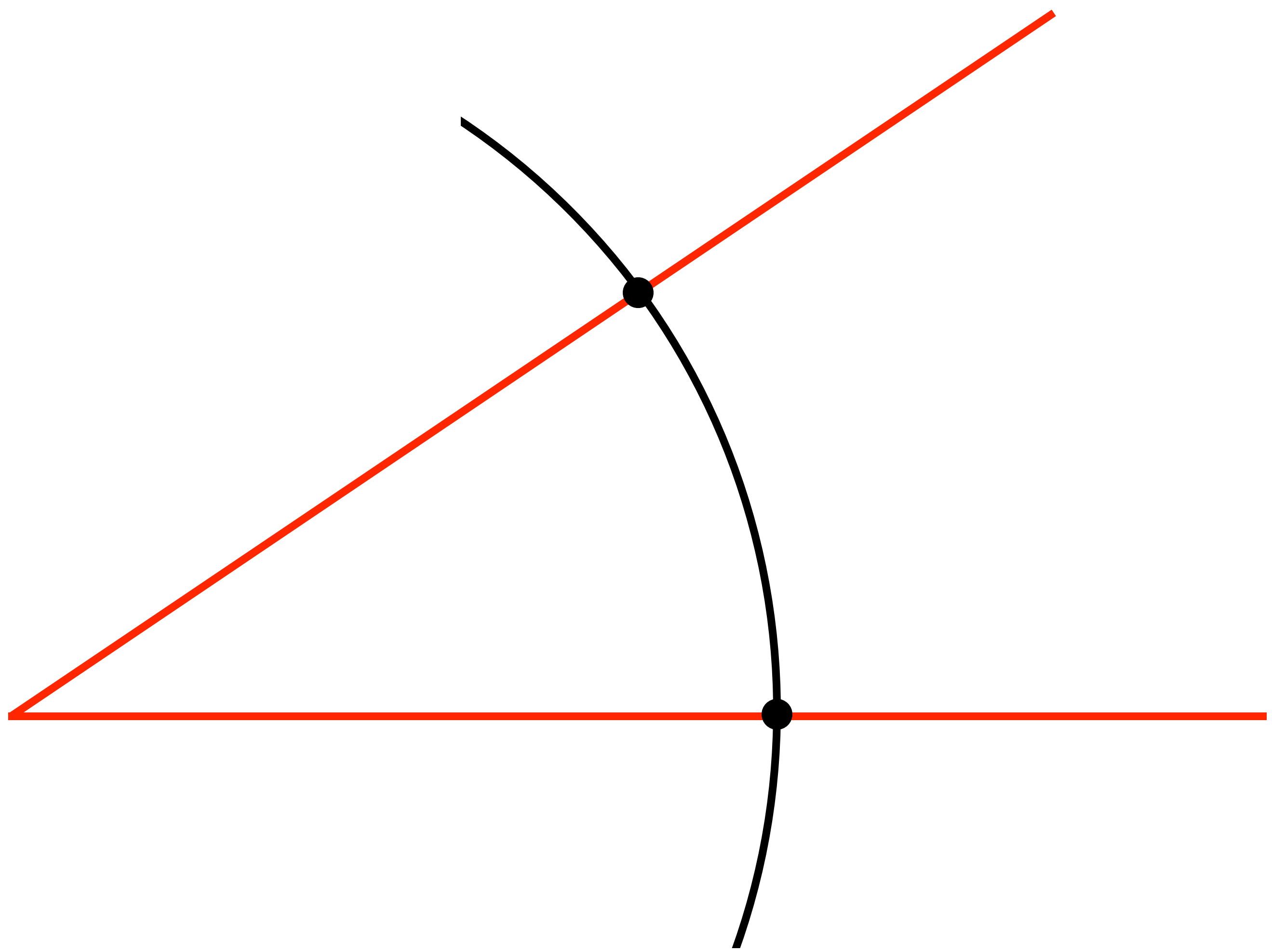

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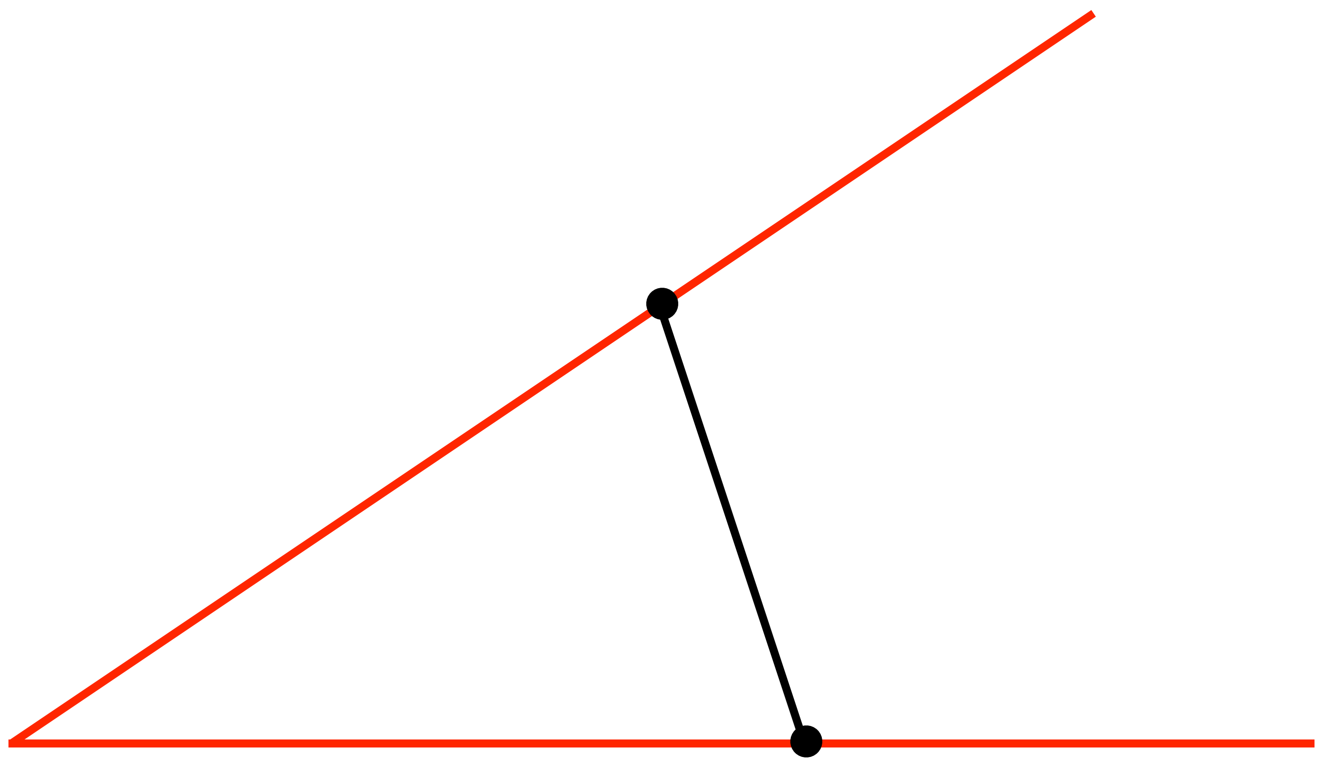

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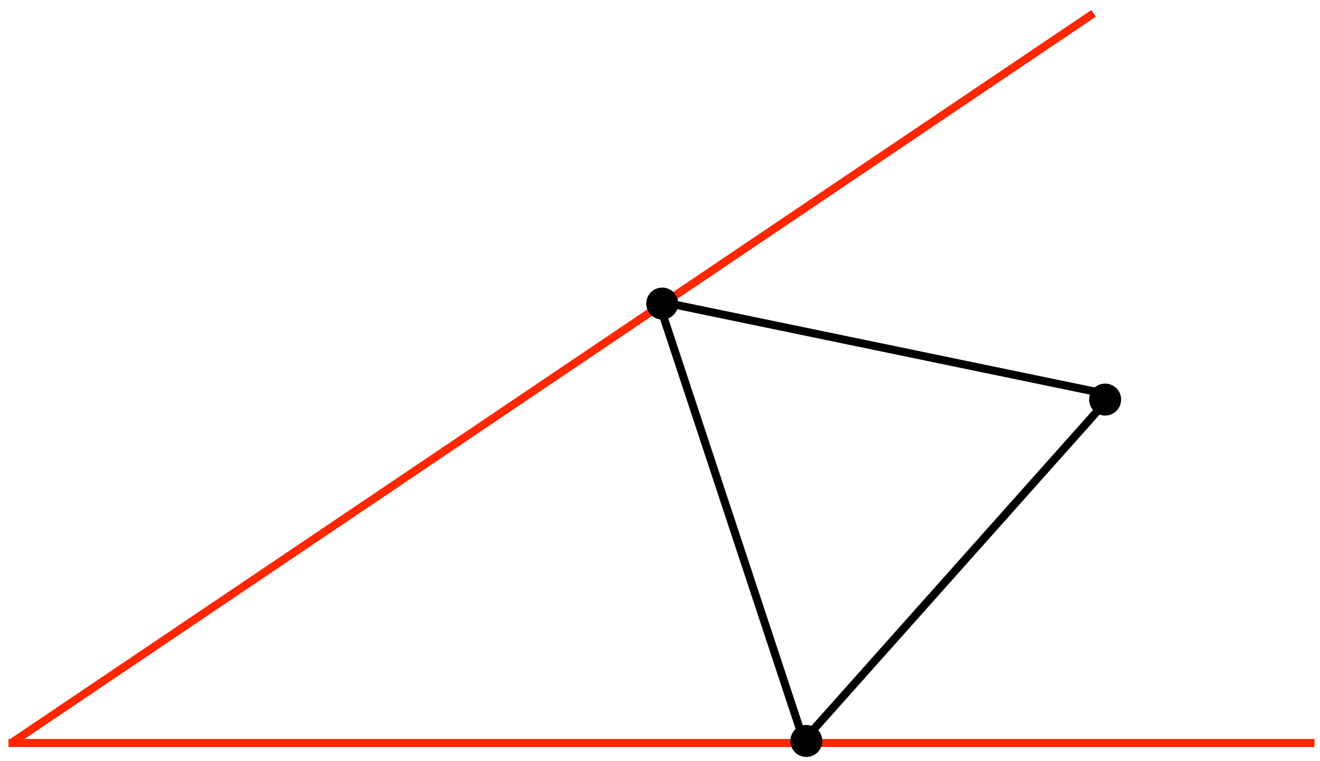

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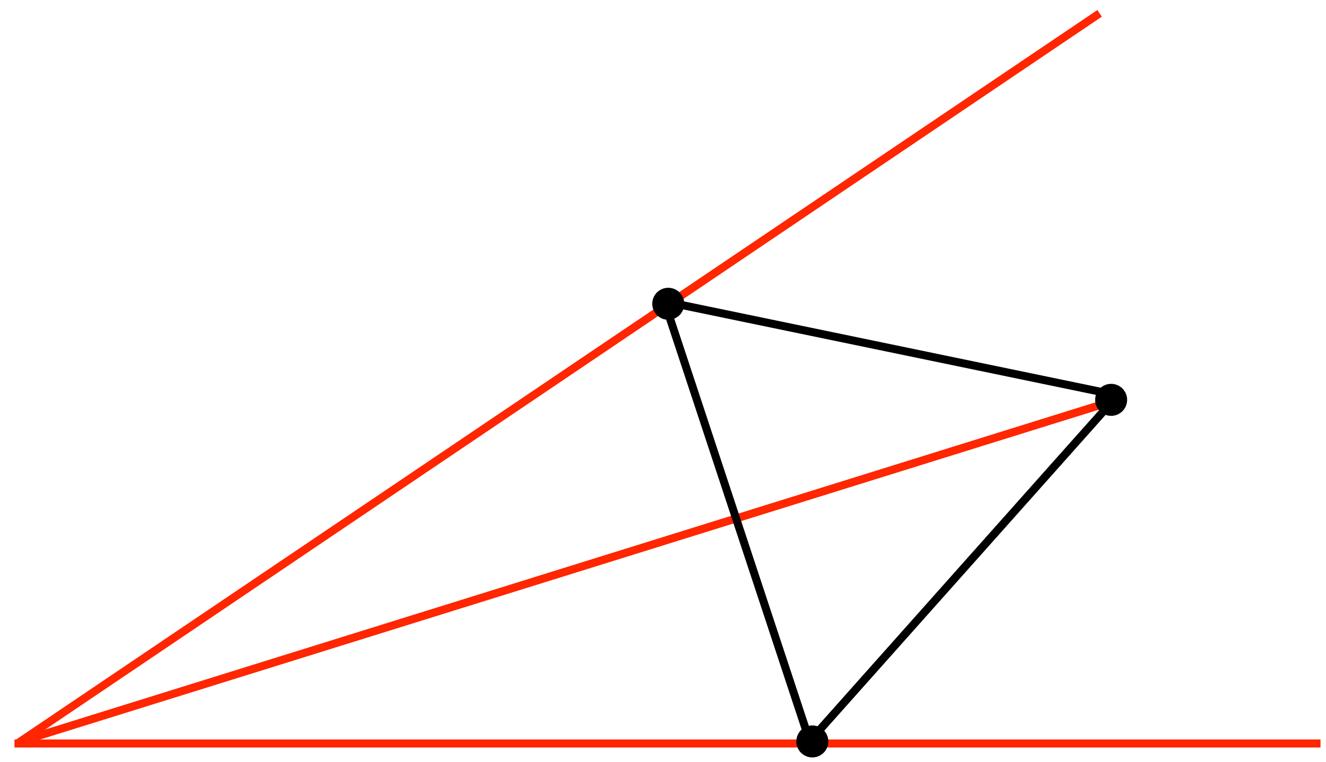

Proposition 10:

To bisect a given finite straight line.

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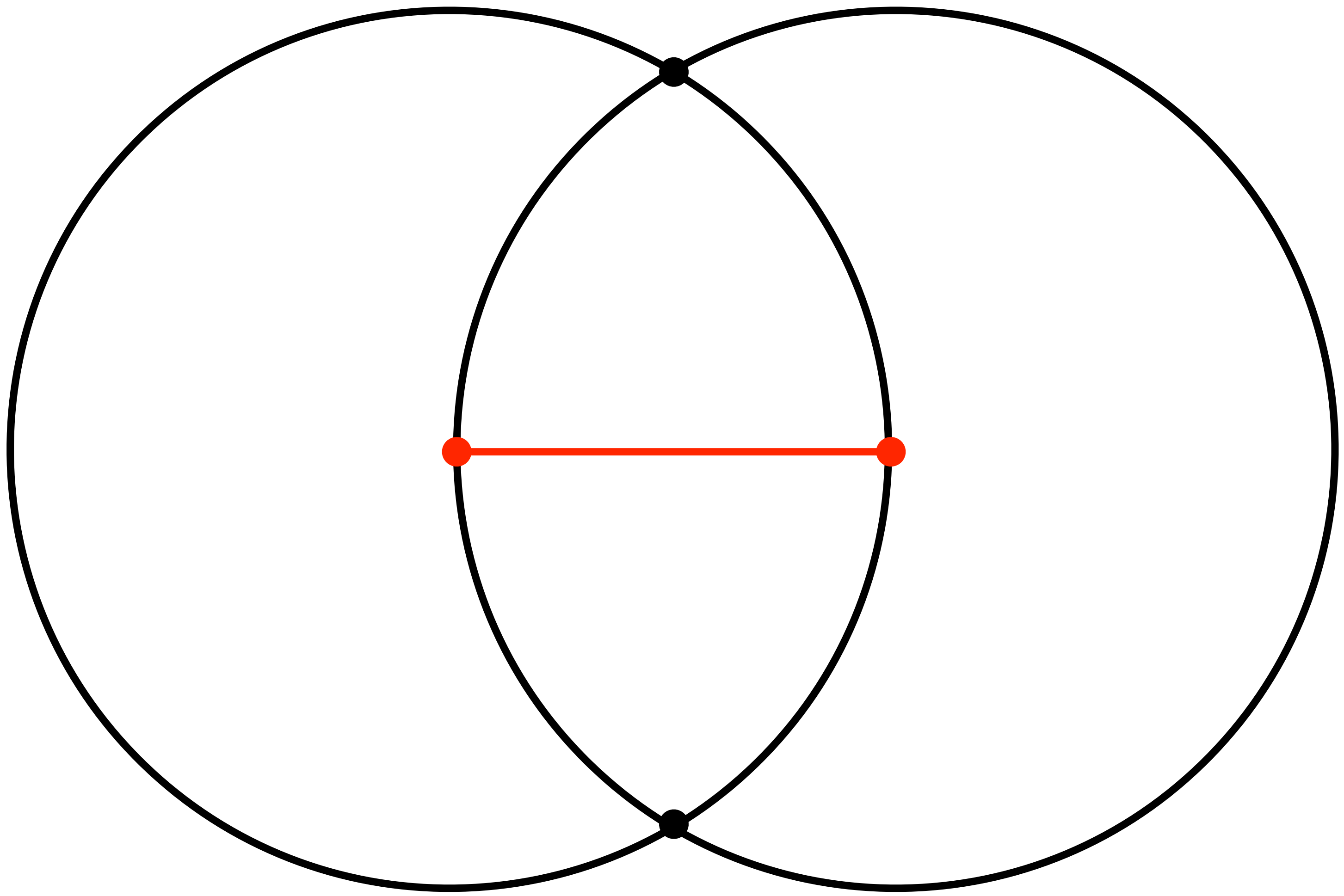

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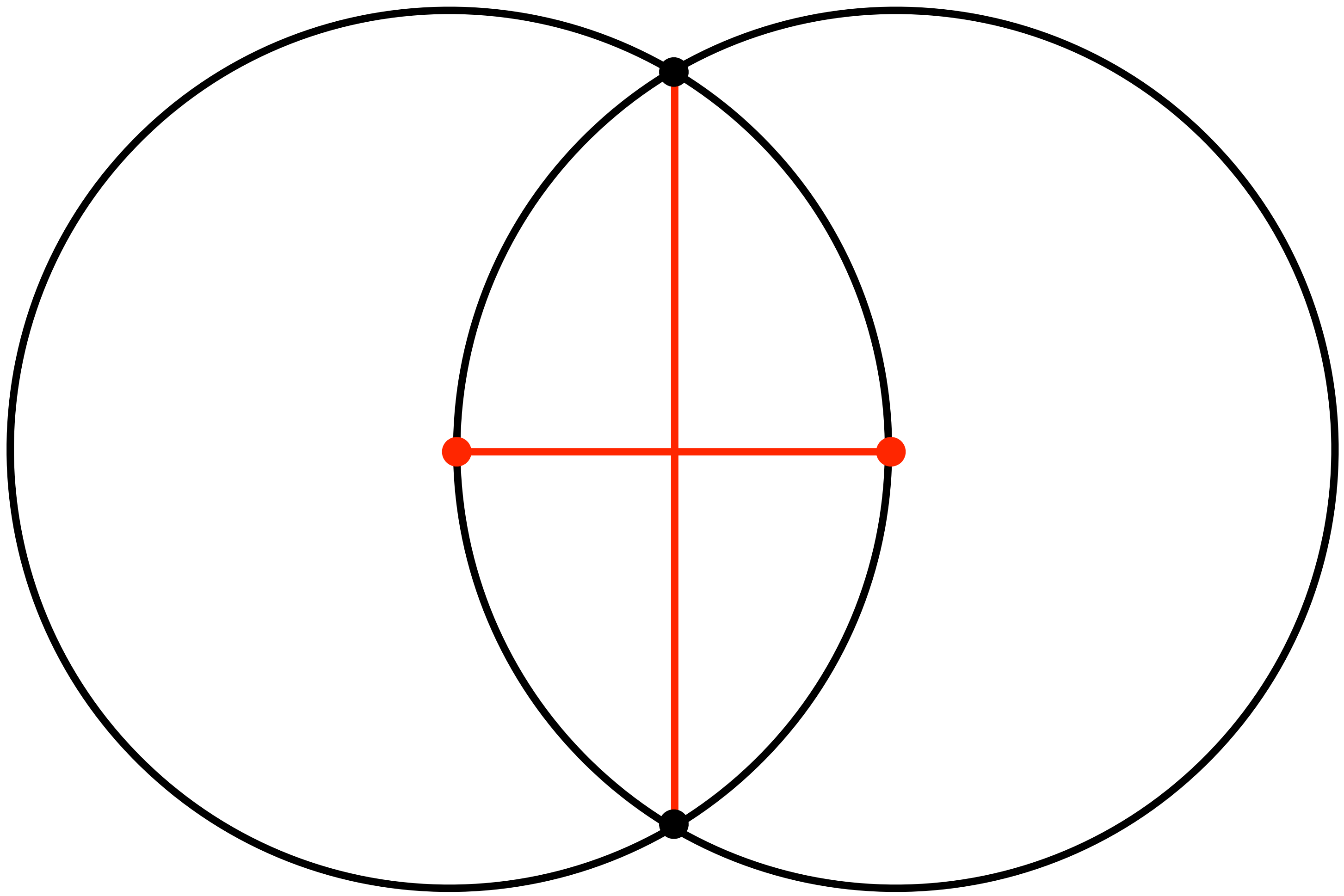


IN a right angled triangle

the square on the hypotenuse is equal to the sum of the squares of the sides, (and).

On , and describe squares, (pr. 46.)

Draw || (pr. 31.)
also draw and .

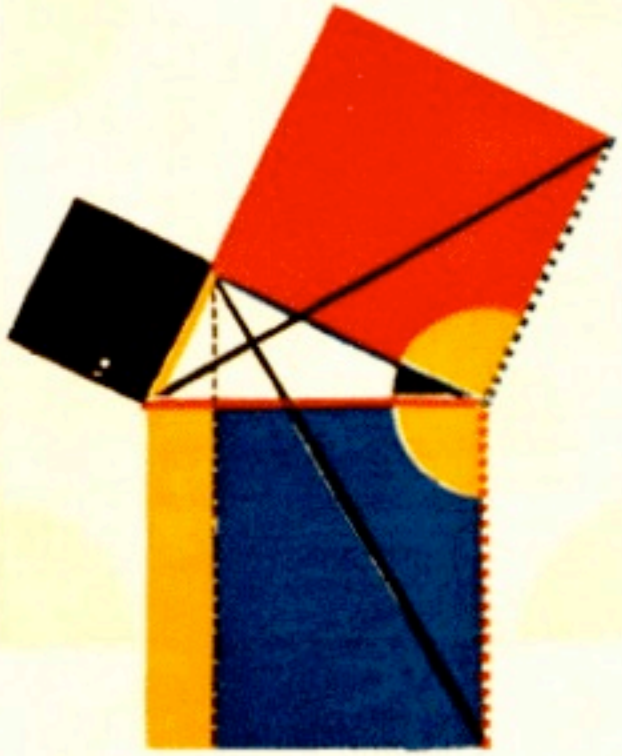
 = ,

To each add  ∴  = ,

= and = ;

∴  = .

Again, because ||



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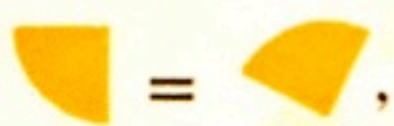
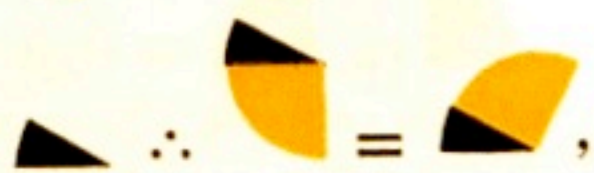
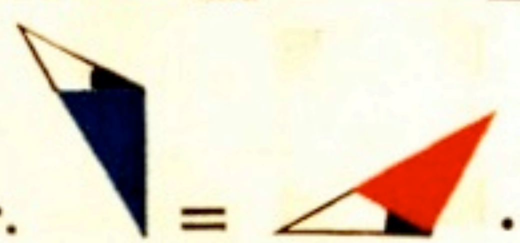


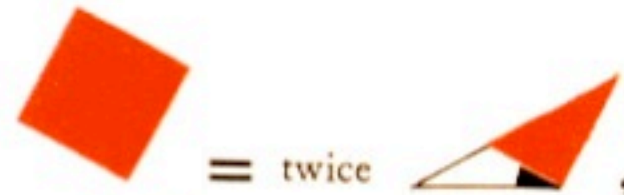
To each add



= and = ;

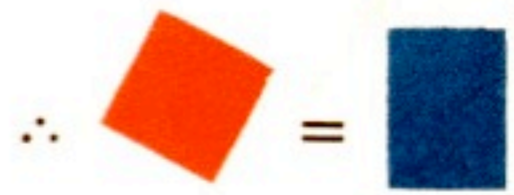


Again, because ||



= twice ,

and = twice ;



∴ = .

In the same manner it may be shown

that = ;

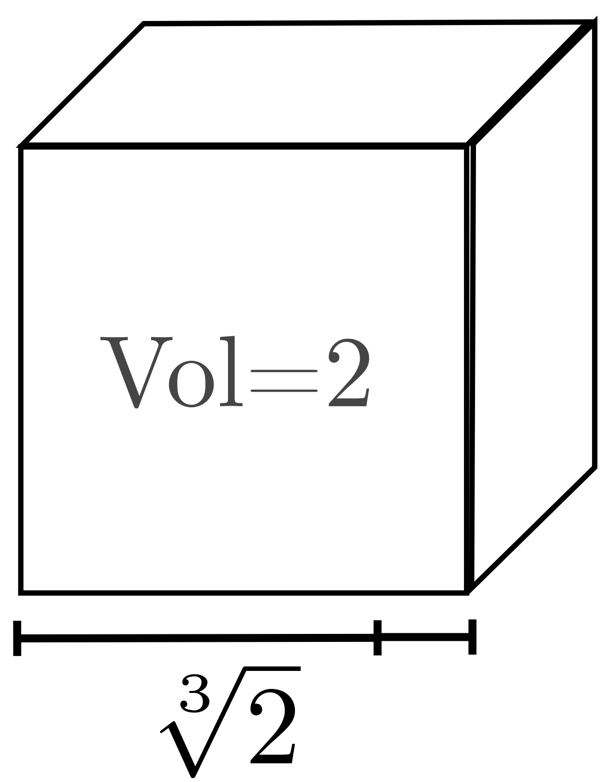
hence = .

Q. E. D.

Double the cube

$$\text{Volume of the cube } = (\text{edge})^3 = 2$$

$$\Rightarrow \text{edge} = \sqrt[3]{2} \simeq 1.25992105...$$



What points can we construct?

What points can we construct?

What points can we construct?

What points can we construct?

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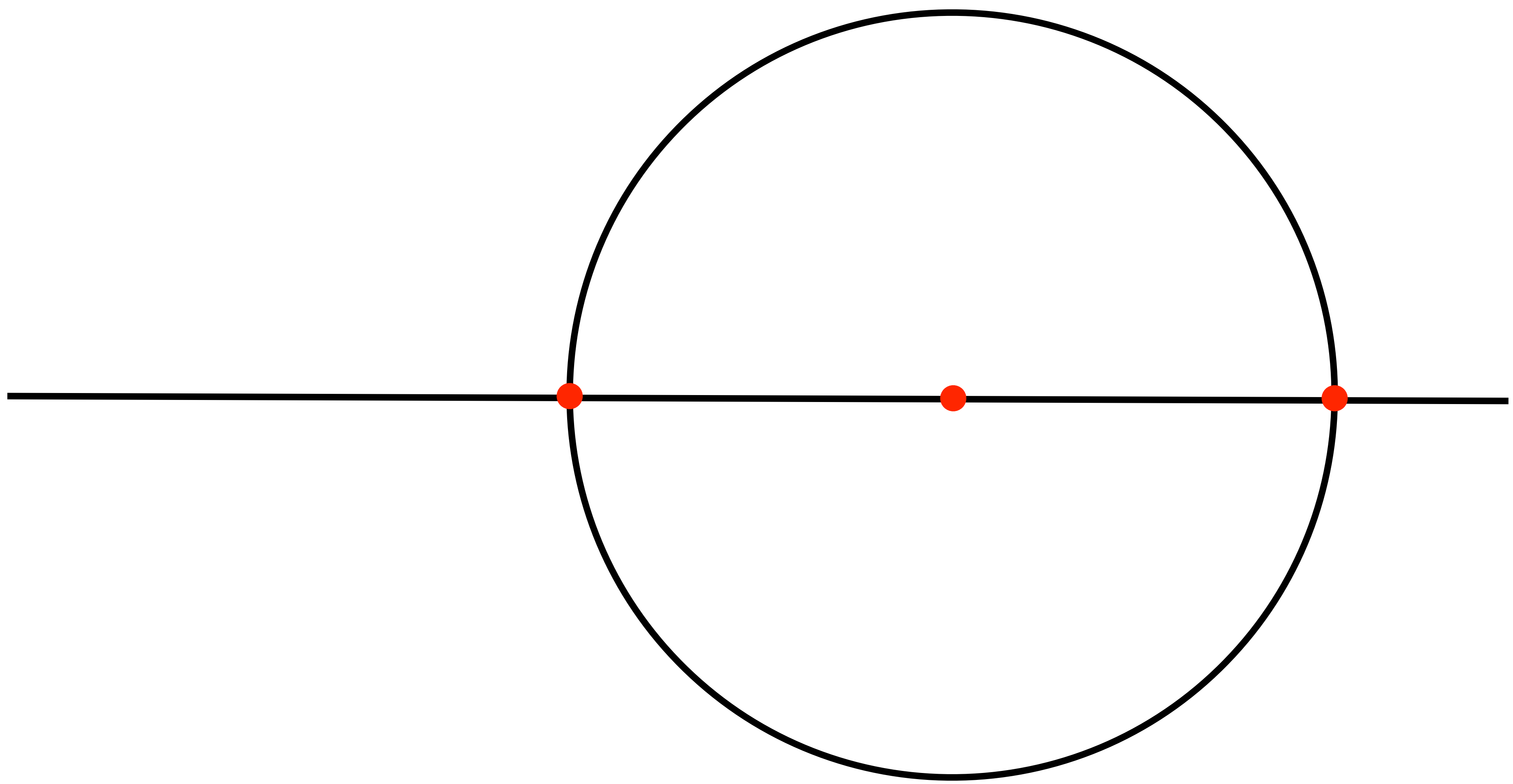

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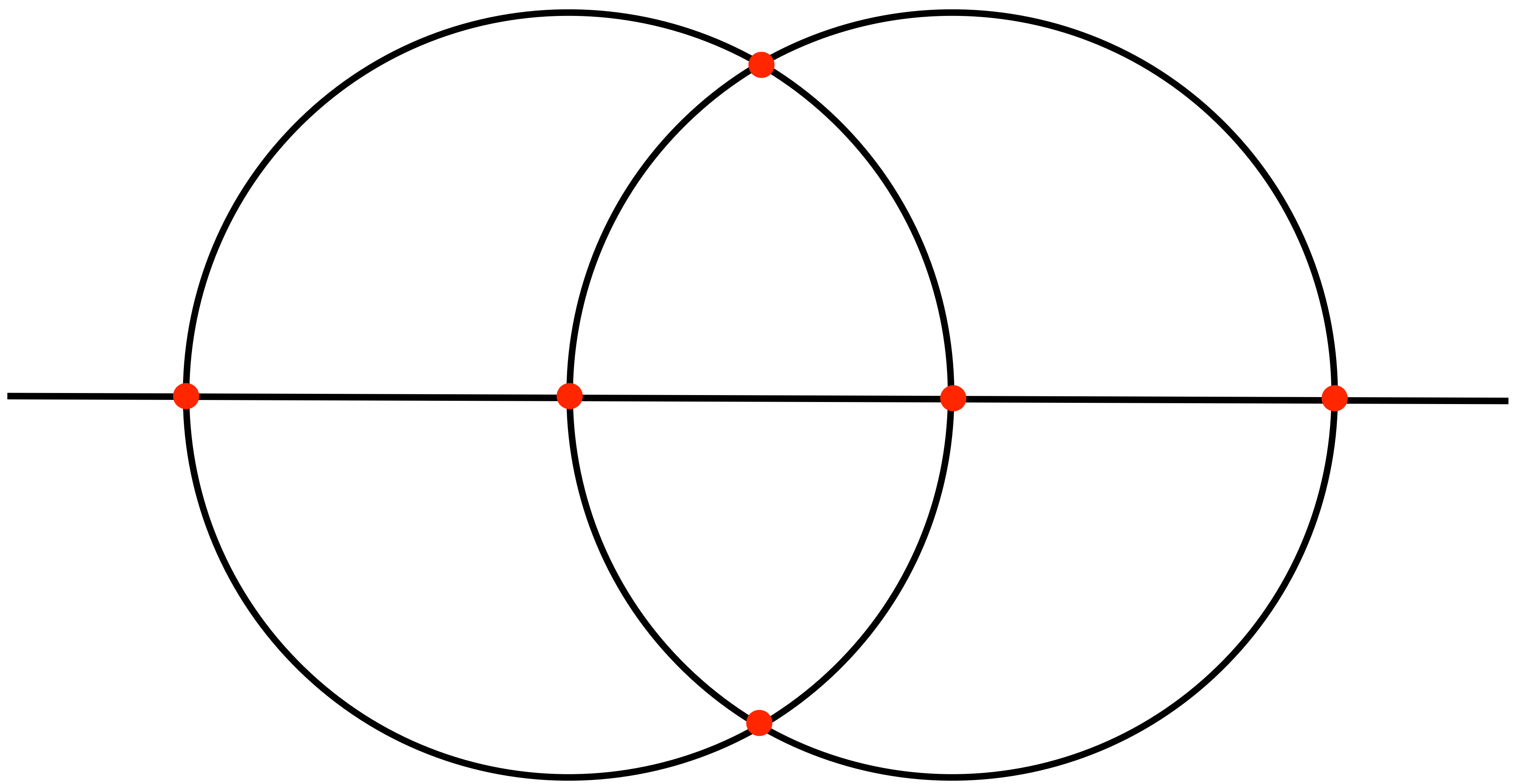

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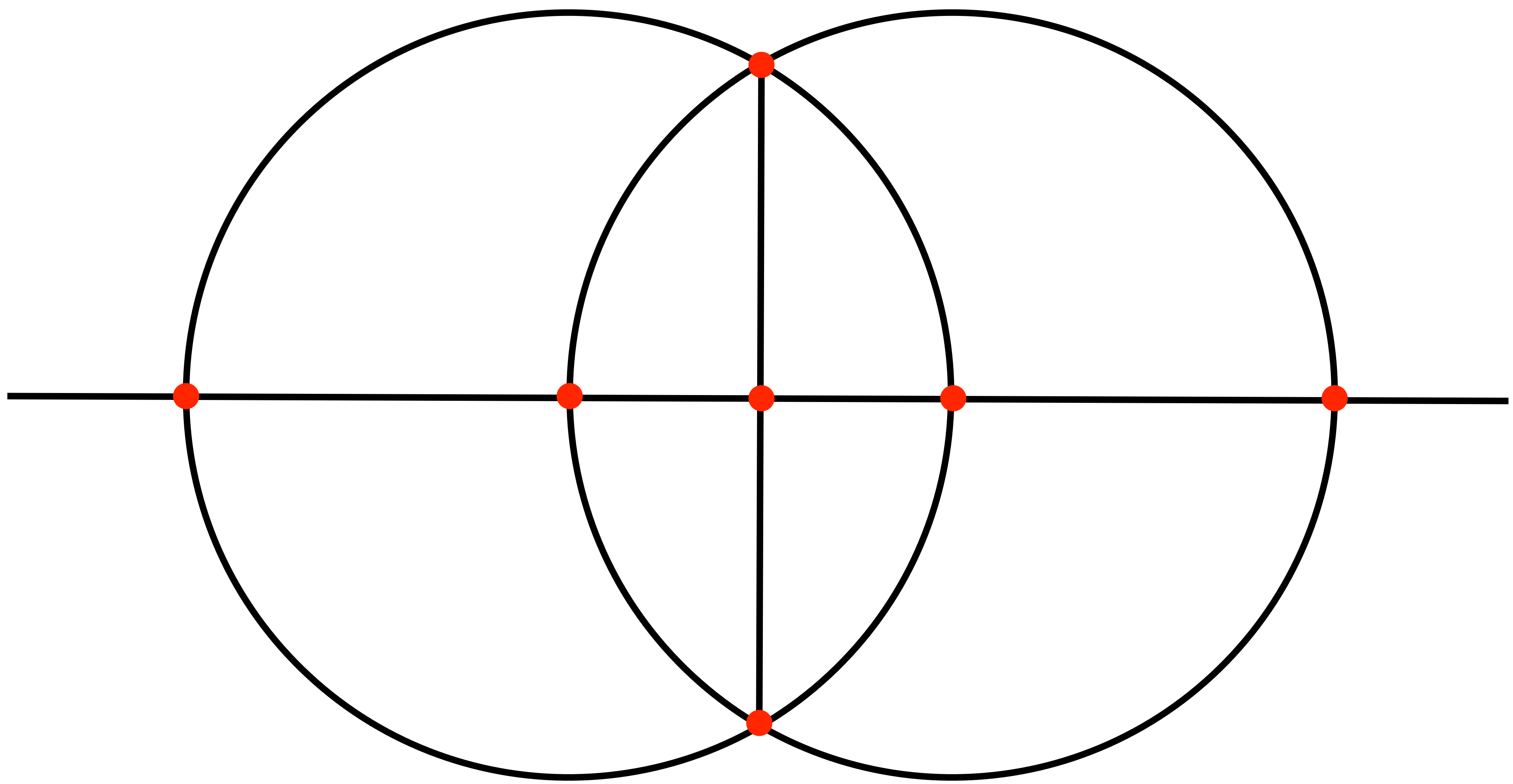
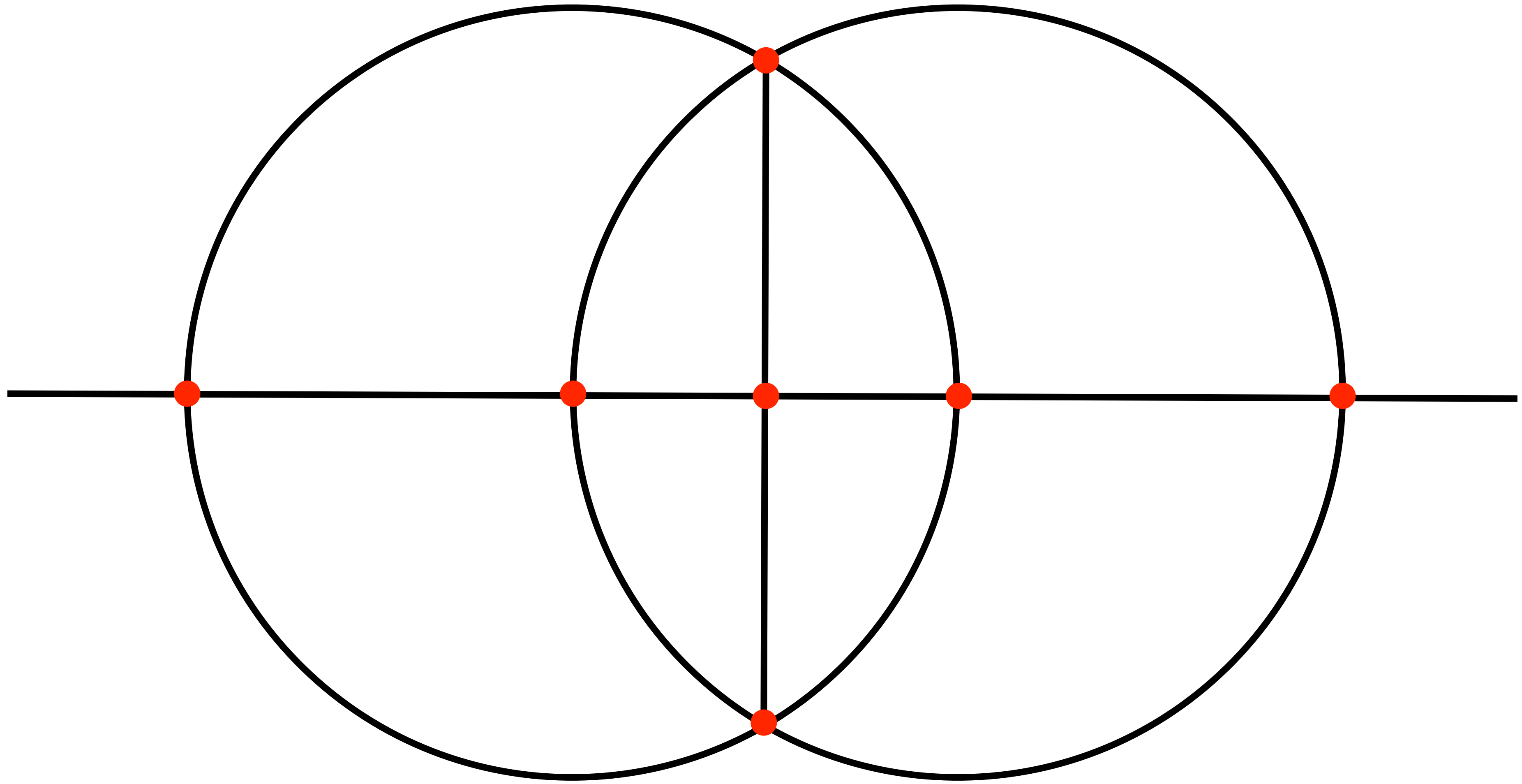


What numbers can we construct?



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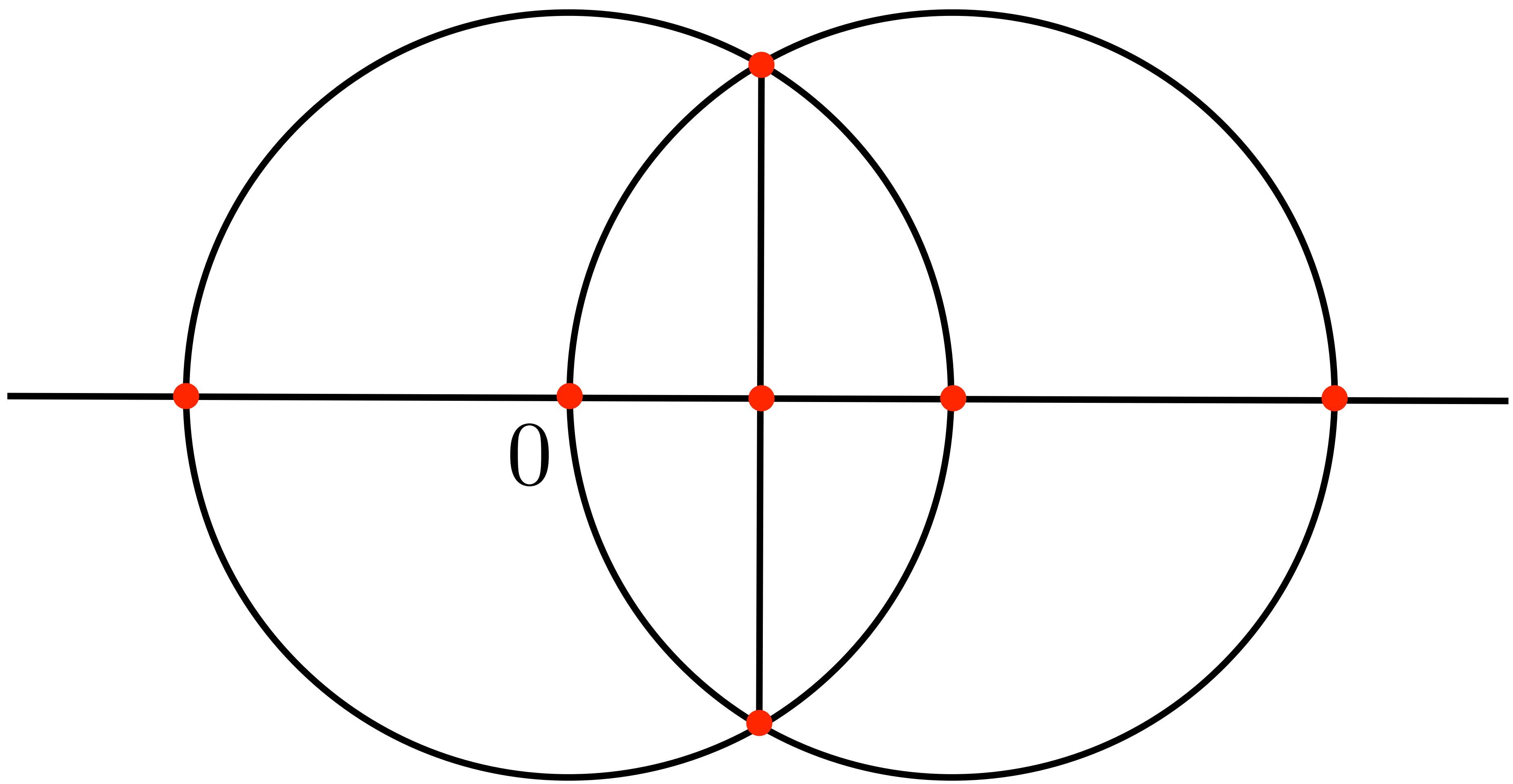

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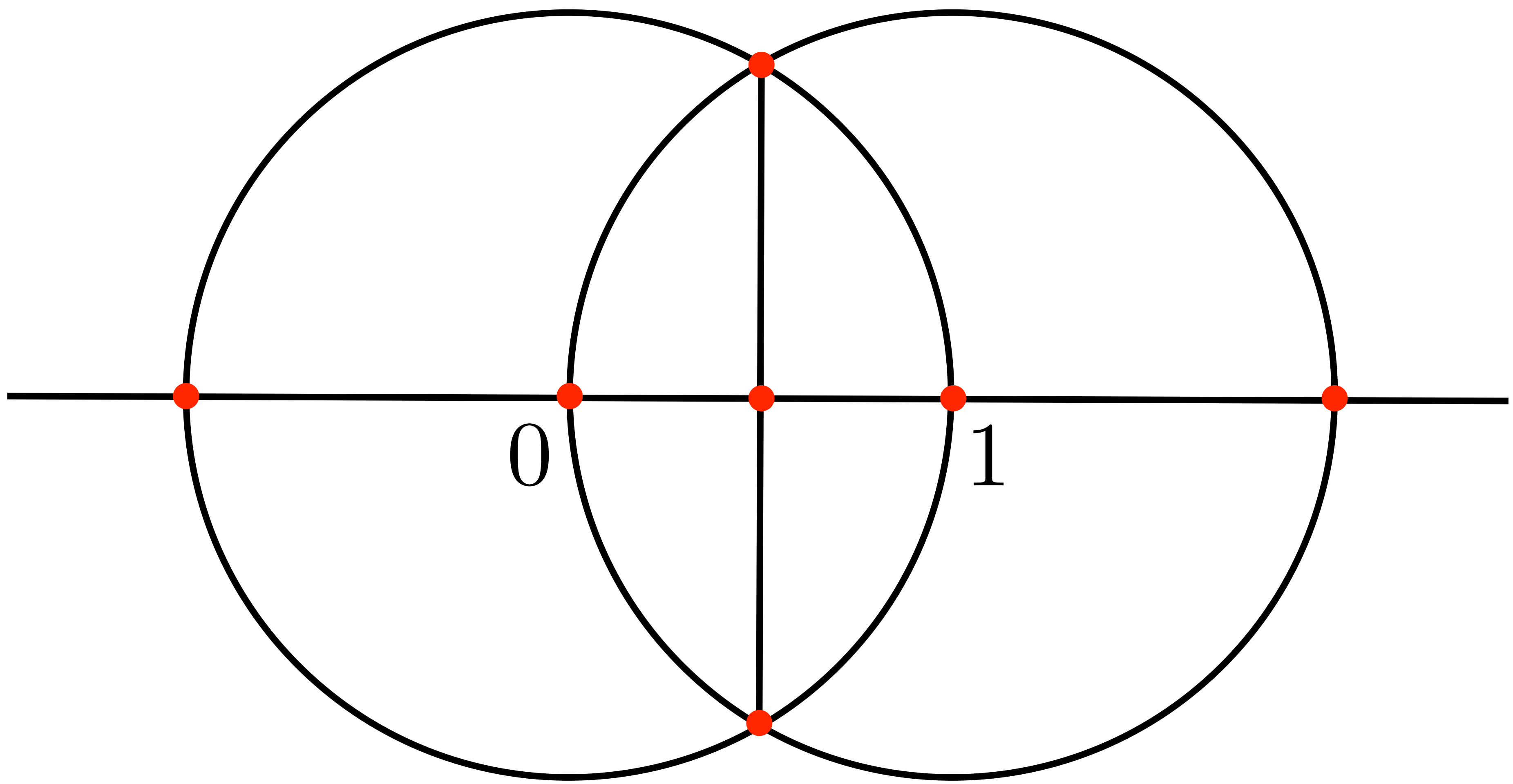

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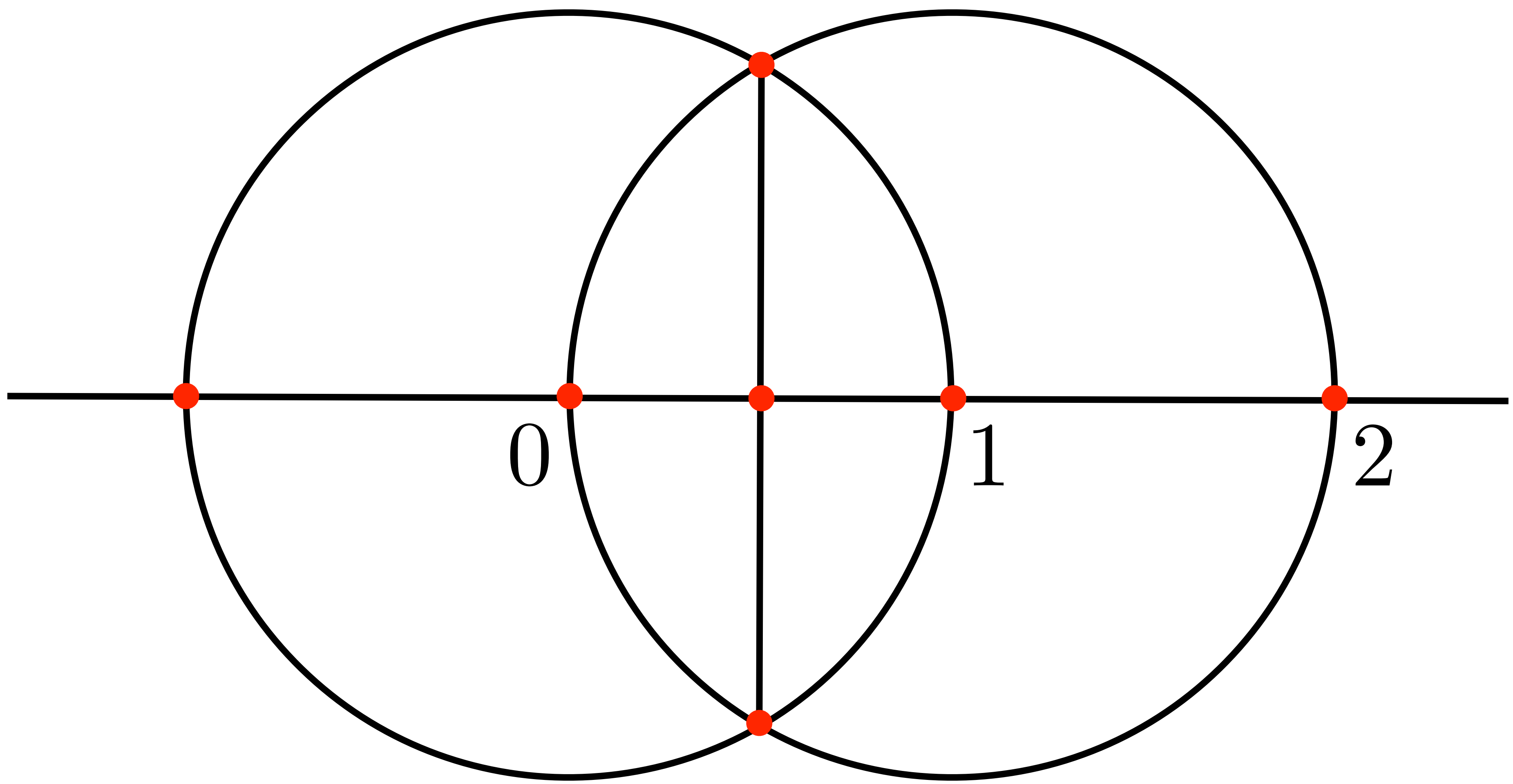

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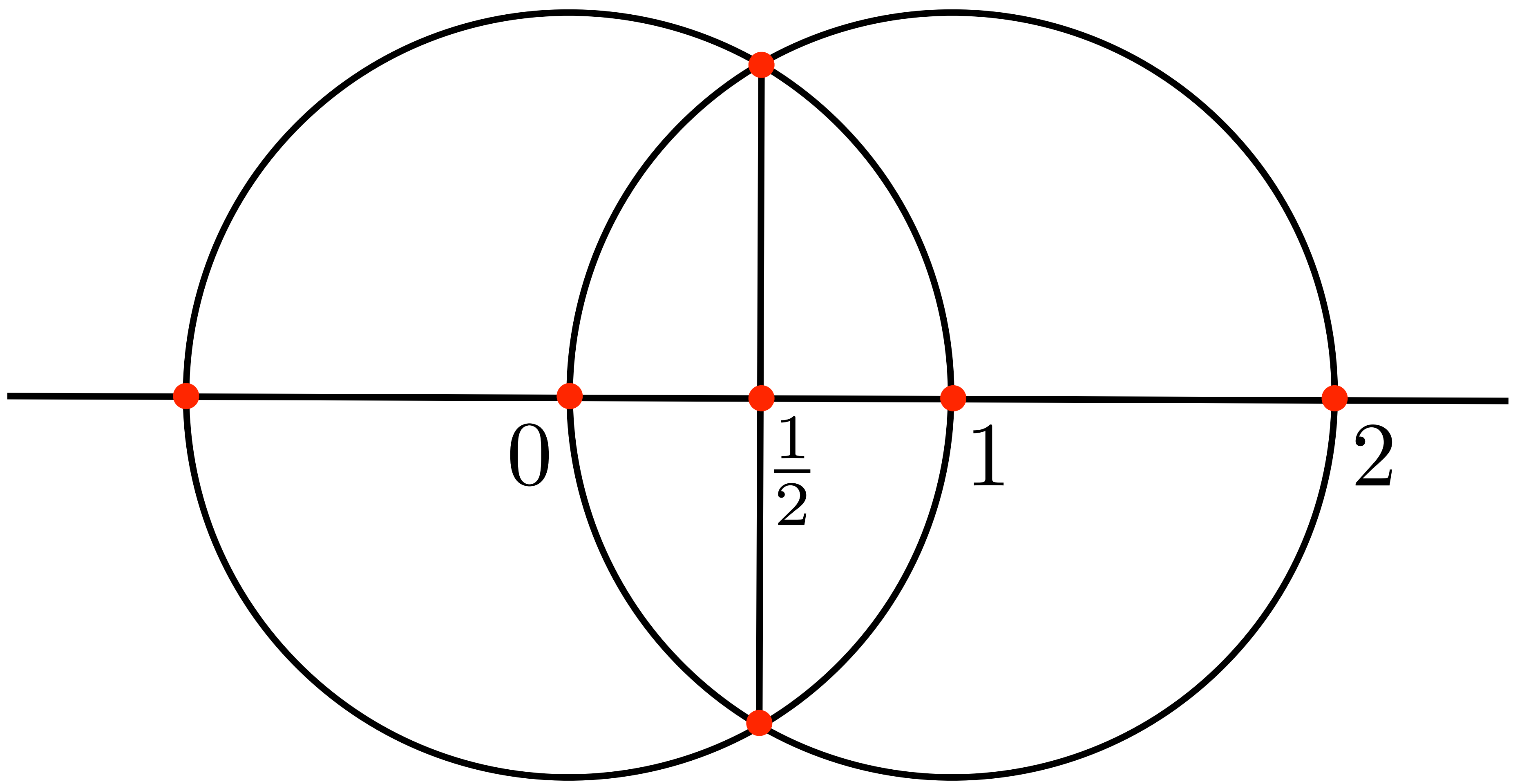

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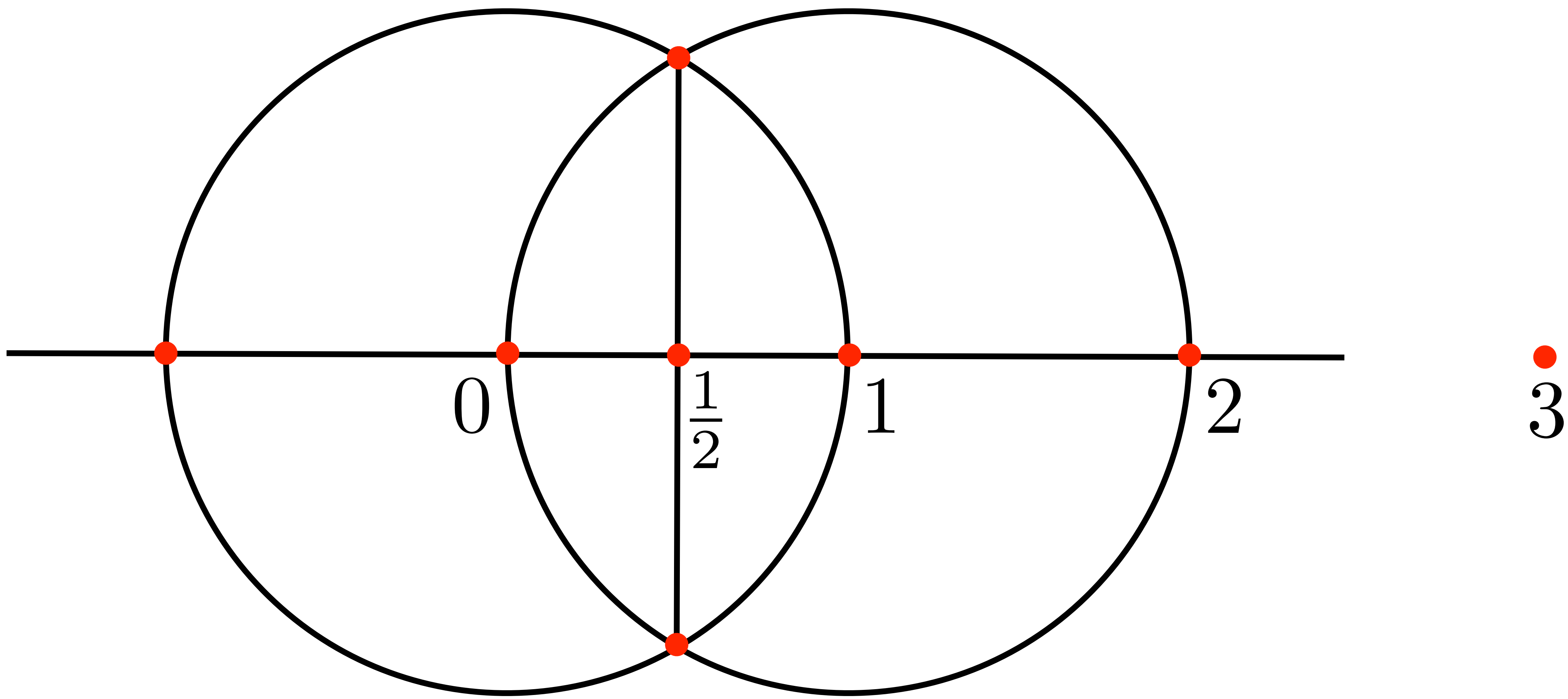

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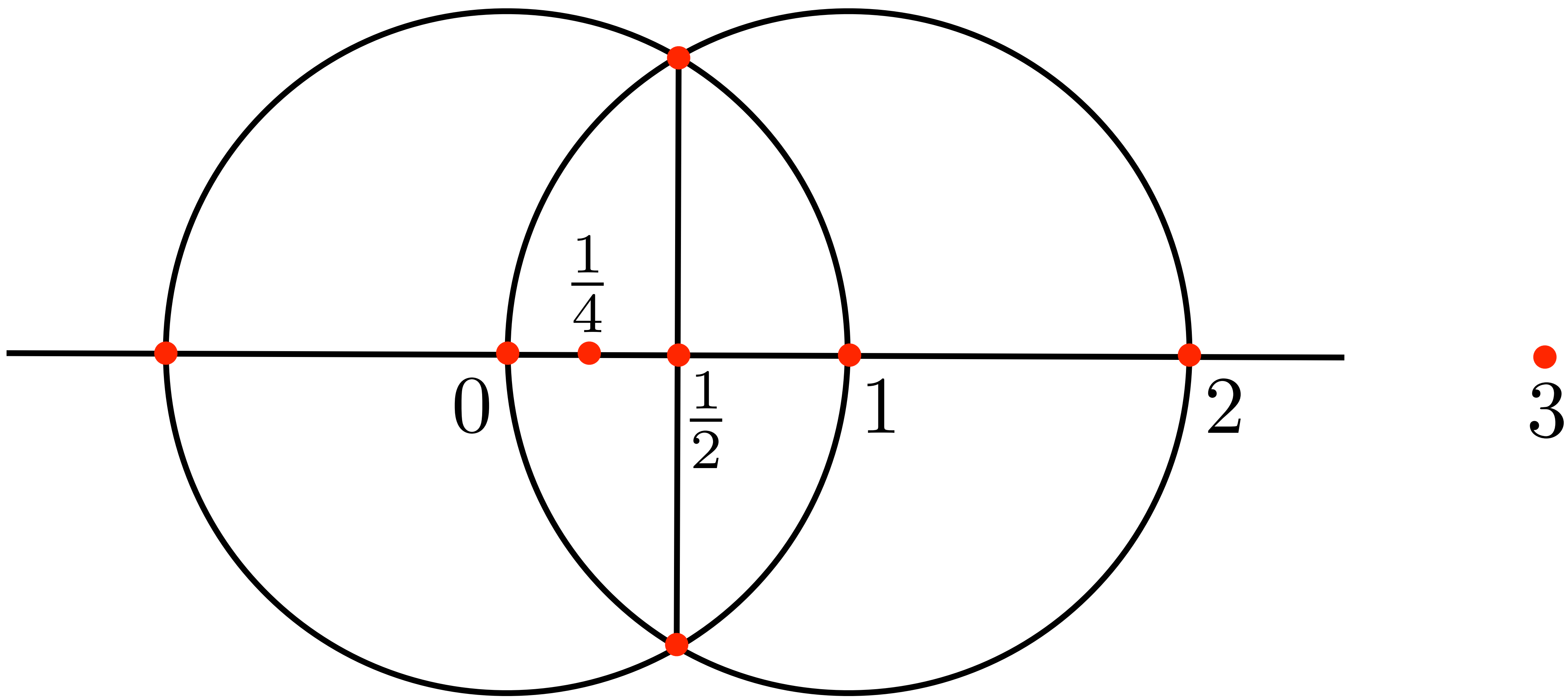

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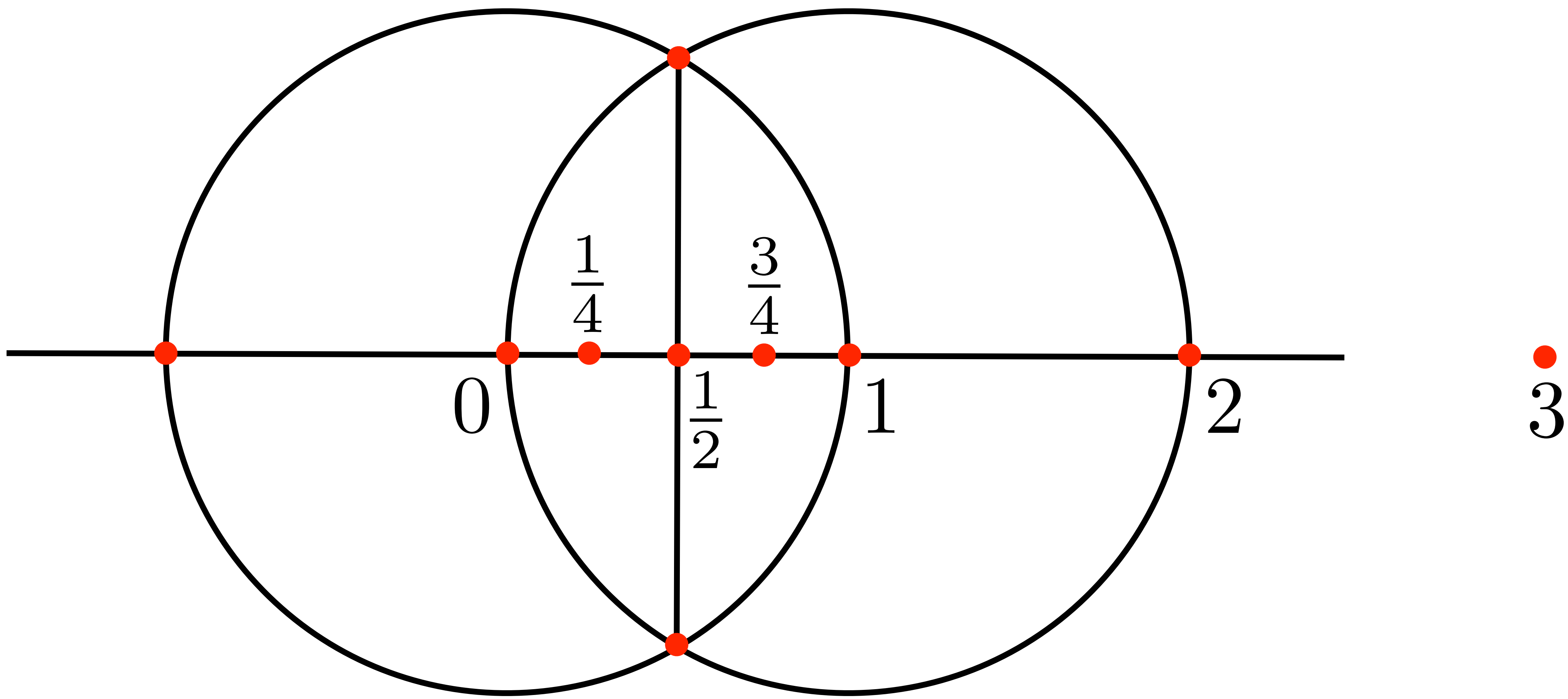

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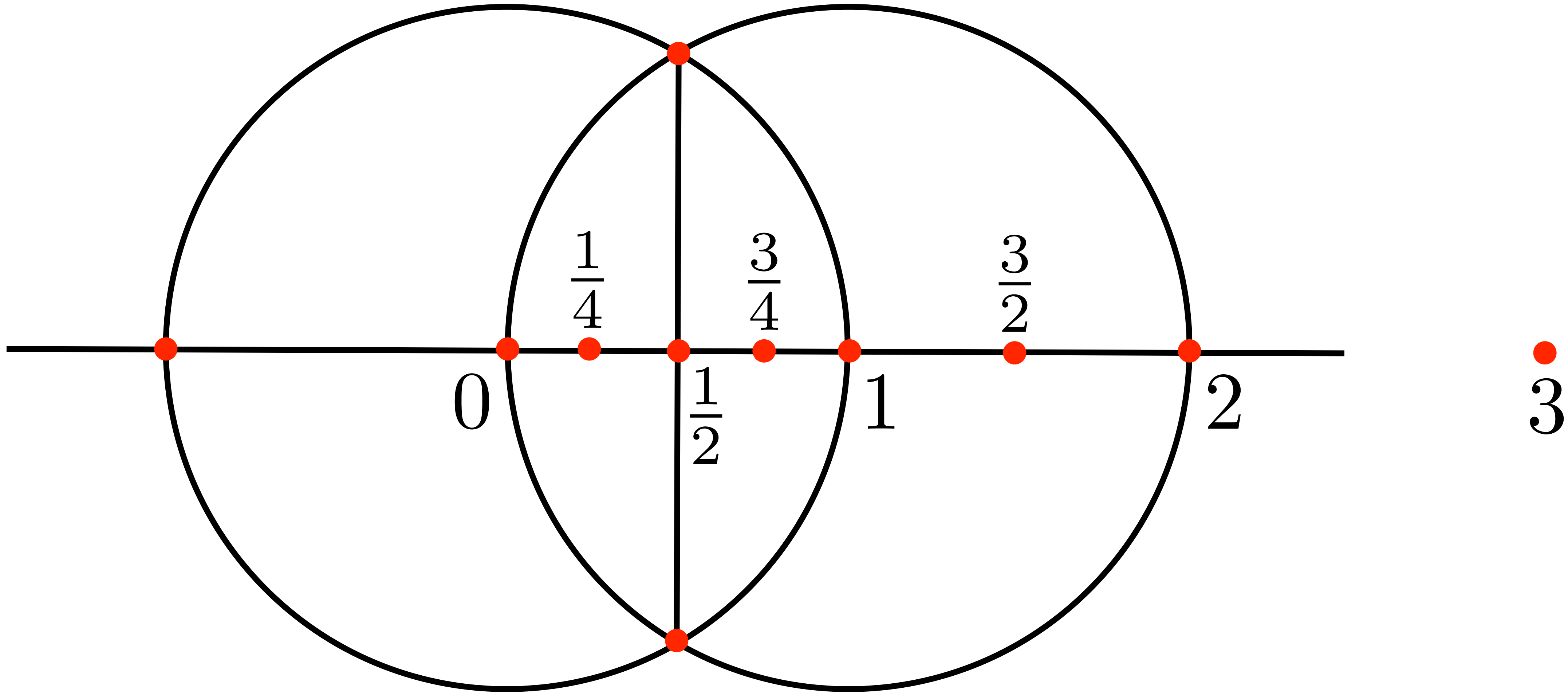

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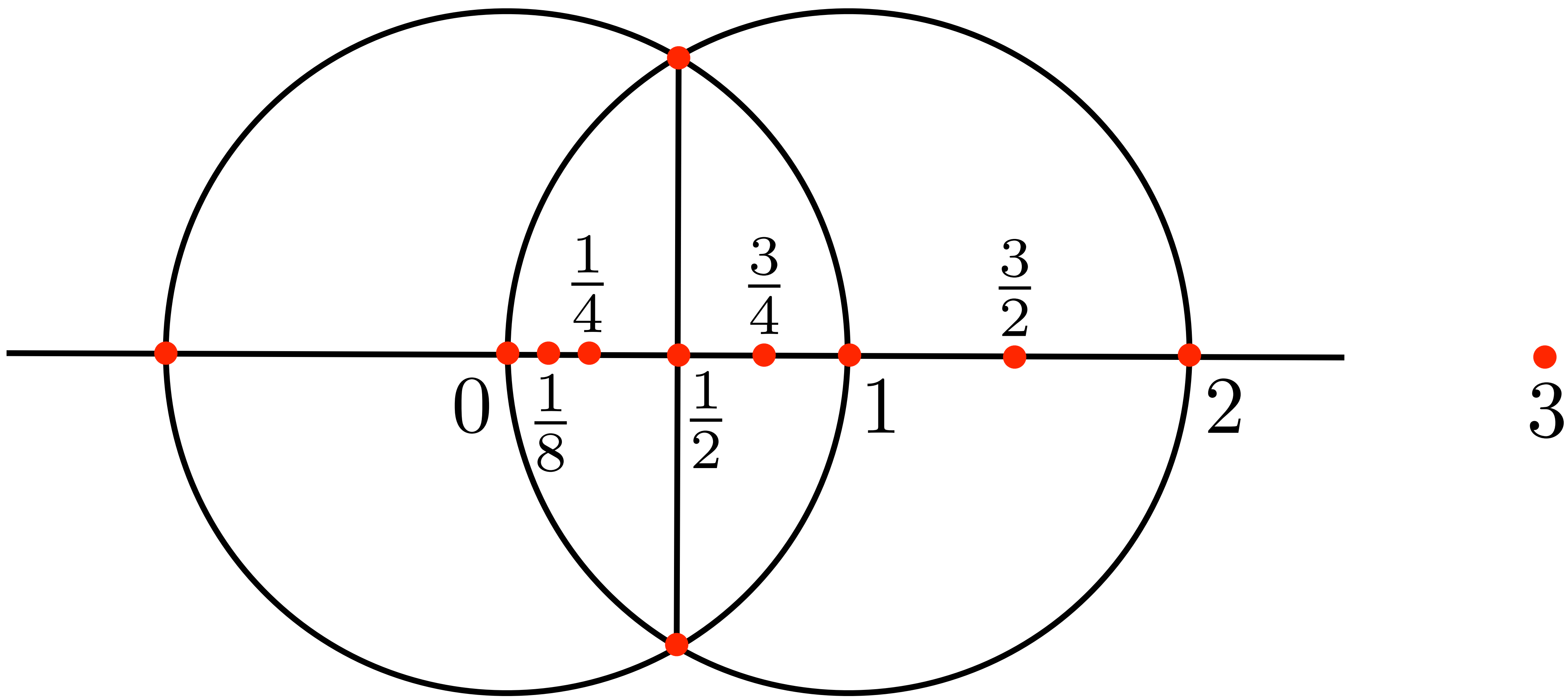

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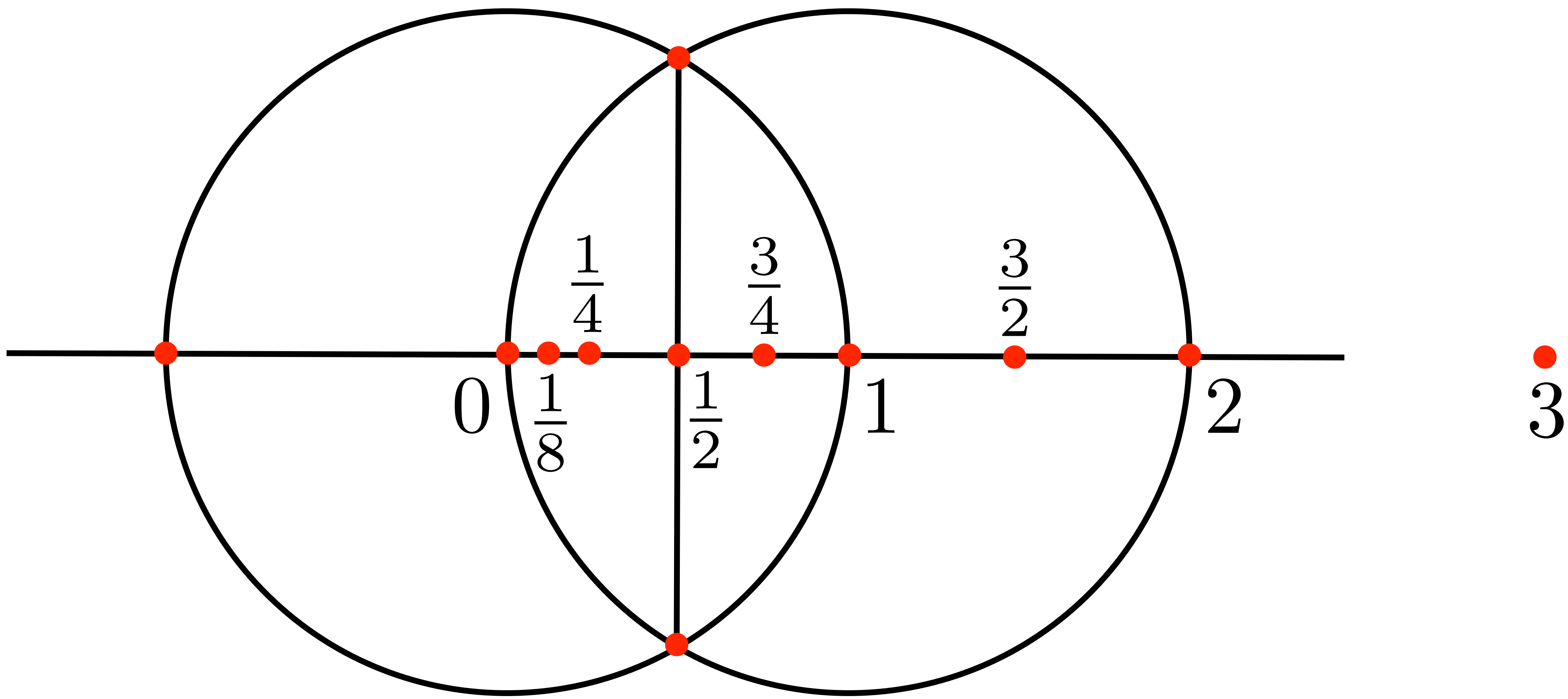
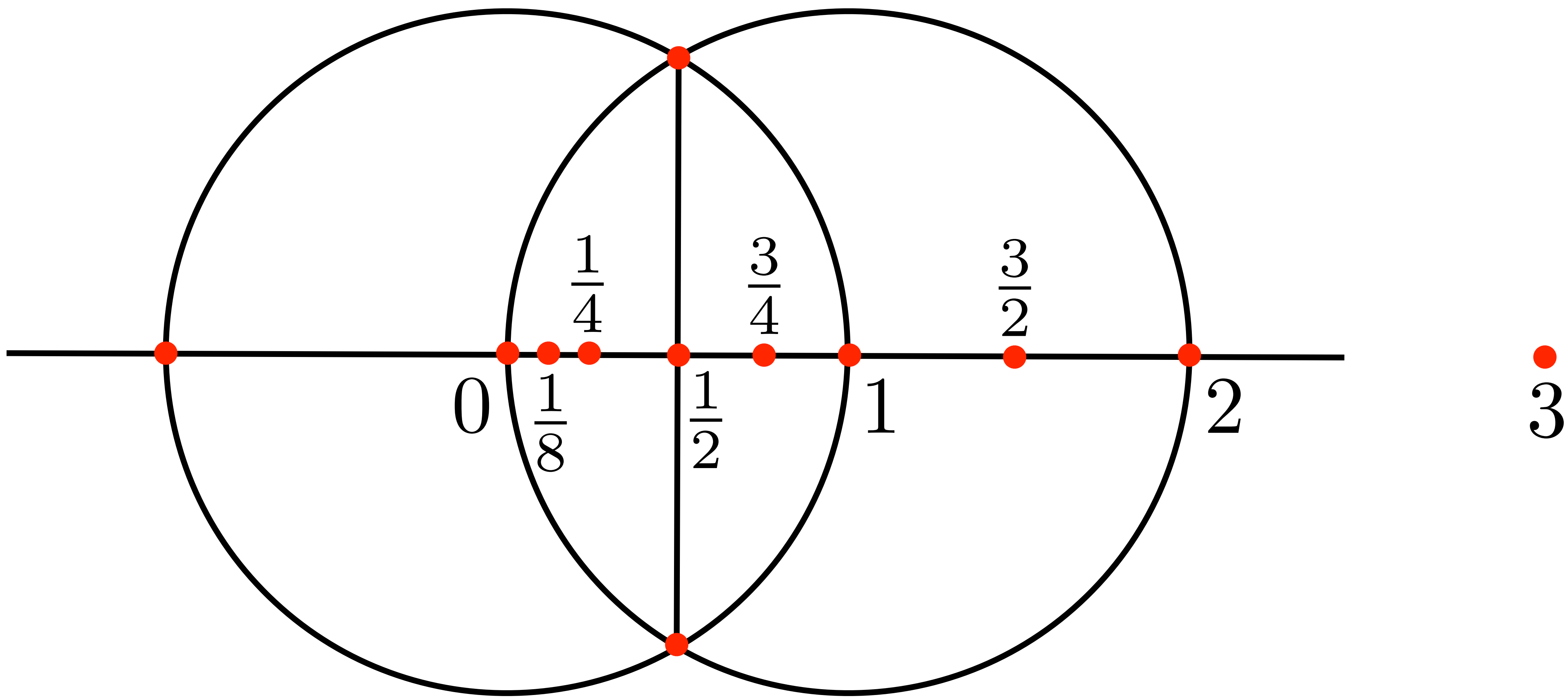


All integers and all fractions with denominator 2^k.

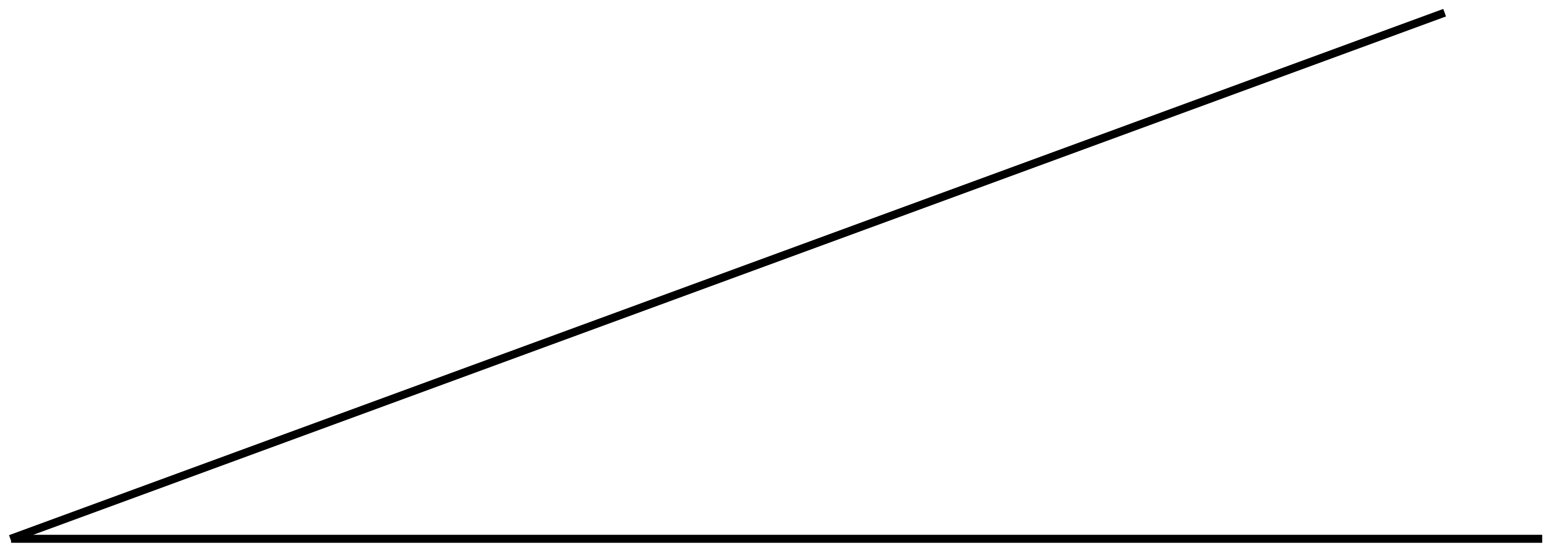
What numbers can we construct?



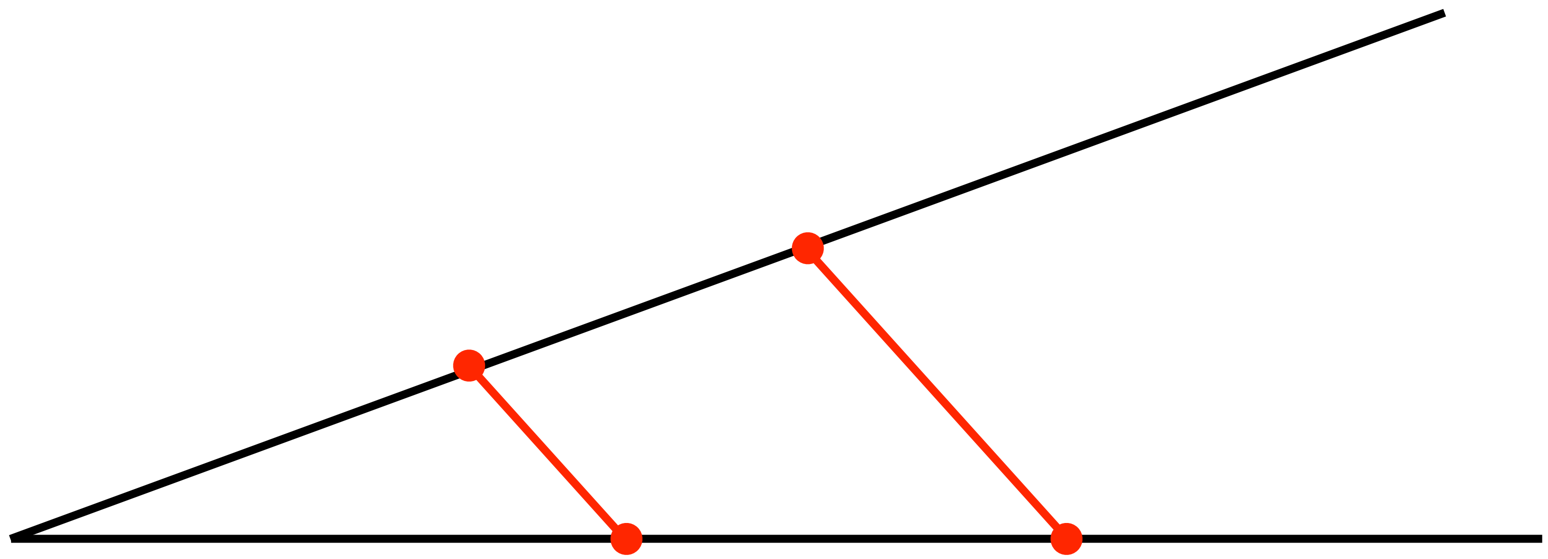
All integers and all fractions with denominator 2^k.

And what else?

Thales' theorem (intercept theorem):



Thales' theorem (intercept theorem):



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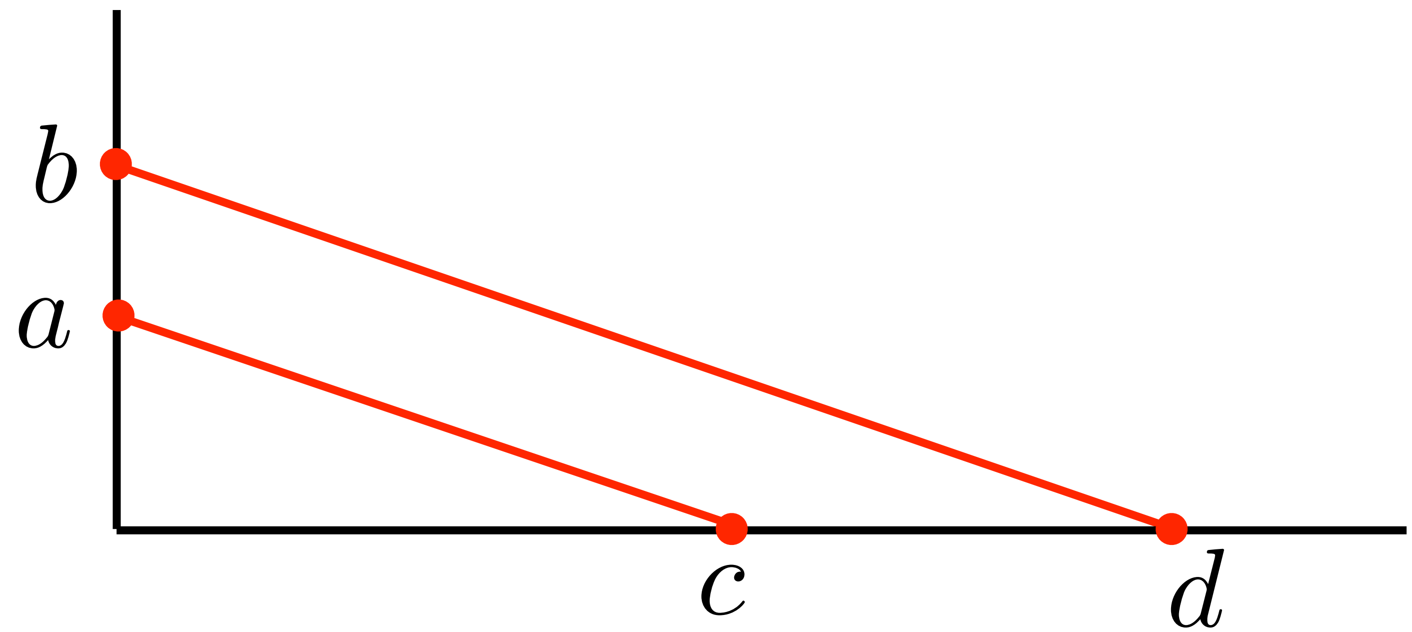

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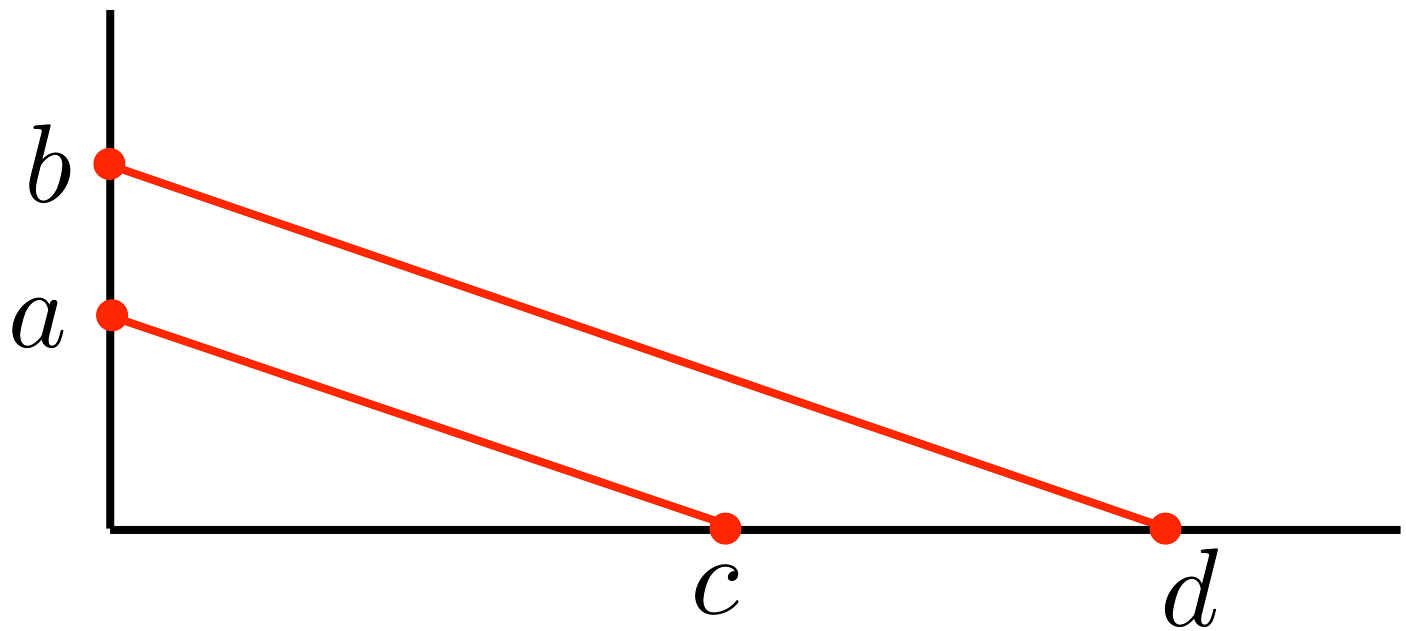
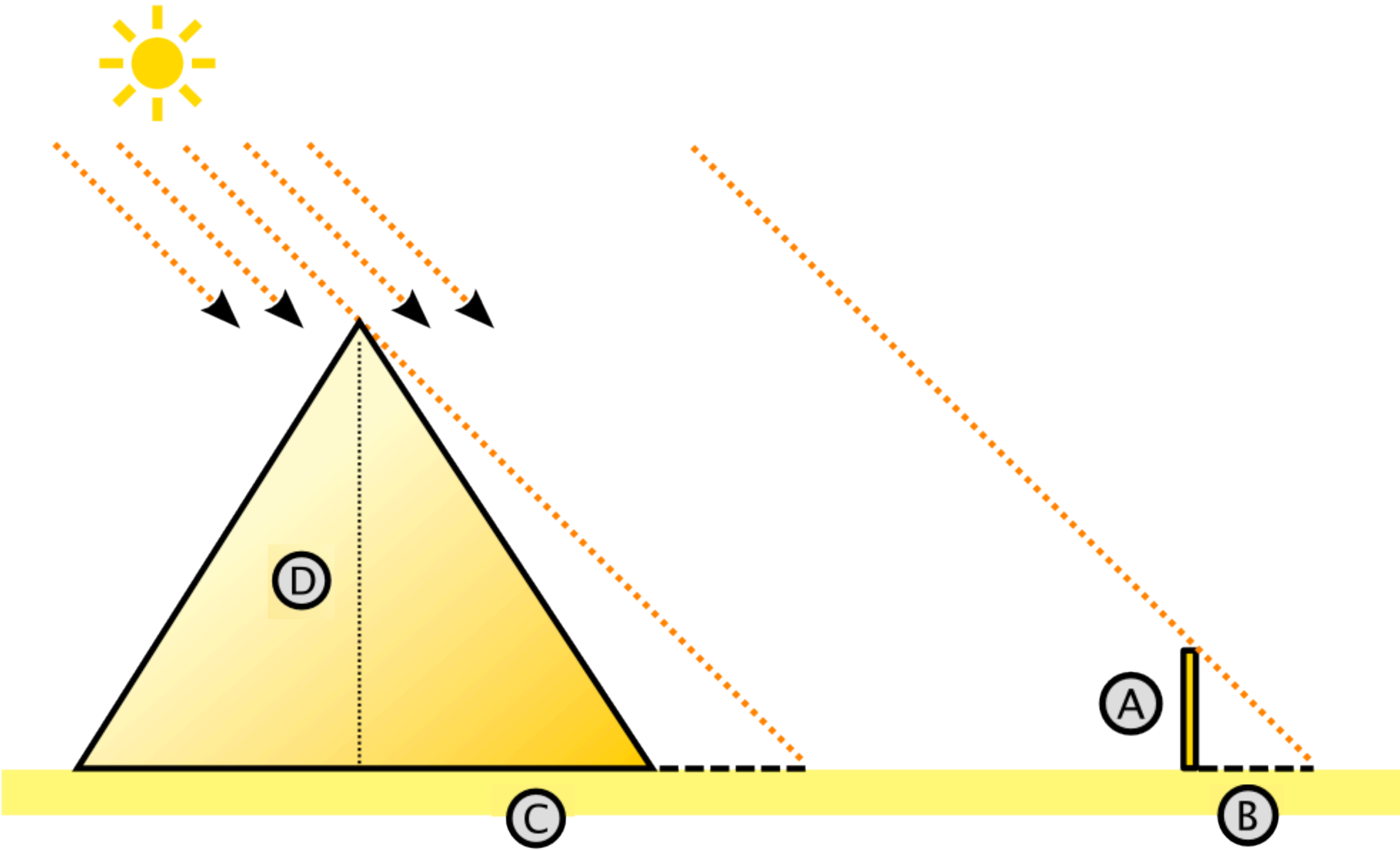
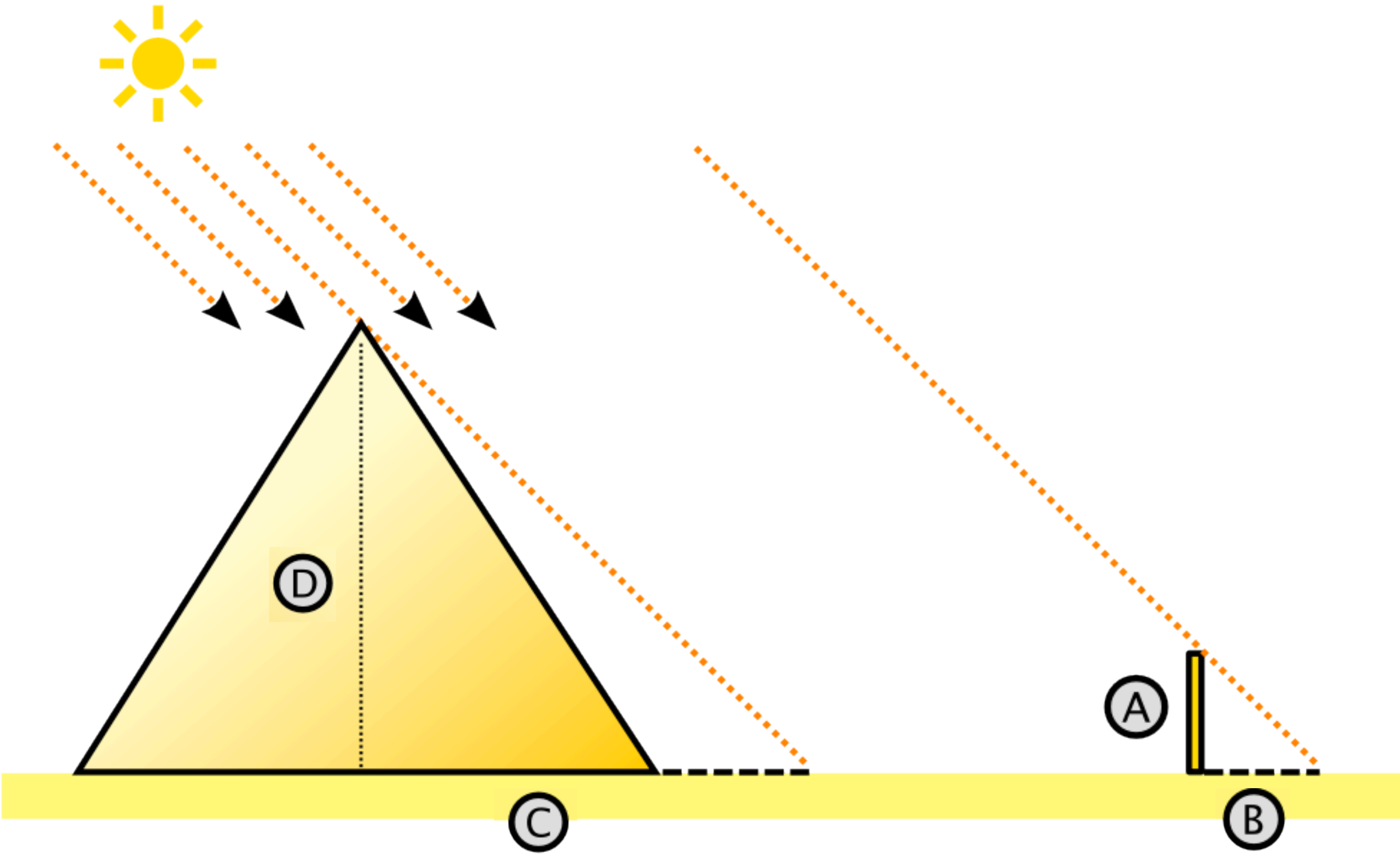


$$\frac{b}{a} = \frac{d}{c}$$

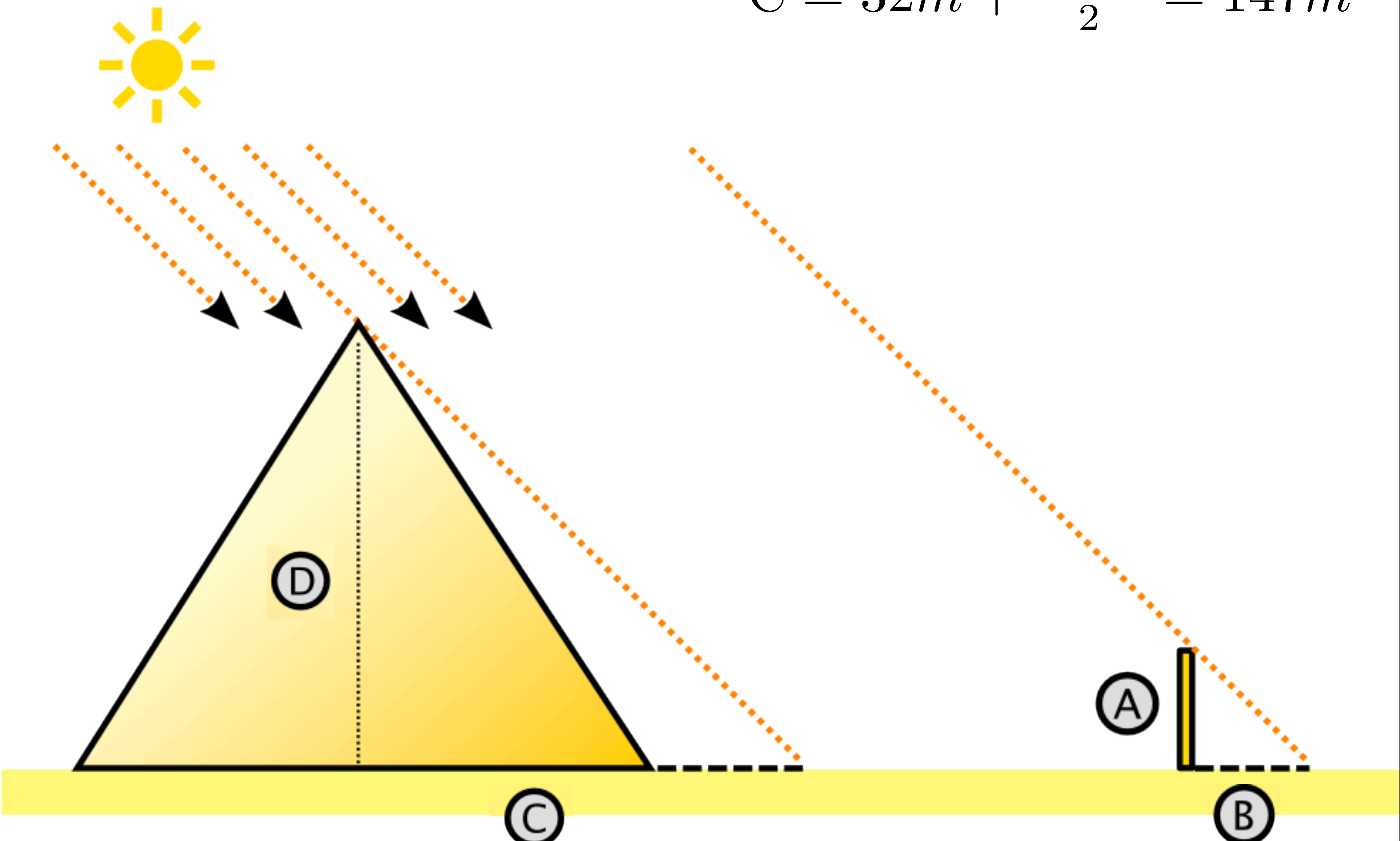


D
C
A
B

$A = B =$ height of Thales



$$A = B = \text{height of Thales}$$

$$C = 32m + \frac{230m}{2} = 147m$$



$$A = B = \text{height of Thales}$$

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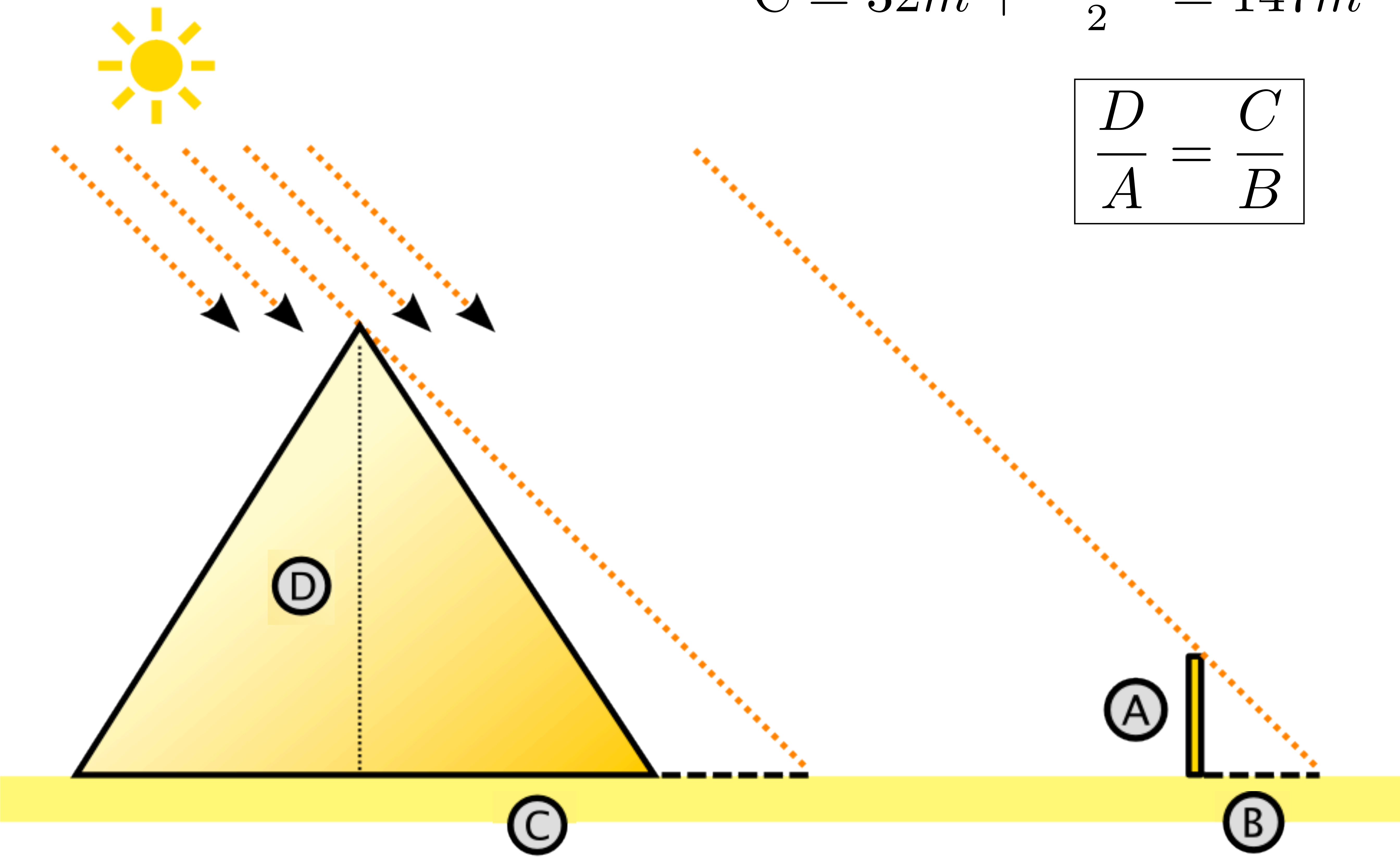
$$\boxed{\frac{D}{A} = \frac{C}{B}}$$



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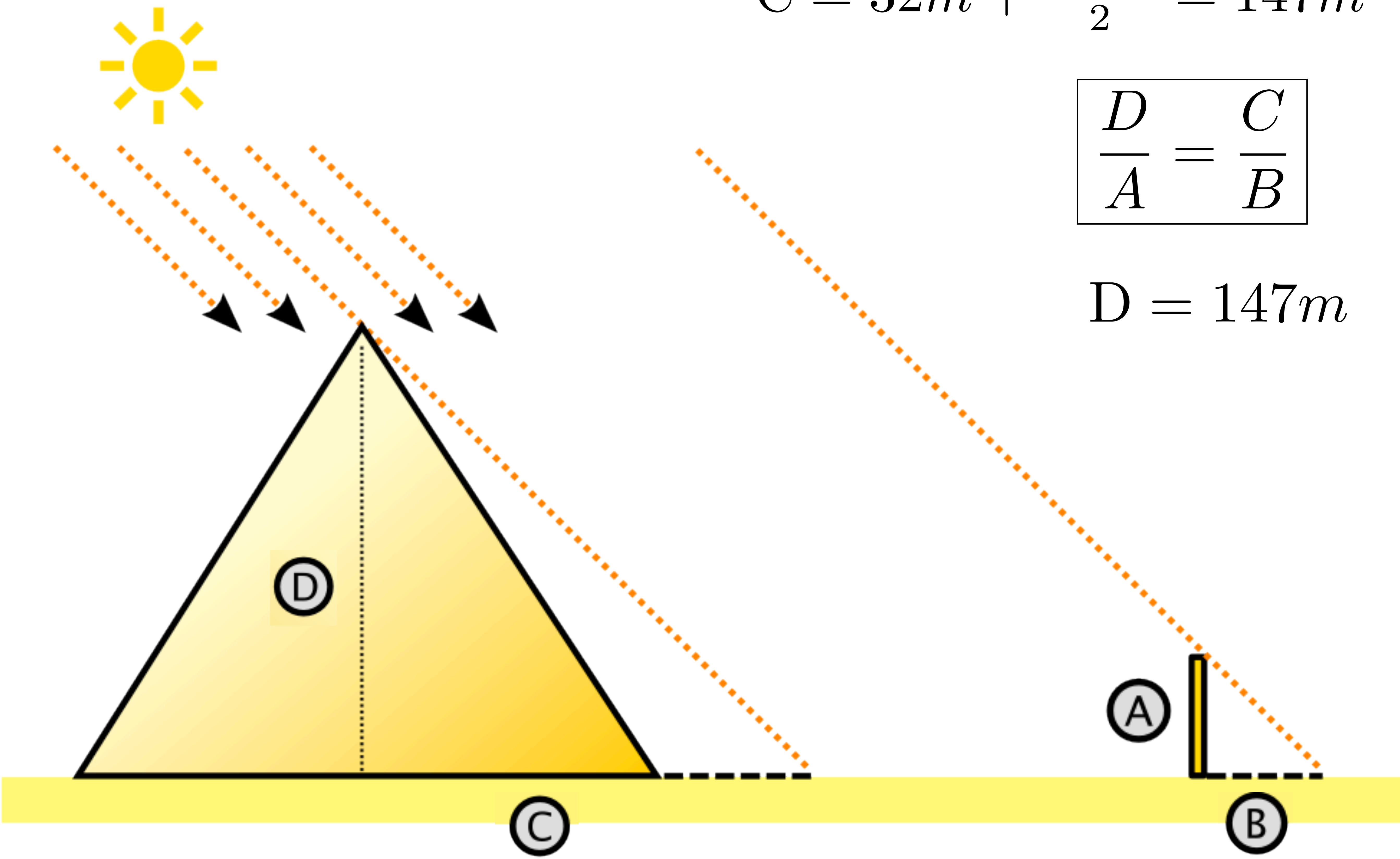
$$D = 147m$$



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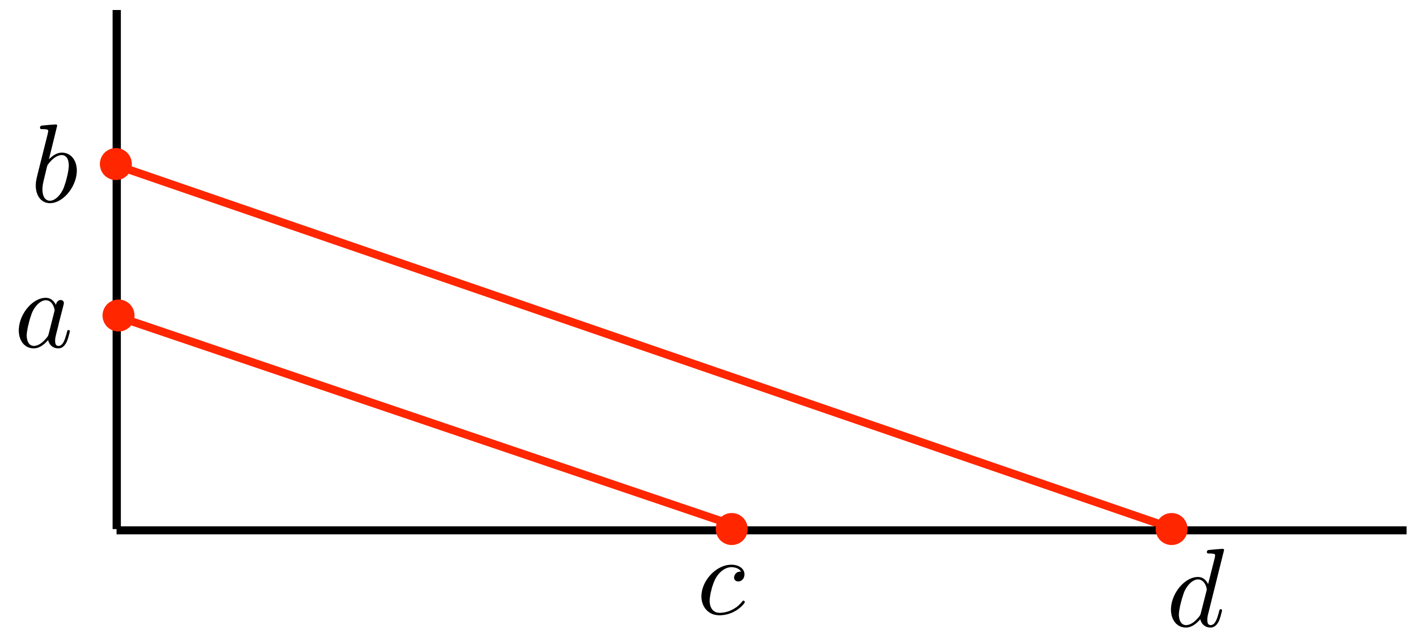

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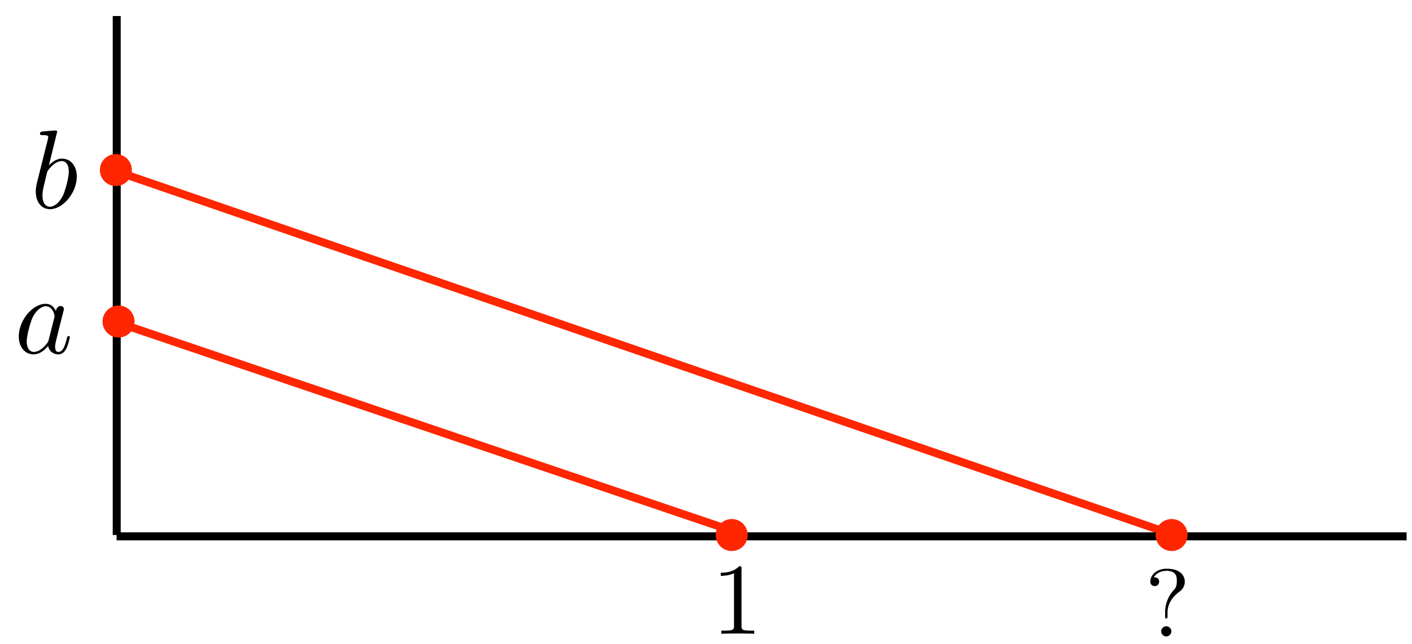
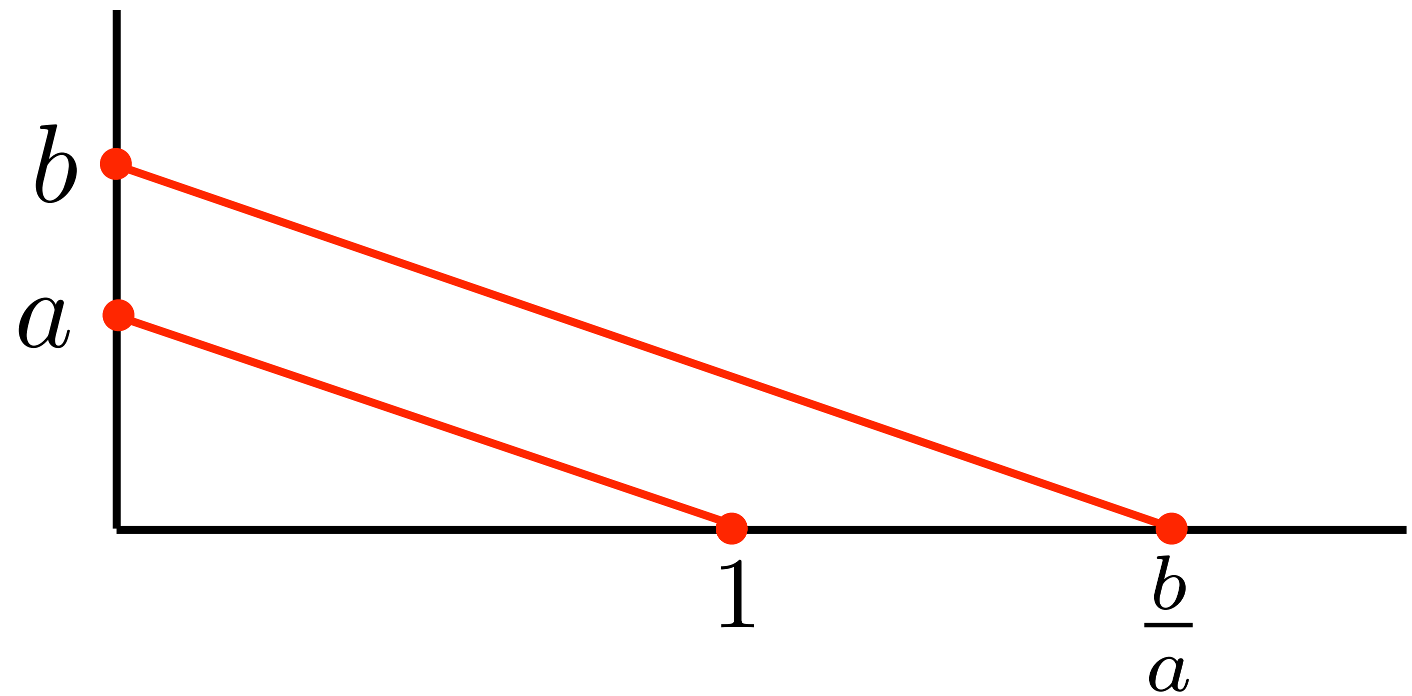
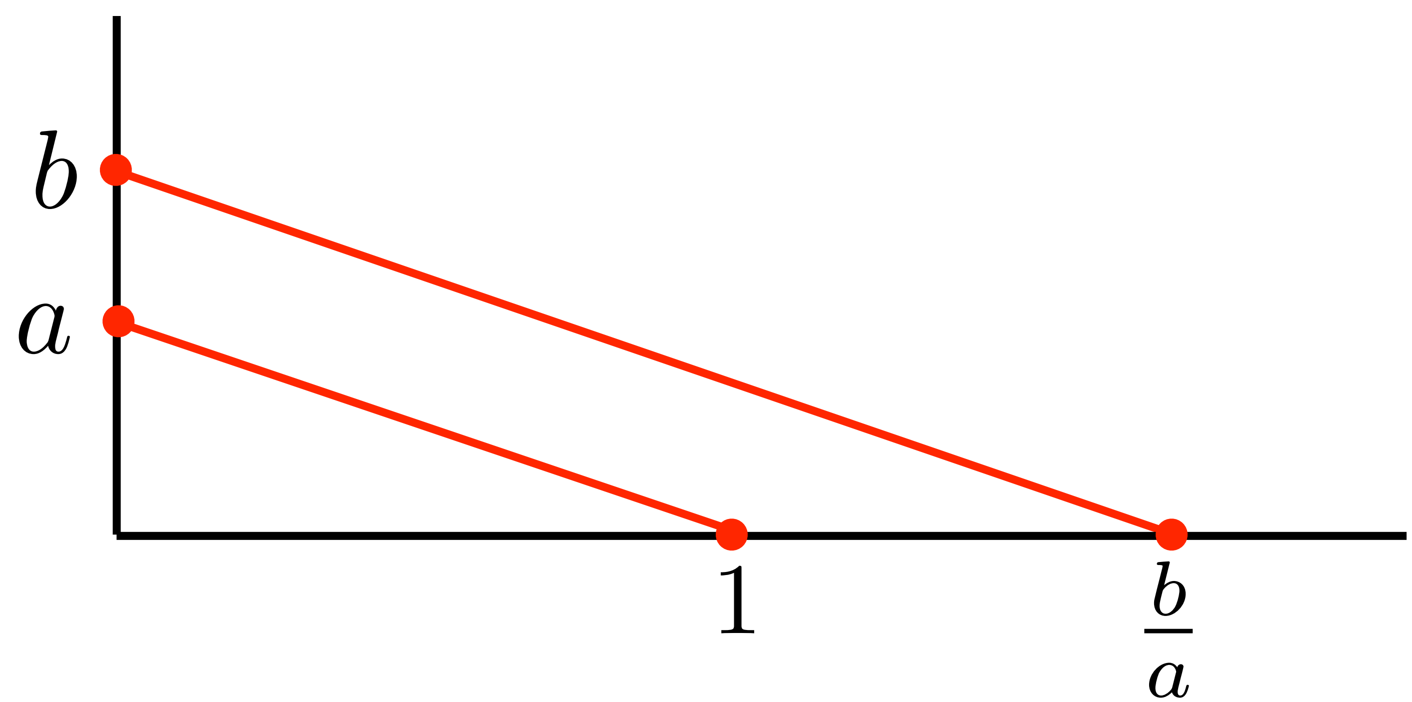


$$\frac{b}{a} = \frac{?}{1}$$



$$\frac{b}{a} = \frac{?}{1}$$

We can construct all fractions!



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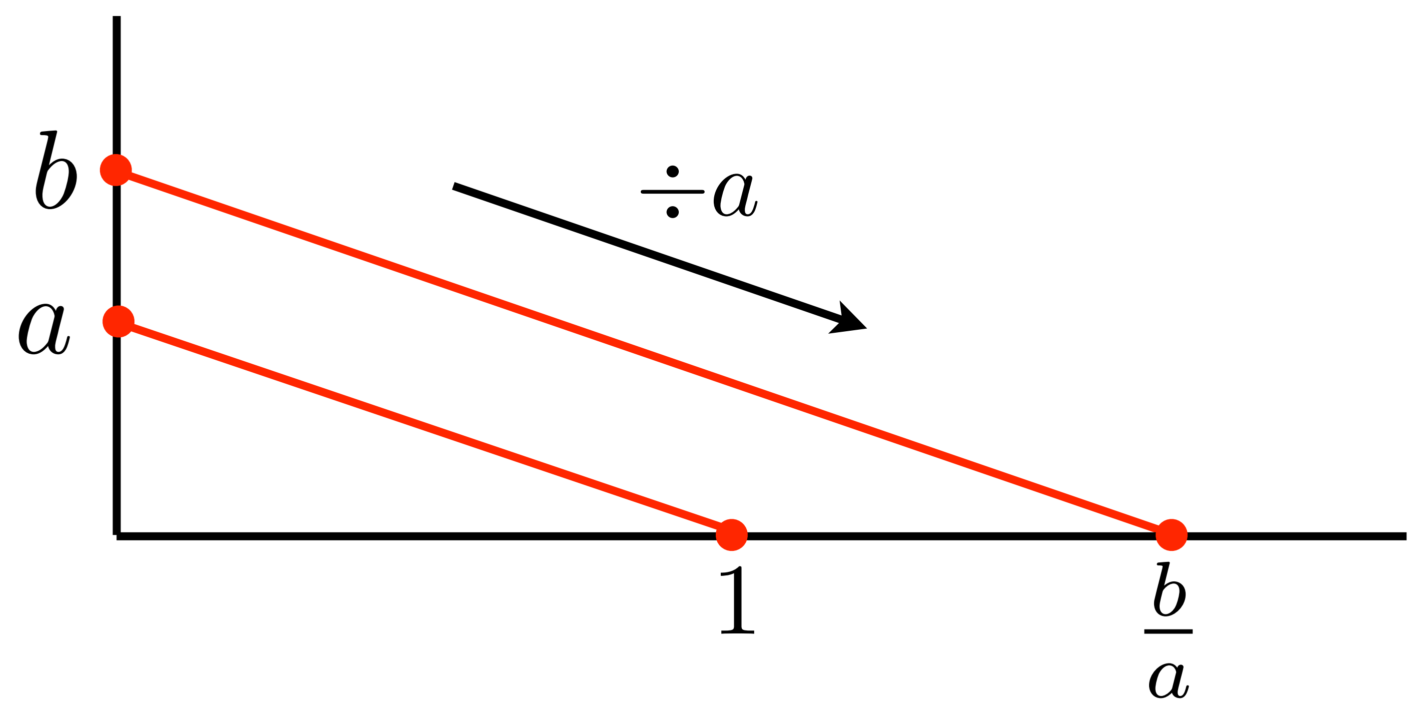

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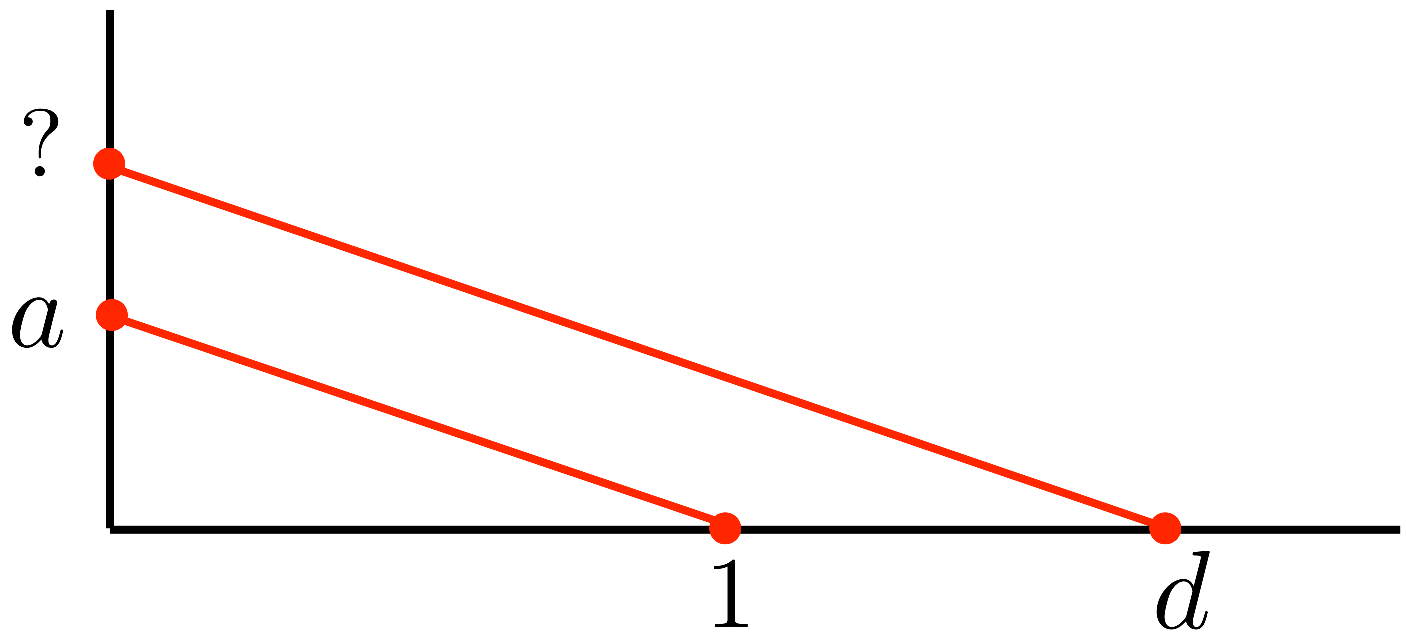

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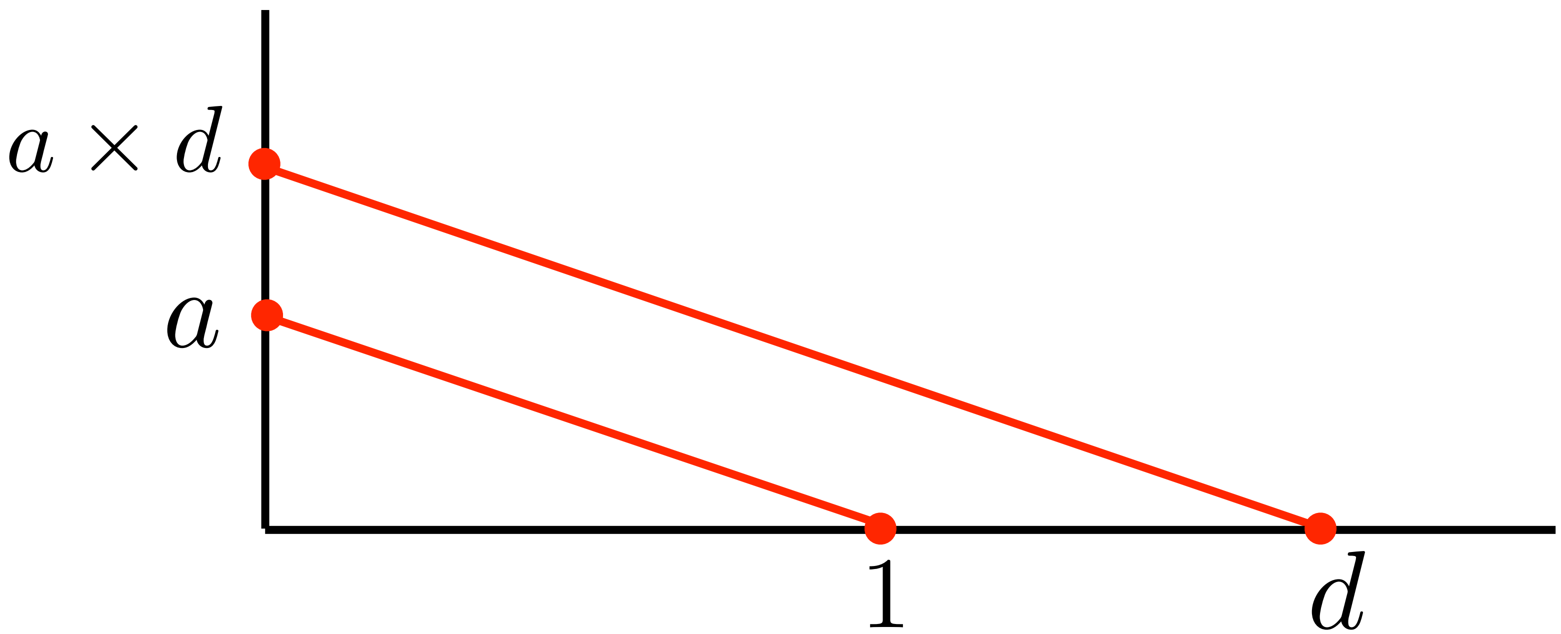

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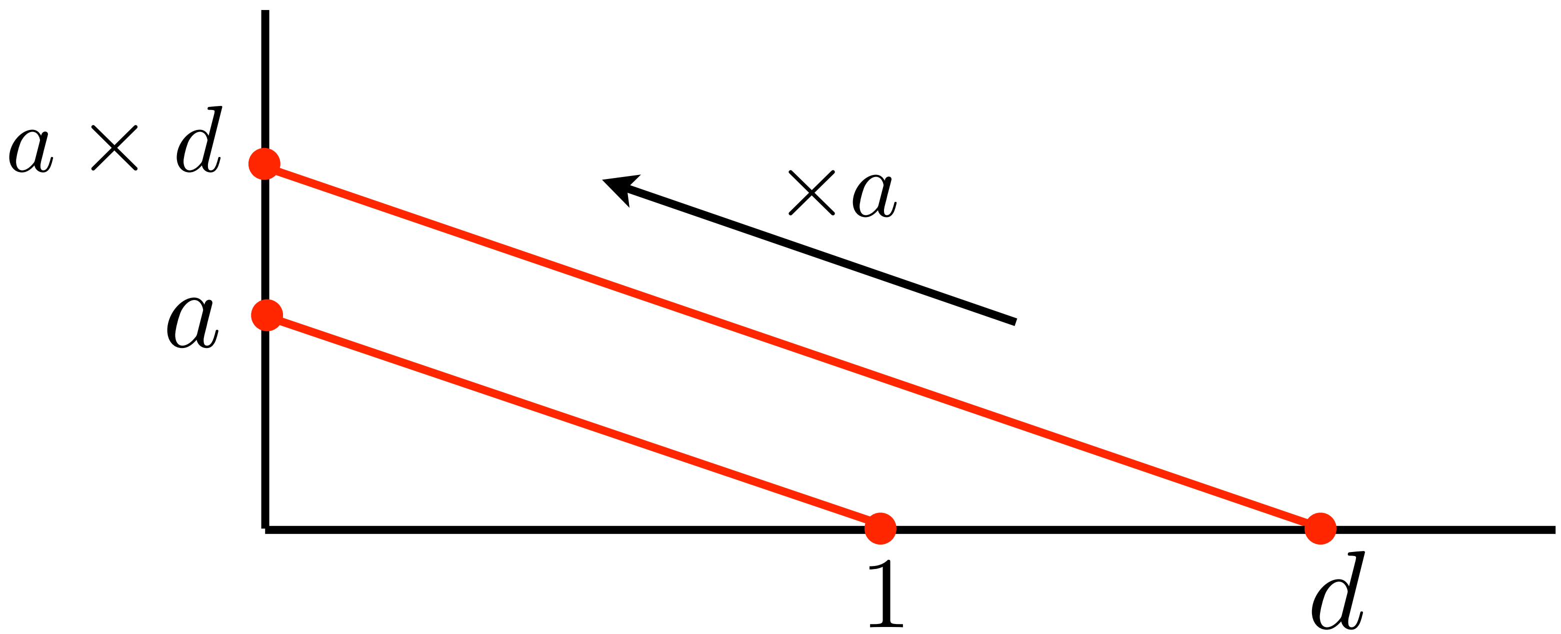
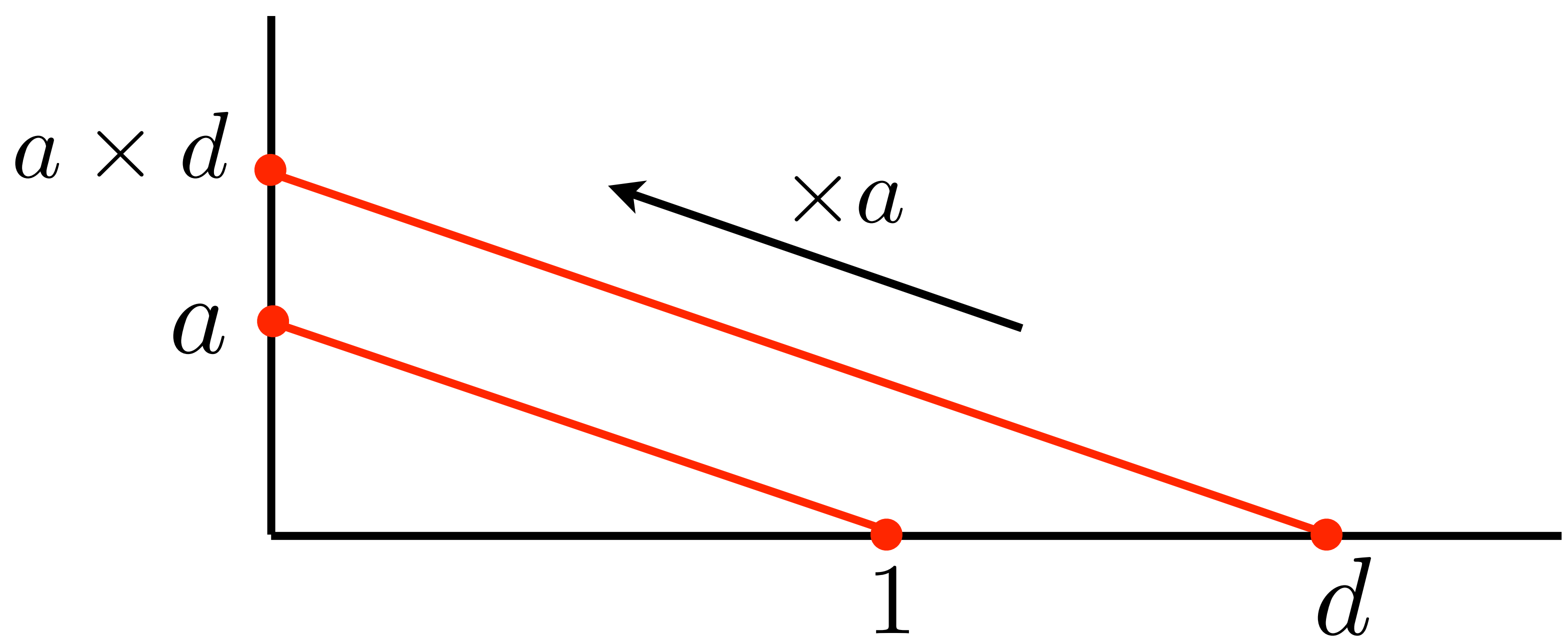


$$\frac{?}{a} = \frac{d}{1}$$

We can multiply, divide, add and subtract.



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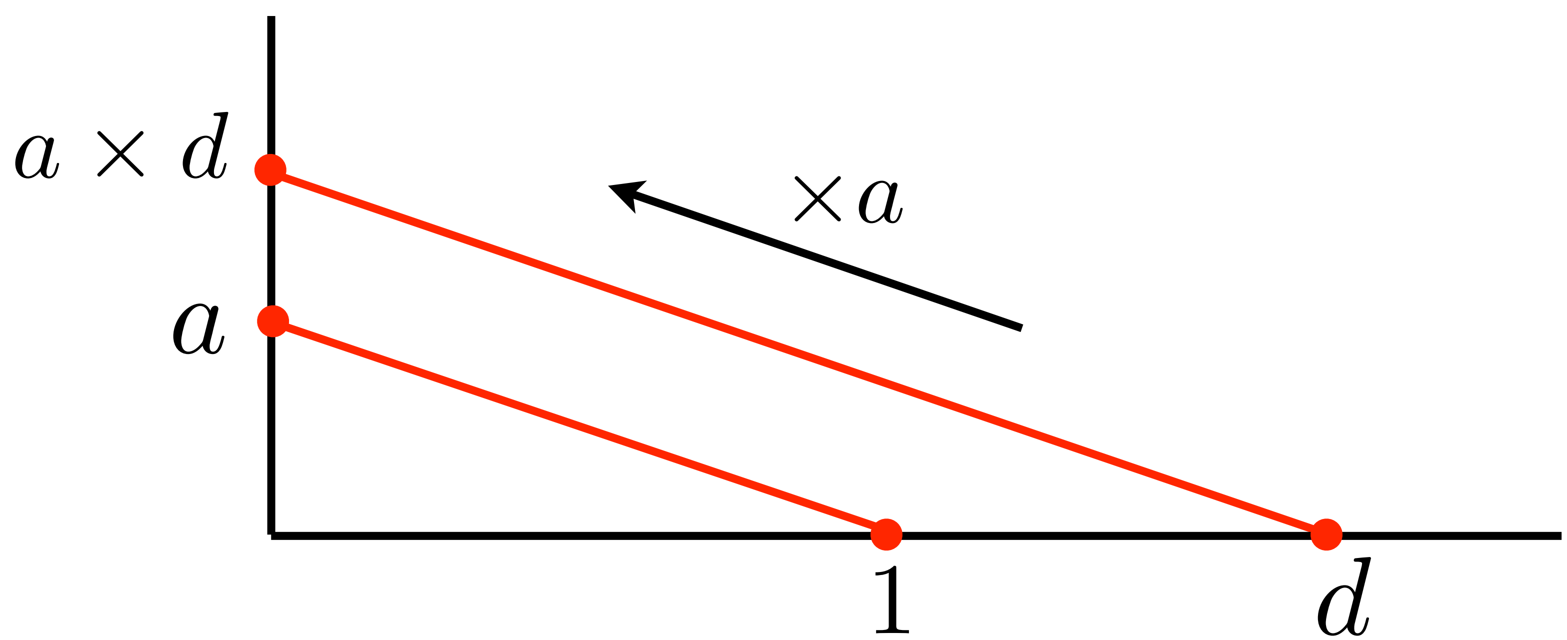
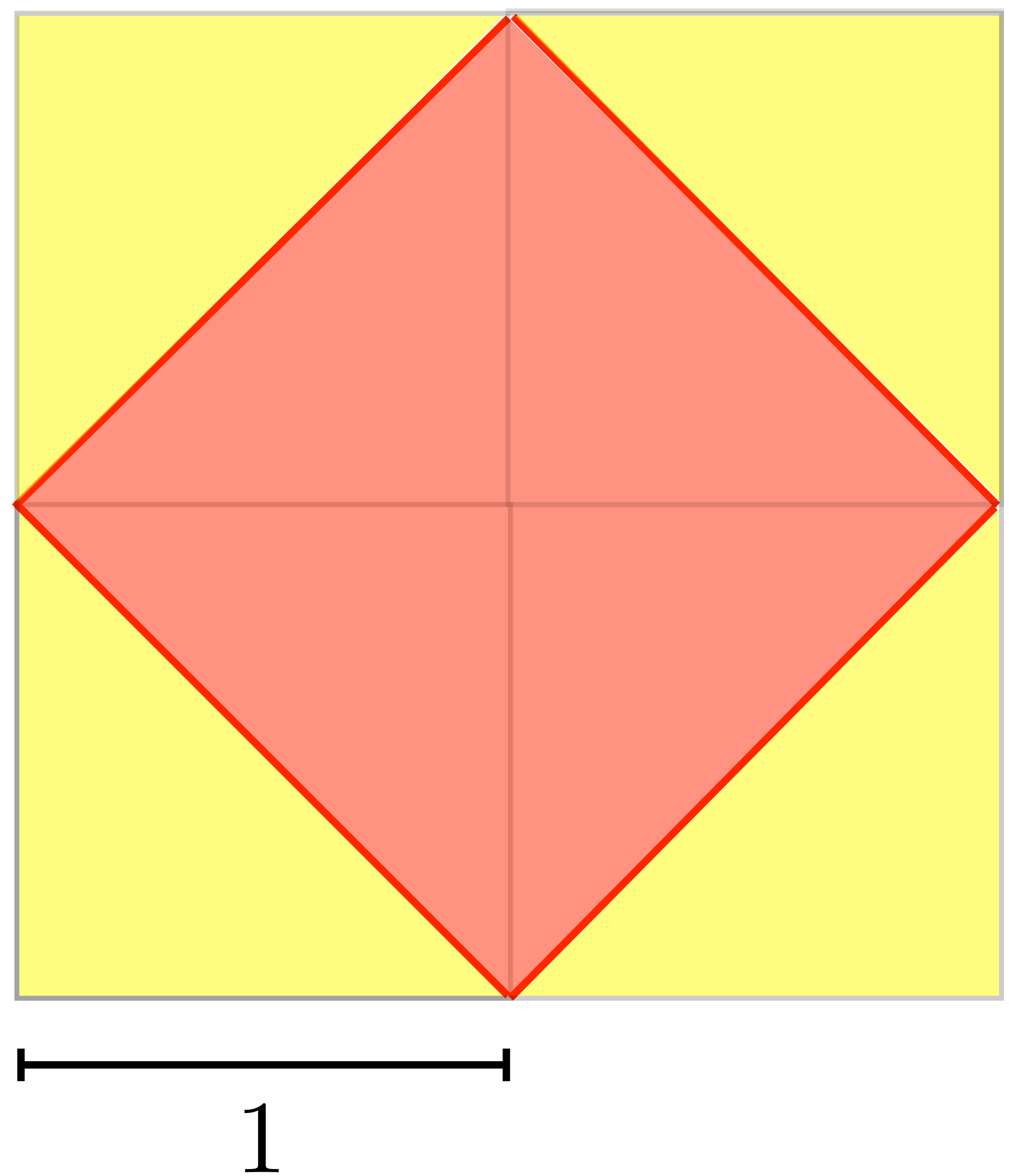
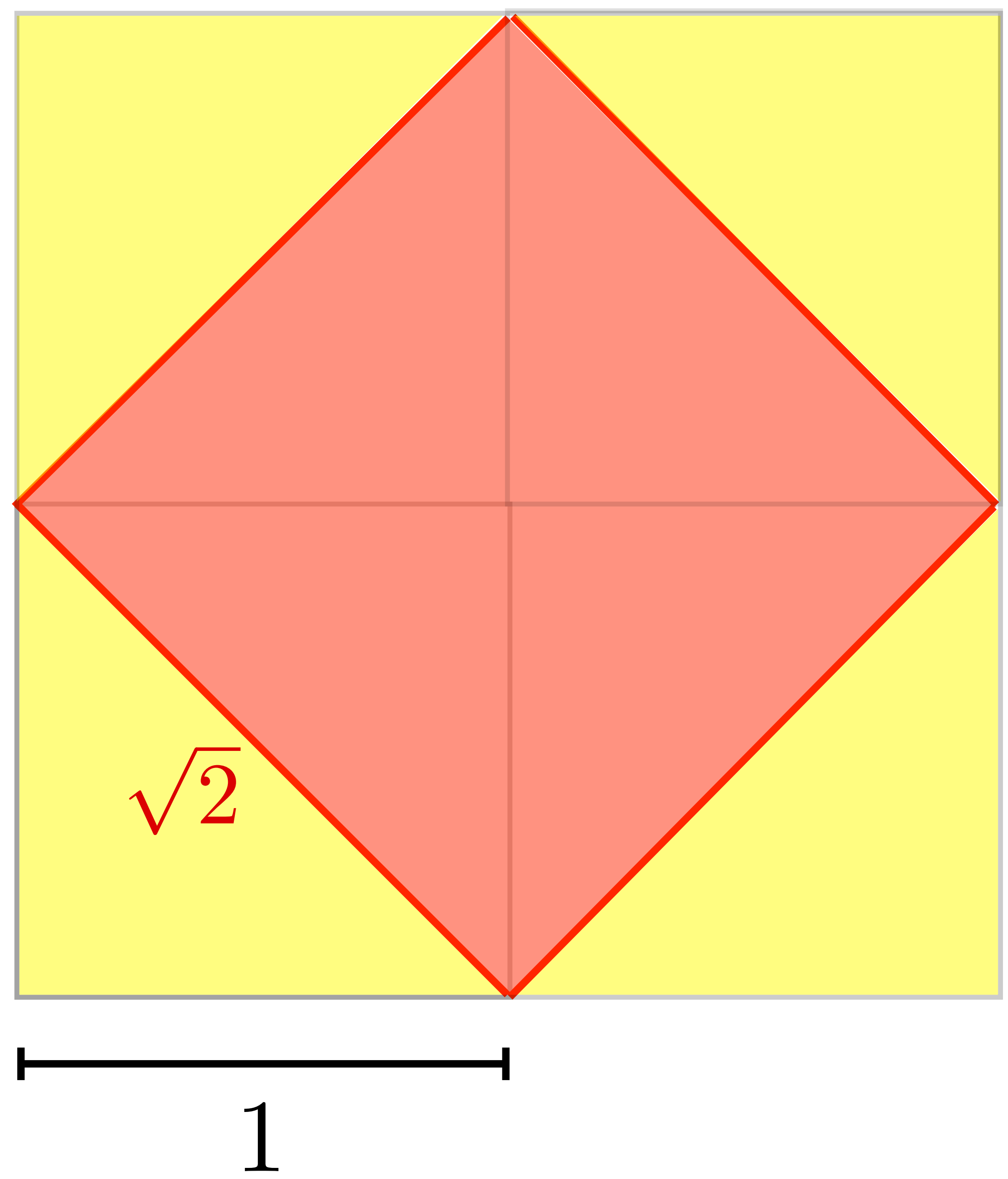


$$\frac{?}{a} = \frac{d}{1}$$

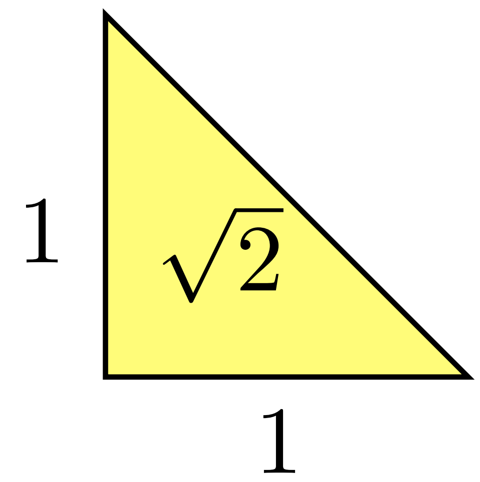
And what else?


1


$\sqrt{2}$
1

All square roots!

All square roots!



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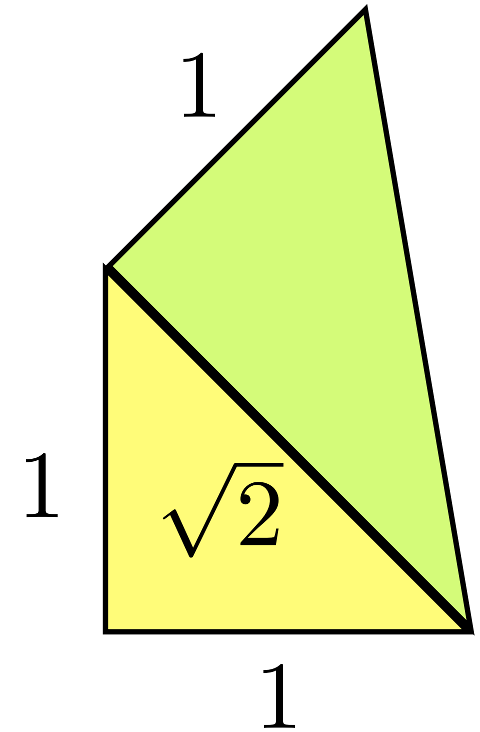

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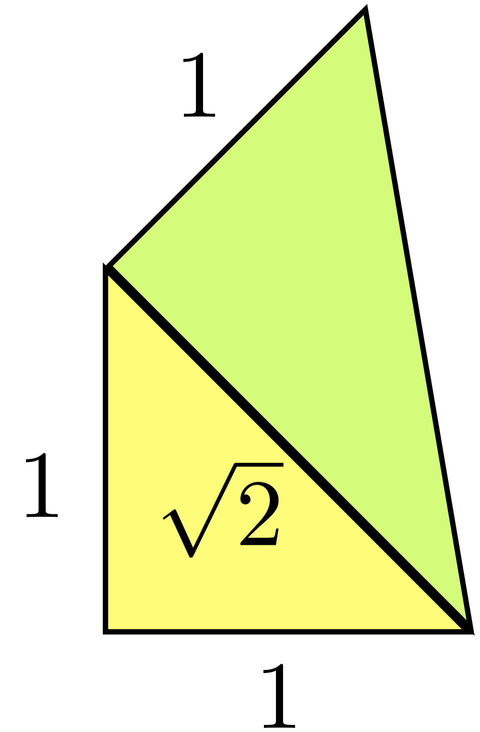
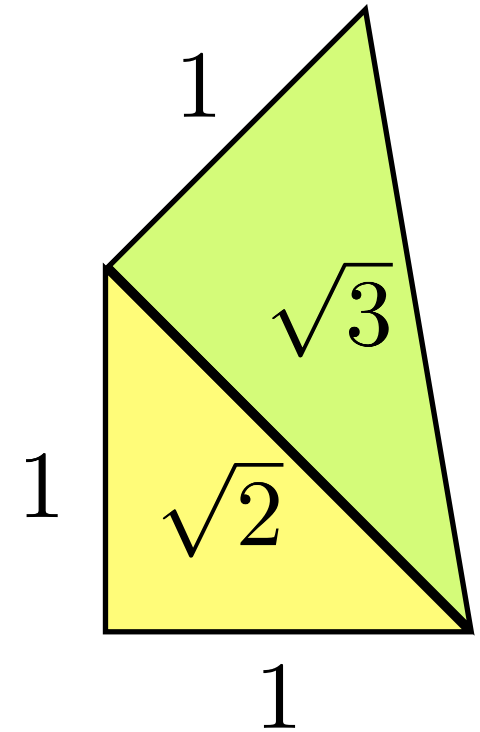


$$1^2 + \sqrt{2}^2 = ?^2$$

All square roots!



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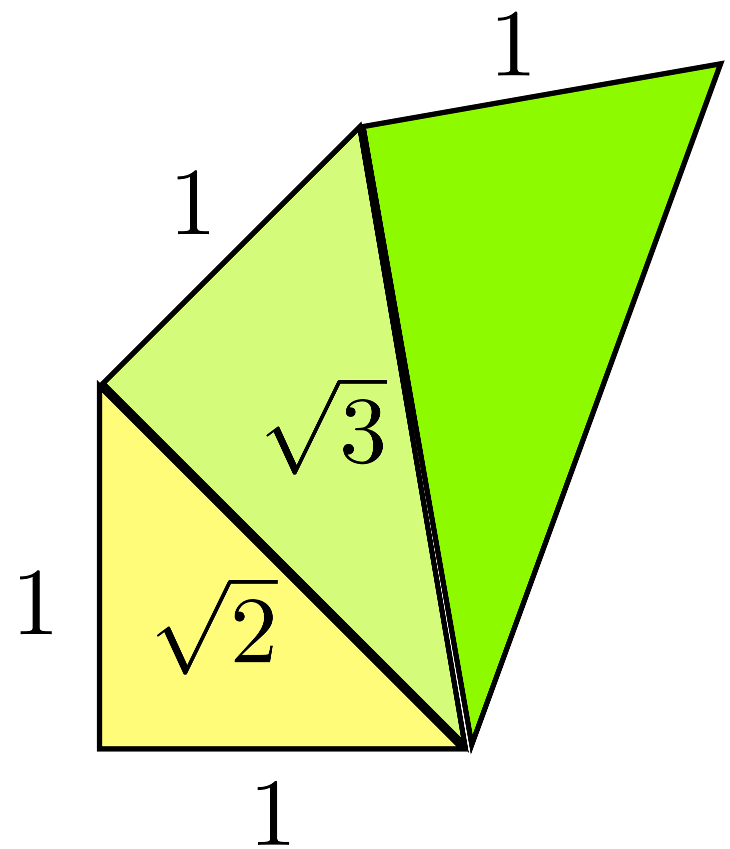

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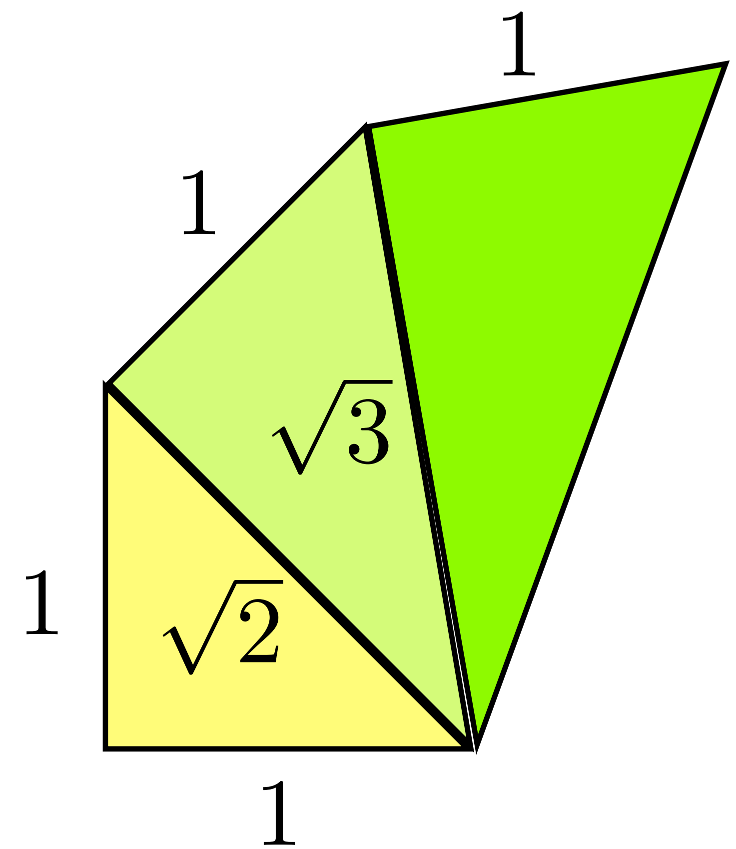
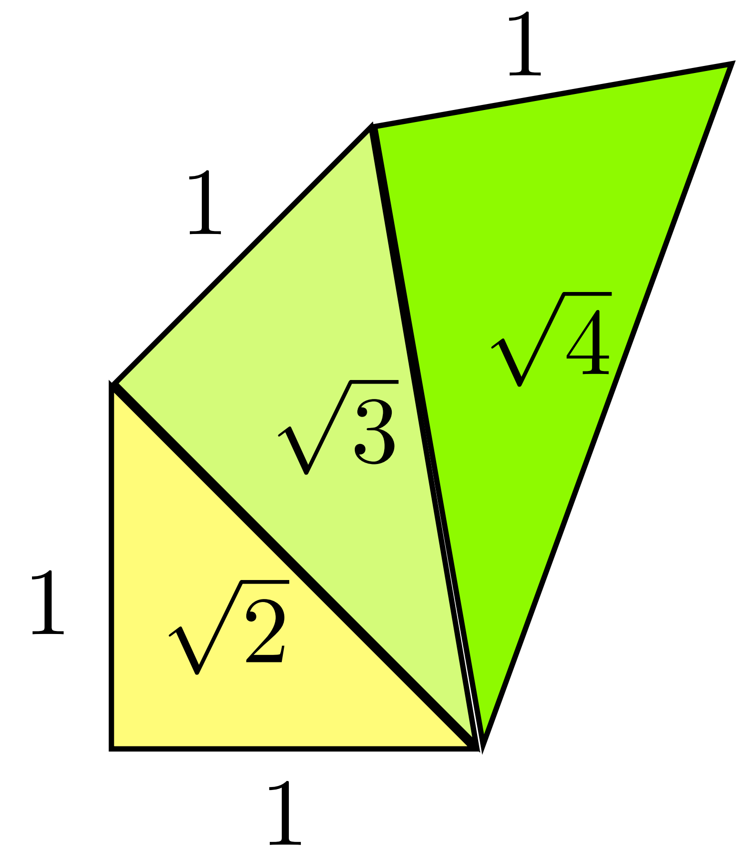


$$1^2 + \sqrt{2}^2 = ?^2$$

$$1^2 + \sqrt{3}^2 = ?^2$$

All square roots!



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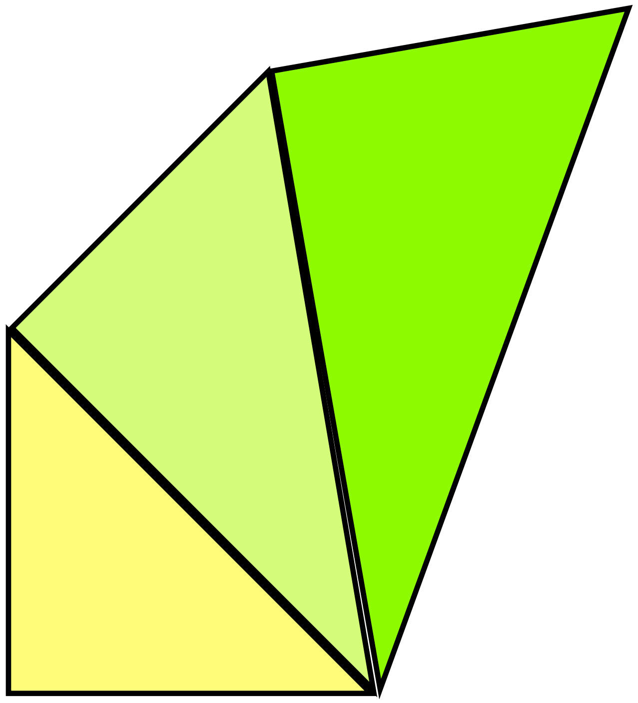

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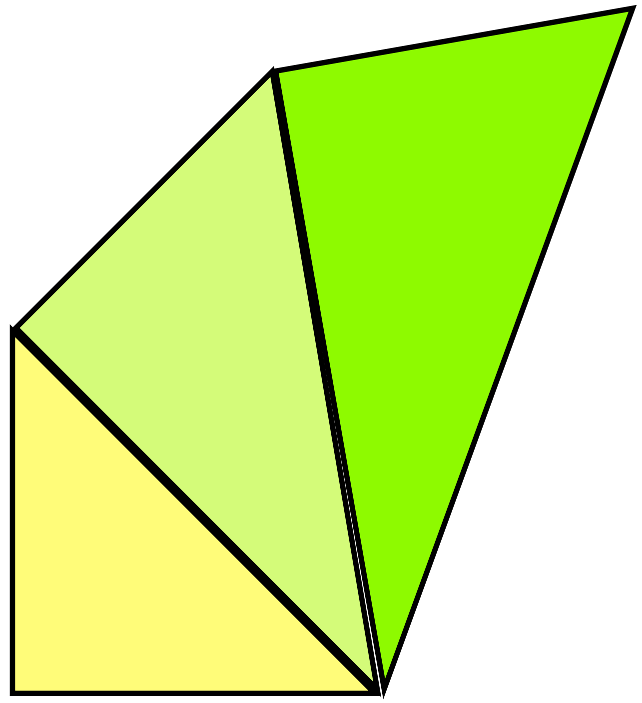
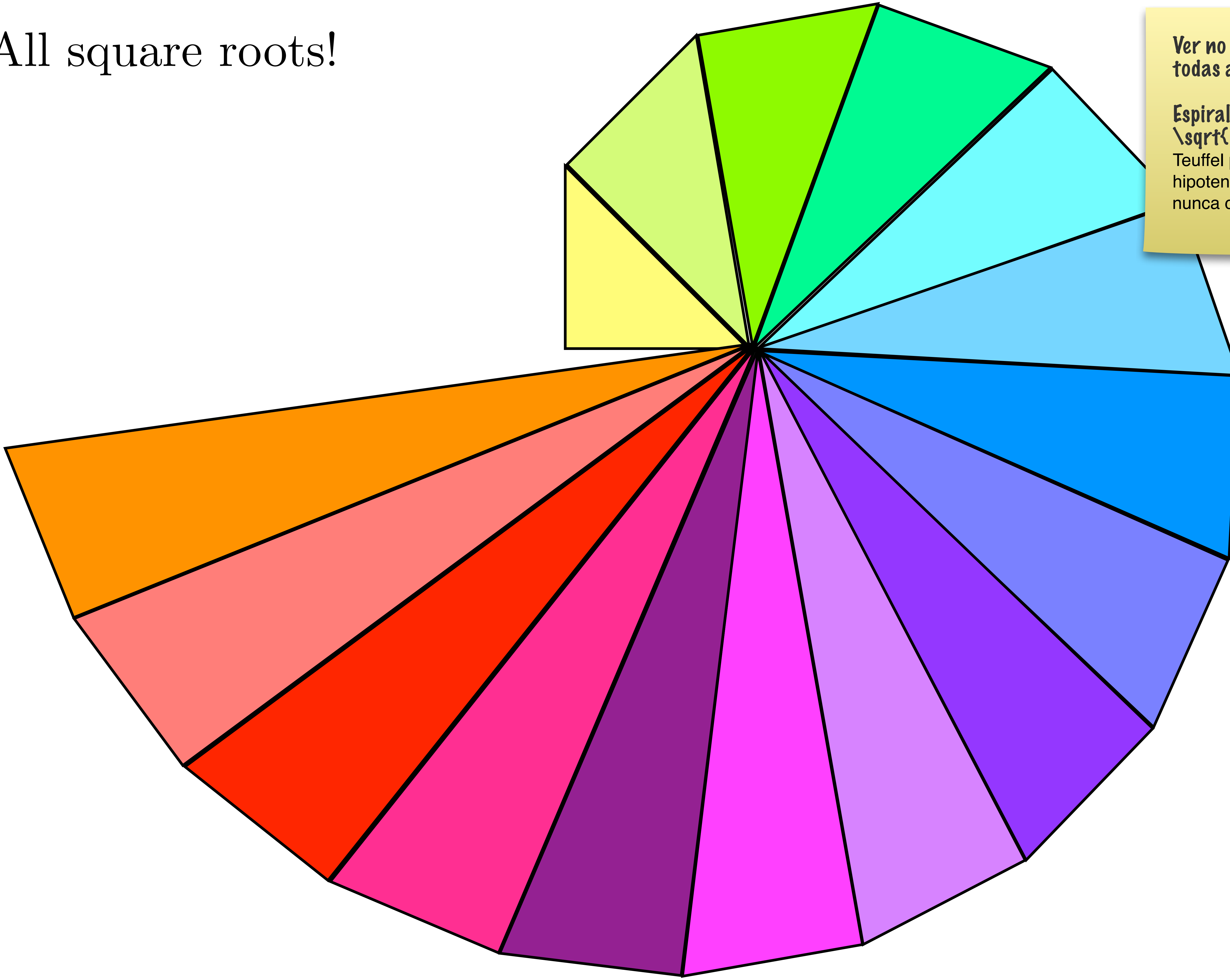


Ver no quadro que
todas as \sqrt{k}

Espiral de Teodoro
\sqrt{17}. Em 19
Teuffel provou que
hipotenusas da esp
nunca colidem.

All square roots!



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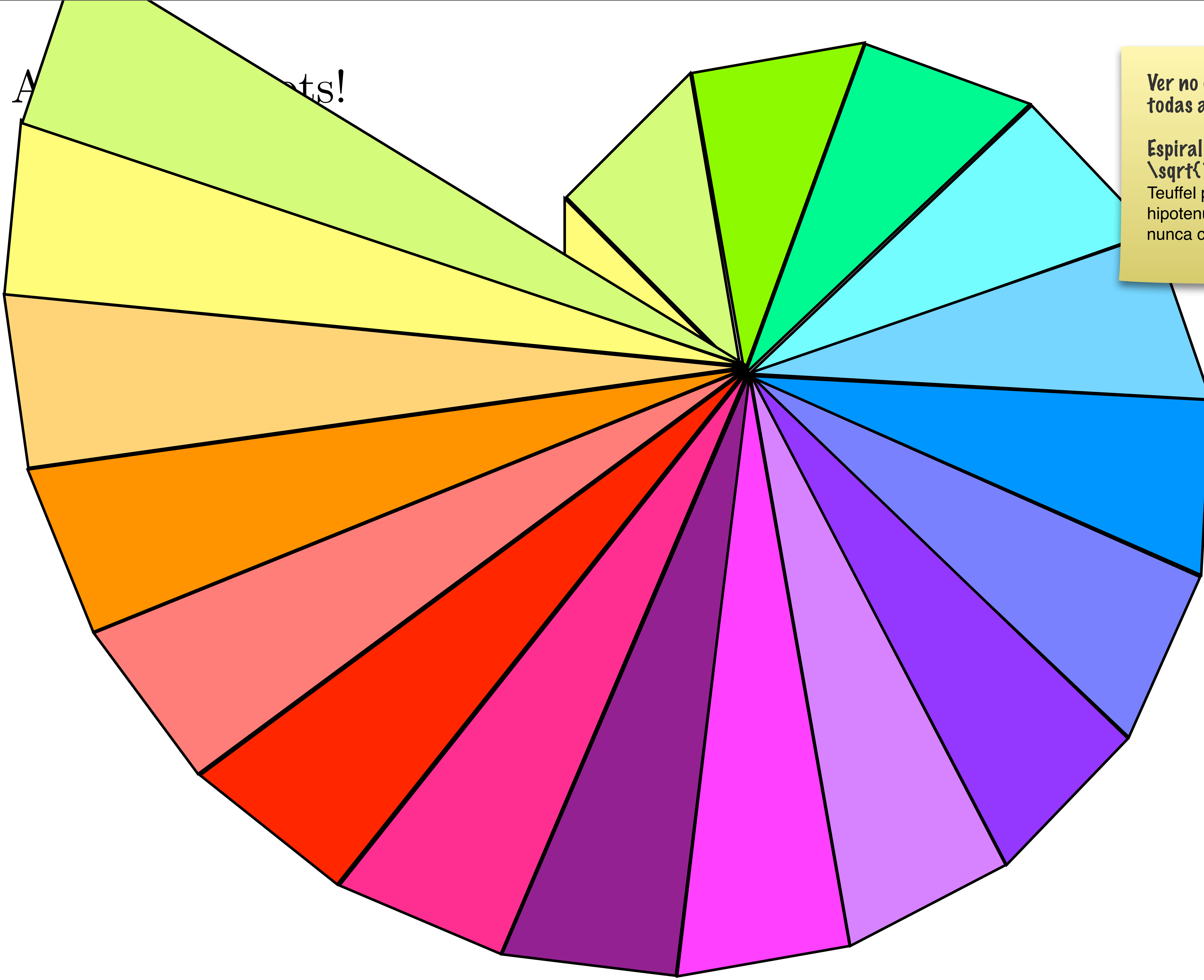
A ... ts!



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We can construct

We can construct $\sqrt{2}$

We can construct $\sqrt{2}$

$\sqrt{3}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$\sqrt{53}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$6\sqrt{7}$

$\sqrt{53}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$6\sqrt{7}$

$1 + 6\sqrt{7}$

$\sqrt{53}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$6\sqrt{7}$

$1 + 6\sqrt{7}$

$\sqrt{53}$

$\frac{2}{3} + 5\sqrt{11}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$1+6\sqrt{7}$

$\sqrt{53}$

$\frac{2}{3}+5\sqrt{11}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$-\frac{9}{13} + \frac{1}{37}\sqrt{10}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$1 + 6\sqrt{7}$

$\sqrt{53}$

$\frac{2}{3} + 5\sqrt{11}$

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$-\frac{9}{13} + \frac{1}{37}\sqrt{10}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$1 + 6\sqrt{7}$

$\frac{5}{6}\sqrt{3} - 2\sqrt{\frac{10}{7}}$

$\sqrt{53}$

$\frac{2}{3} + 5\sqrt{11}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$$-\frac{9}{13}+\frac{1}{37}\sqrt{10}$$

$$\frac{4-2\sqrt{15}}{22}$$

$$6\sqrt{7}$$

$$1+6\sqrt{7}$$

$$\frac{5}{6}\sqrt{3}-2\sqrt{\frac{10}{7}}$$

$$\frac{8}{\sqrt{5}}$$

$$\sqrt{53}$$

$$\frac{2}{3}+5\sqrt{11}$$

We can construct $\sqrt{2}$

$\sqrt{3}$

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$$\sqrt{\frac{23}{41}}$$

$$\sqrt{53}$$

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$6\sqrt{7}$

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$\sqrt{\frac{23}{41}}$

$\sqrt{53}$

$\frac{\sqrt{2}}{\sqrt{3}}$

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$-4+12\sqrt{\frac{19}{92}}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$-\frac{9}{13} + \frac{1}{37}\sqrt{10}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$\frac{3-\sqrt{22}}{-22+\frac{4}{5}\sqrt{67}}$

$1 + 6\sqrt{7}$

$\frac{5}{6}\sqrt{3} - 2\sqrt{\frac{10}{7}}$

$\frac{8}{\sqrt{5}}$

$\sqrt{\frac{23}{41}}$

$\sqrt{53}$

$\frac{\sqrt{2}}{\sqrt{3}}$

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$\frac{3-\sqrt{22}}{-22+\frac{4}{5}\sqrt{67}}$

$\frac{5}{6}\sqrt{3}-2\sqrt{\frac{10}{7}}$

$\frac{8}{\sqrt{5}}$

$\sqrt{\frac{23}{41}}$

$\sqrt{53}$

$\frac{\sqrt{2}}{\sqrt{3}}$

$\frac{2}{3}+5\sqrt{11}$

$-4+12\sqrt{\frac{19}{92}}$

$\frac{\sqrt{\frac{21}{100}}}{20}-\frac{52}{5}\sqrt{26}+\frac{43}{\sqrt{7}}+\frac{38}{83}\sqrt{99}$

We can construct $\sqrt{2}$

$\sqrt{3}$

$-\frac{9}{13} + \frac{1}{37}\sqrt{10}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$1 + 6\sqrt{7}$

$\frac{3-\sqrt{22}}{-22+\frac{4}{5}\sqrt{67}}$

$\frac{5}{6}\sqrt{3} - 2\sqrt{\frac{10}{7}}$

etc.

$\frac{8}{\sqrt{5}}$

$\sqrt{\frac{23}{41}}$

$\sqrt{53}$

$\frac{\sqrt{2}}{\sqrt{3}}$

$\frac{2}{3} + 5\sqrt{11}$

$-4 + 12\sqrt{\frac{19}{92}}$

$\frac{\sqrt{\frac{21}{100}}}{20} - \frac{52}{5}\sqrt{26} + \frac{43}{\sqrt{7}} + \frac{38}{83}\sqrt{99}$

We can construct $\sqrt{2}$

$\sqrt{3}$

And what about $\sqrt{1+3\sqrt{2}}$?

$-\frac{9}{13}+\frac{1}{37}\sqrt{10}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$1+6\sqrt{7}$

$\frac{3-\sqrt{22}}{-22+\frac{4}{5}\sqrt{67}}$

$\frac{5}{6}\sqrt{3}-2\sqrt{\frac{10}{7}}$

etc.

$\frac{8}{\sqrt{5}}$

$\sqrt{\frac{23}{41}}$

$\sqrt{53}$

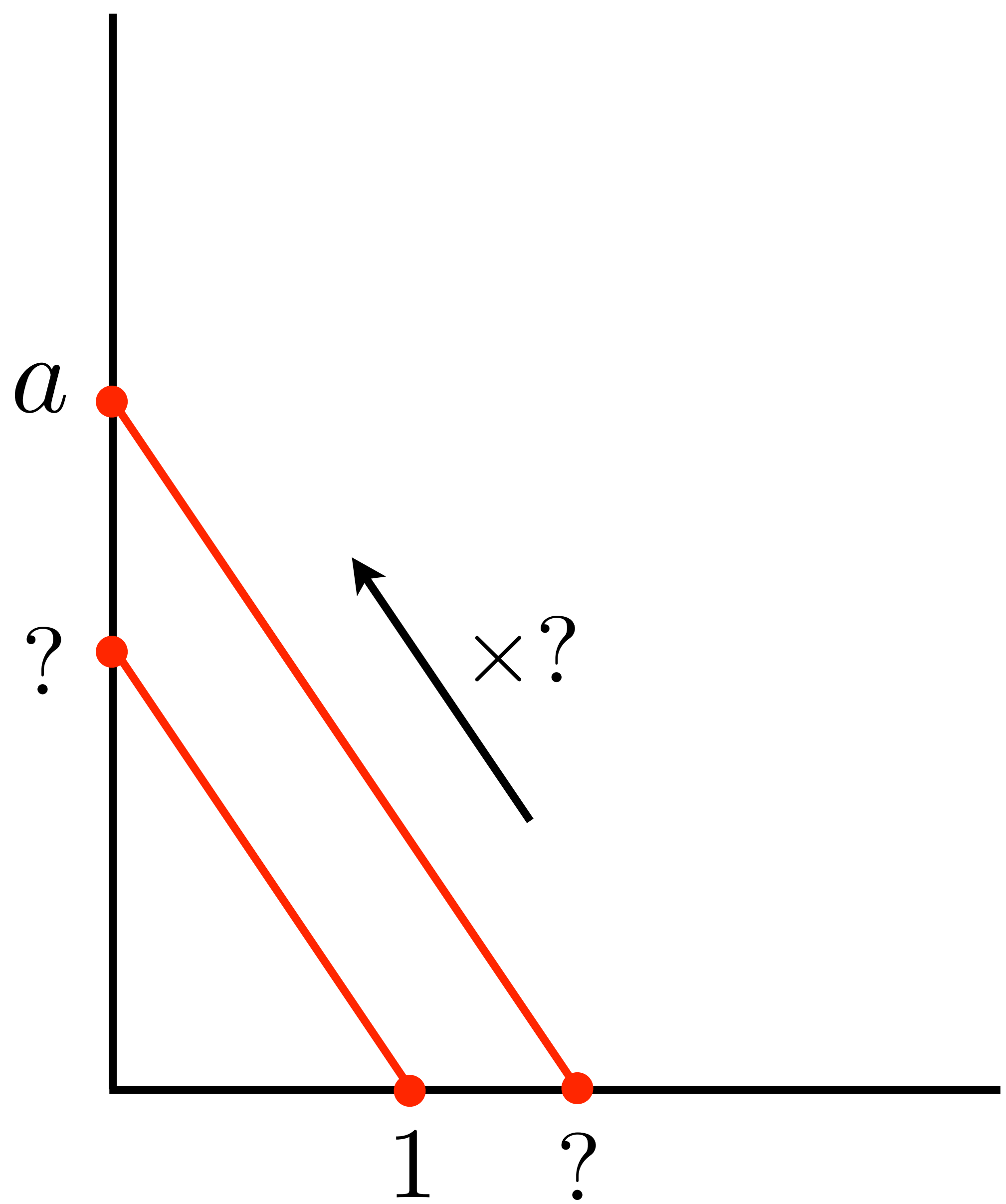
$\frac{\sqrt{2}}{\sqrt{3}}$

$\frac{2}{3}+5\sqrt{11}$

$-4+12\sqrt{\frac{19}{92}}$

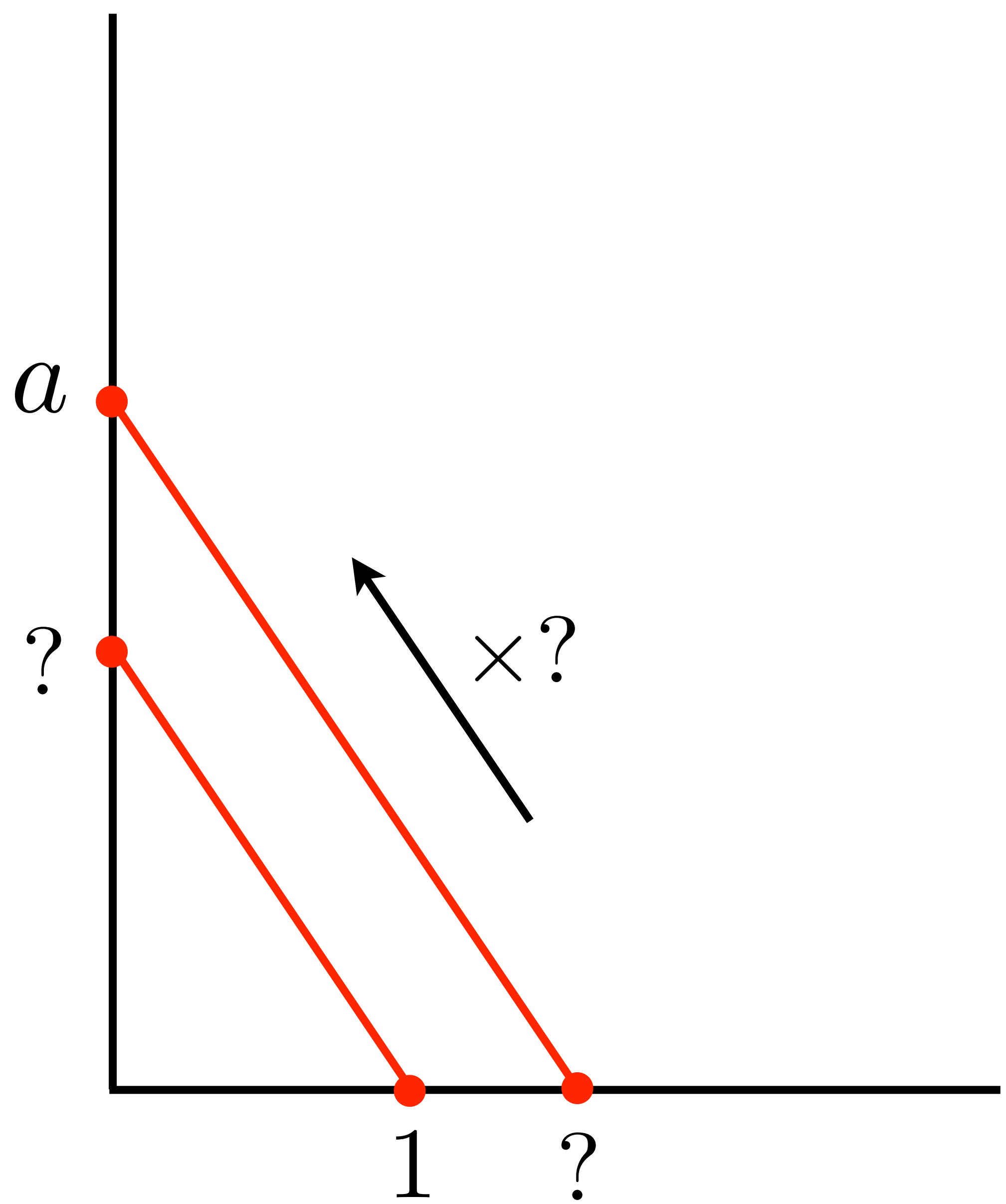
$\frac{\sqrt{\frac{21}{100}}}{20}-\frac{52}{5}\sqrt{26}+\frac{43}{\sqrt{7}}+\frac{38}{83}\sqrt{99}$

All square roots?



$$\frac{a}{?} = \frac{?}{1}$$

All square roots?

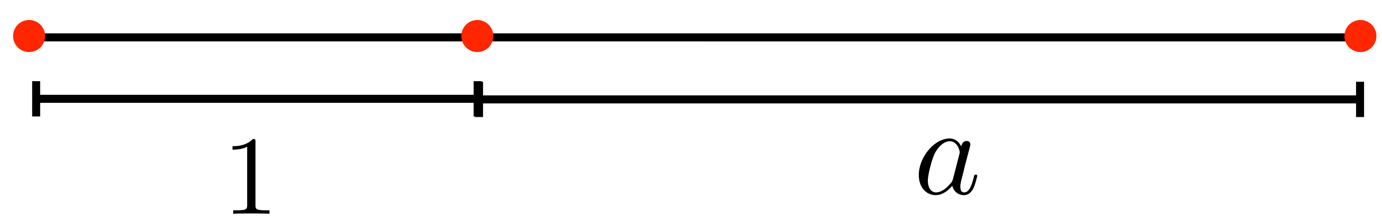


$$\frac{a}{?} = \frac{?}{1}$$

$$? = \sqrt{a}$$

Sunday, March 3, 13

All square roots!



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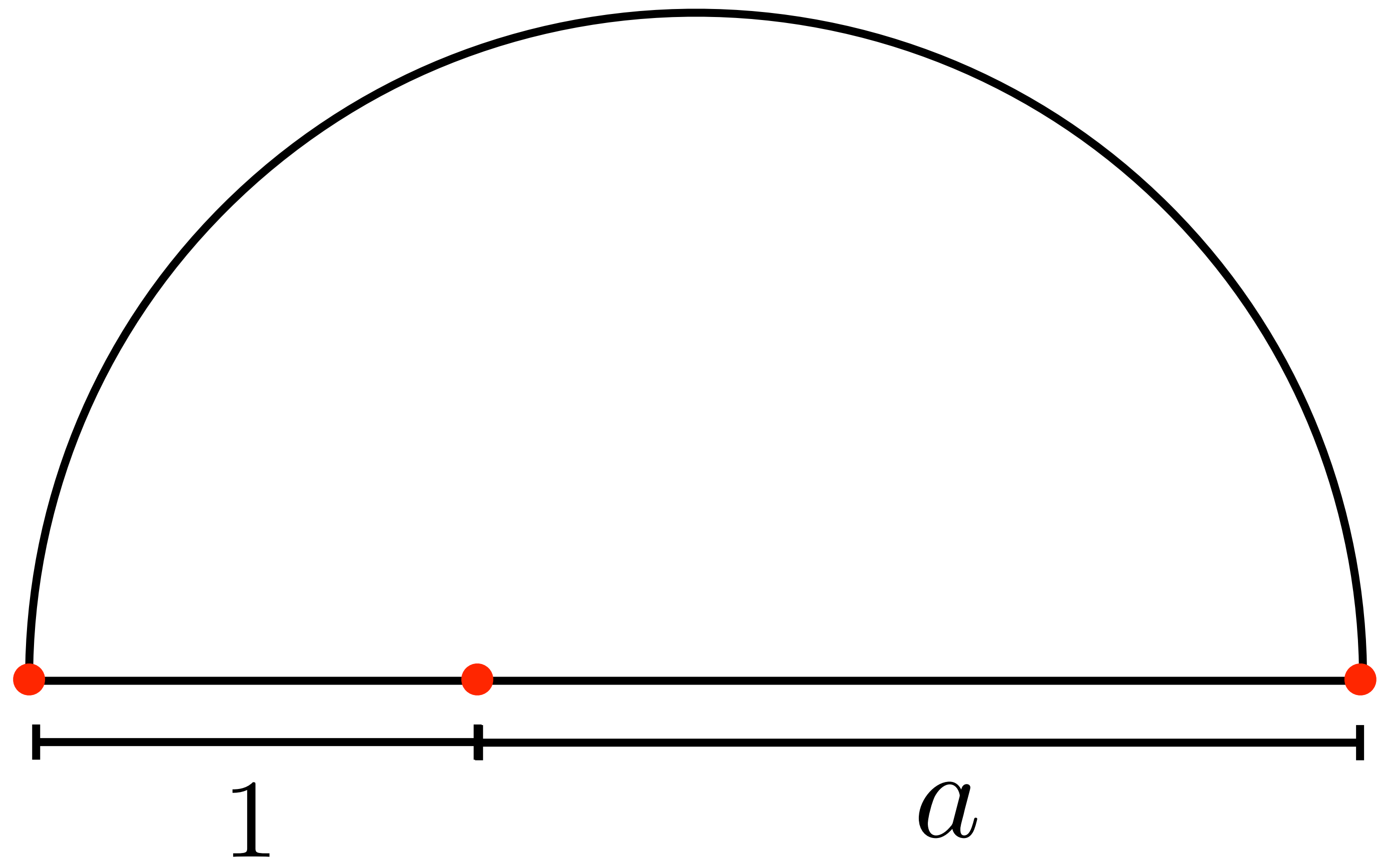

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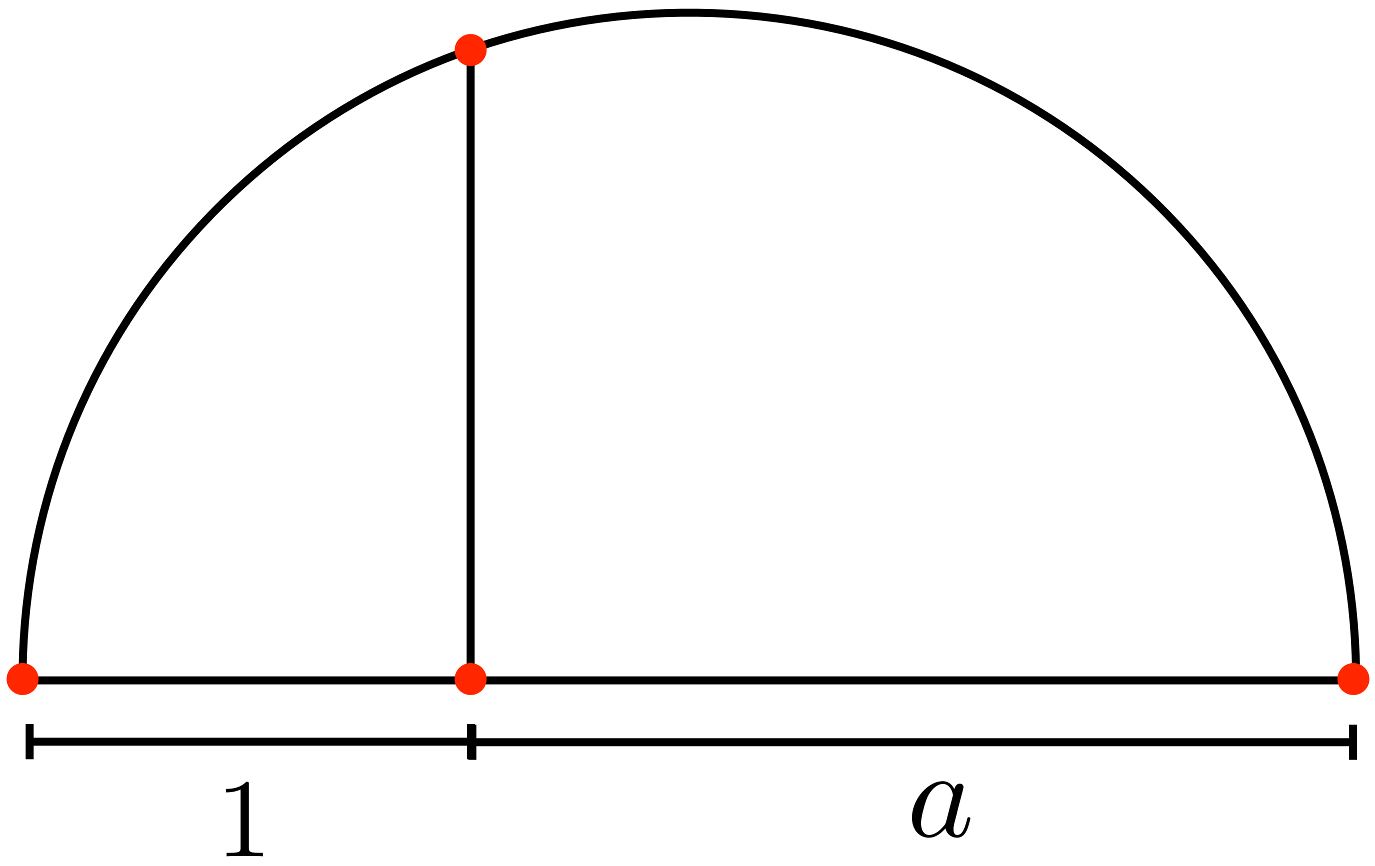

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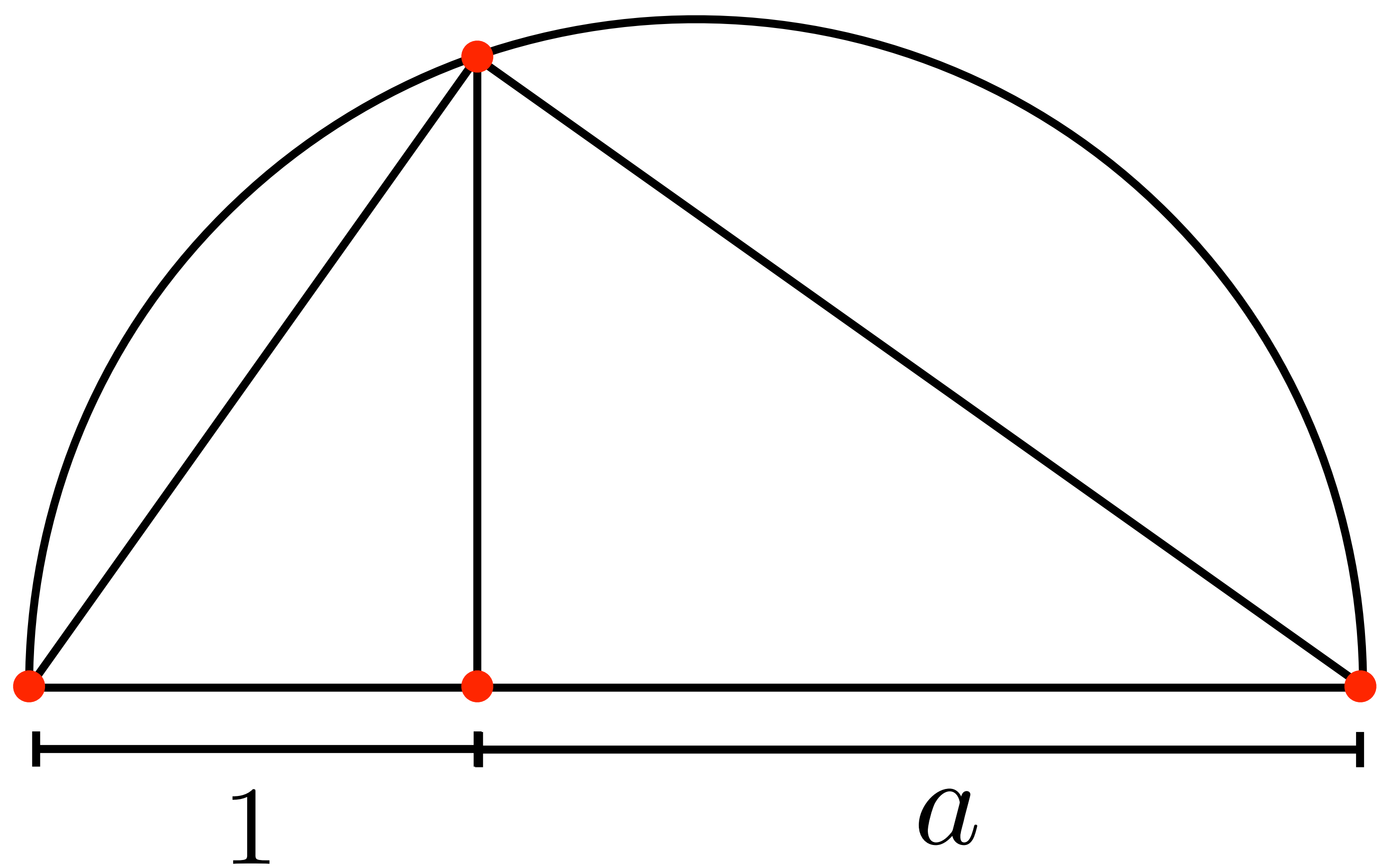

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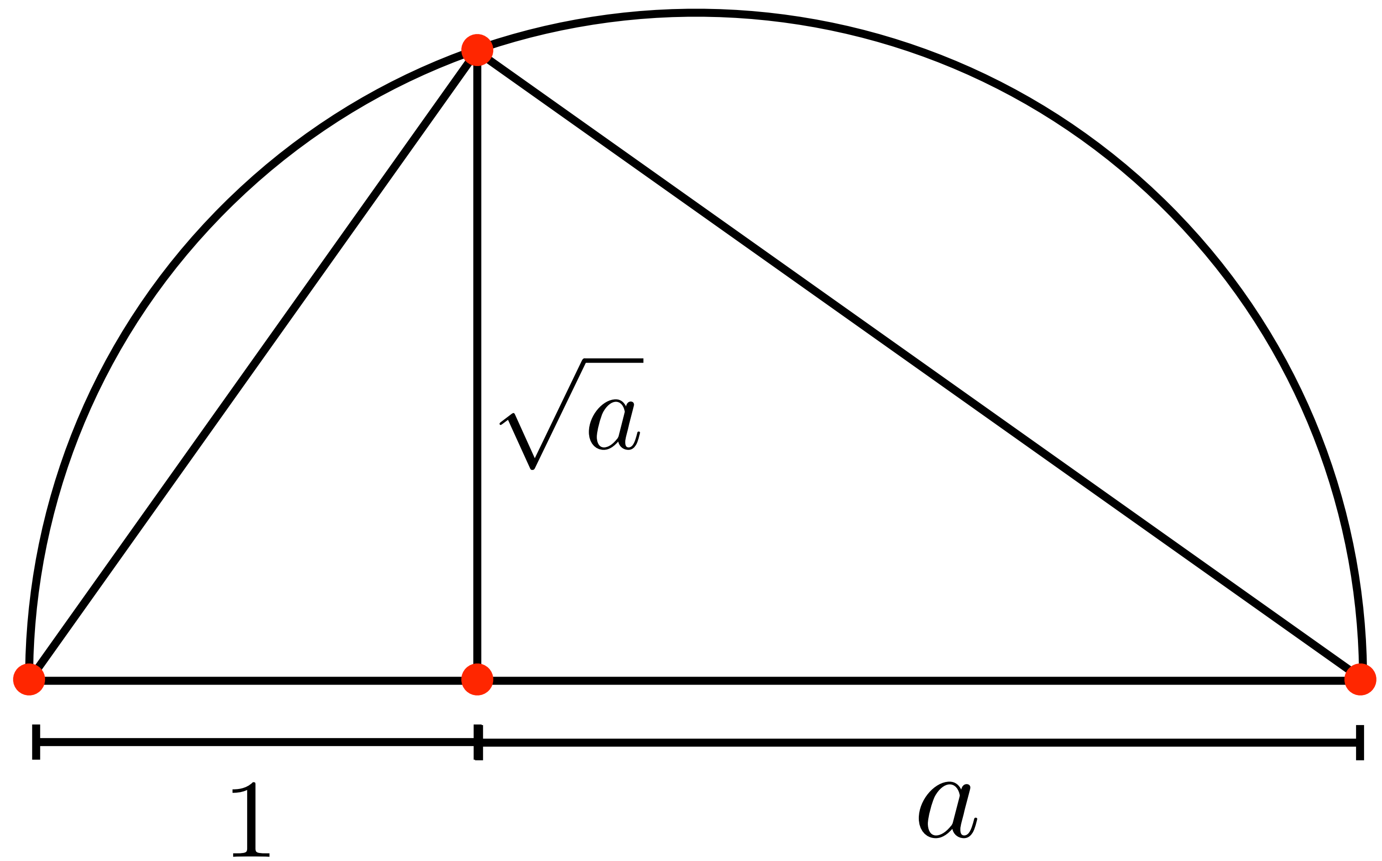

We can construct $\sqrt{2}$

$\sqrt{3}$

$-\frac{9}{13} + \frac{1}{37}\sqrt{10}$

$\frac{4-2\sqrt{15}}{22}$

$6\sqrt{7}$

$1 + 6\sqrt{7}$

$\frac{3-\sqrt{22}}{-22+\frac{4}{5}\sqrt{67}}$

$\frac{5}{6}\sqrt{3} - 2\sqrt{\frac{10}{7}}$

etc.

$\frac{8}{\sqrt{5}}$

$\sqrt{\frac{23}{41}}$

$\sqrt{53}$

$\frac{\sqrt{2}}{\sqrt{3}}$

$\frac{2}{3} + 5\sqrt{11}$

$-4 + 12\sqrt{\frac{19}{92}}$

$\frac{\sqrt{\frac{21}{100}}}{20} - \frac{52}{5}\sqrt{26} + \frac{43}{\sqrt{7}} + \frac{38}{83}\sqrt{99}$

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$\sqrt[8]{47}$

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And what else?

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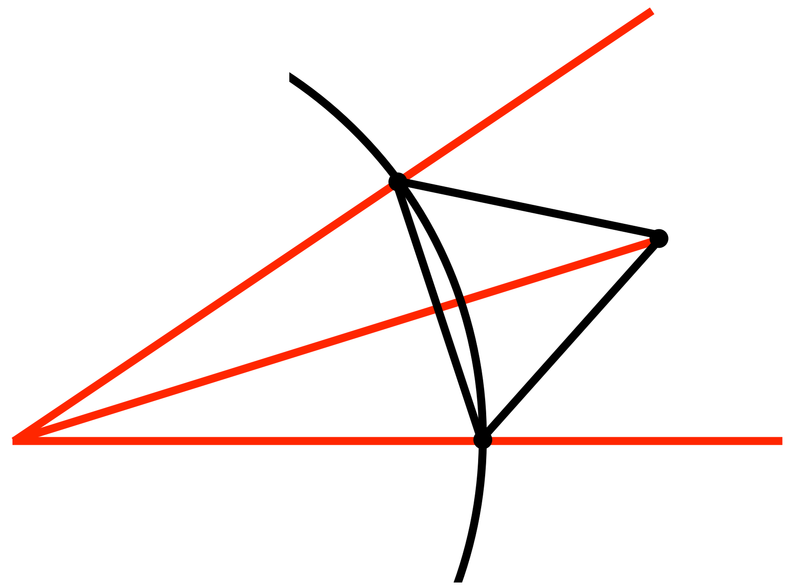
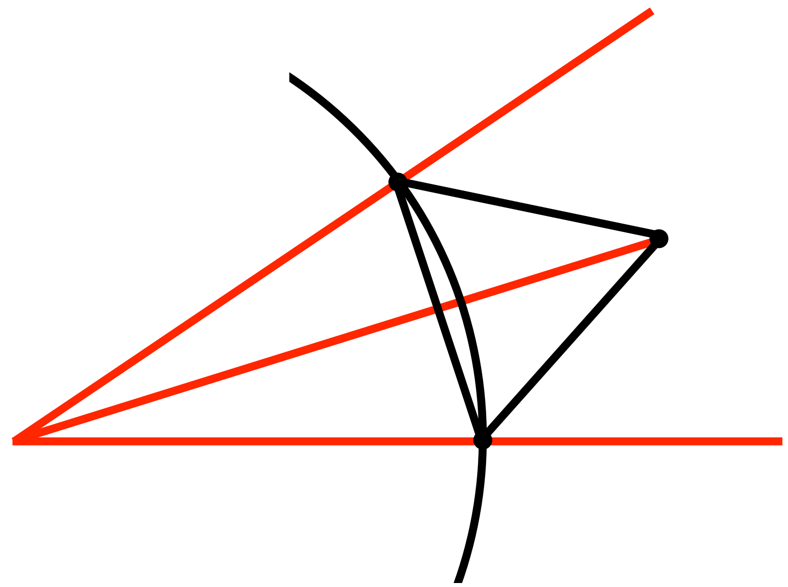
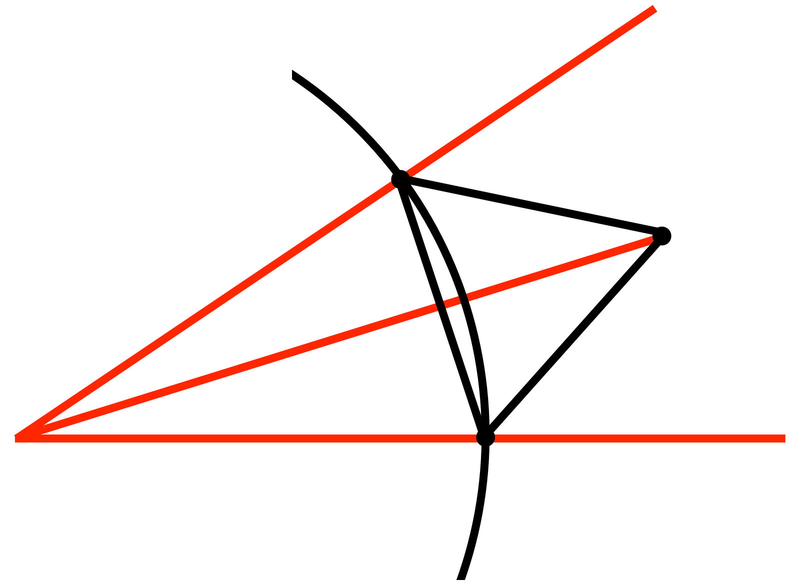
And what else? Nothing else! $\quad \frac{\sqrt{\frac{21}{100}}}{20}-\frac{52}{5}\sqrt{26}+\frac{43}{\sqrt{7}}+\frac{38}{83}\sqrt{99}$





In general, it's impossible to trisect an angle θ with ruler and compass.



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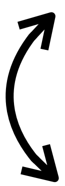
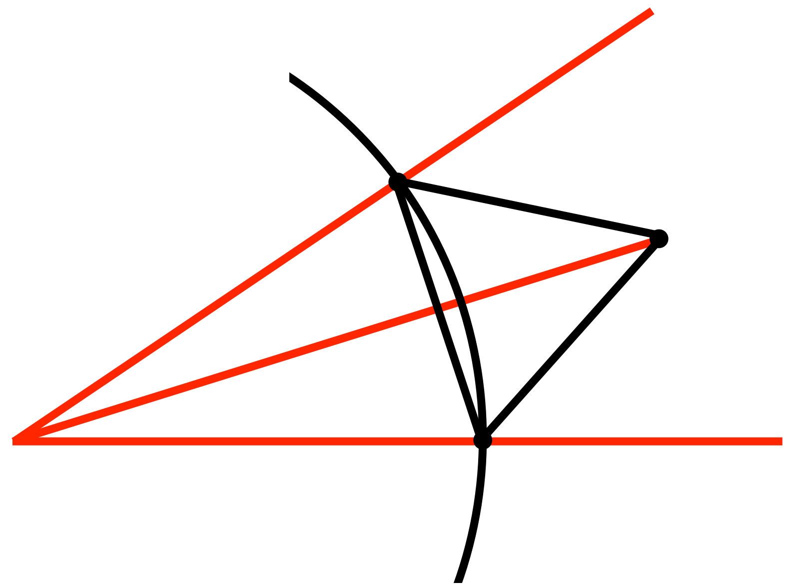


In general, the number $\cos \frac{\theta}{3}$ is not constructible with ruler and compass.



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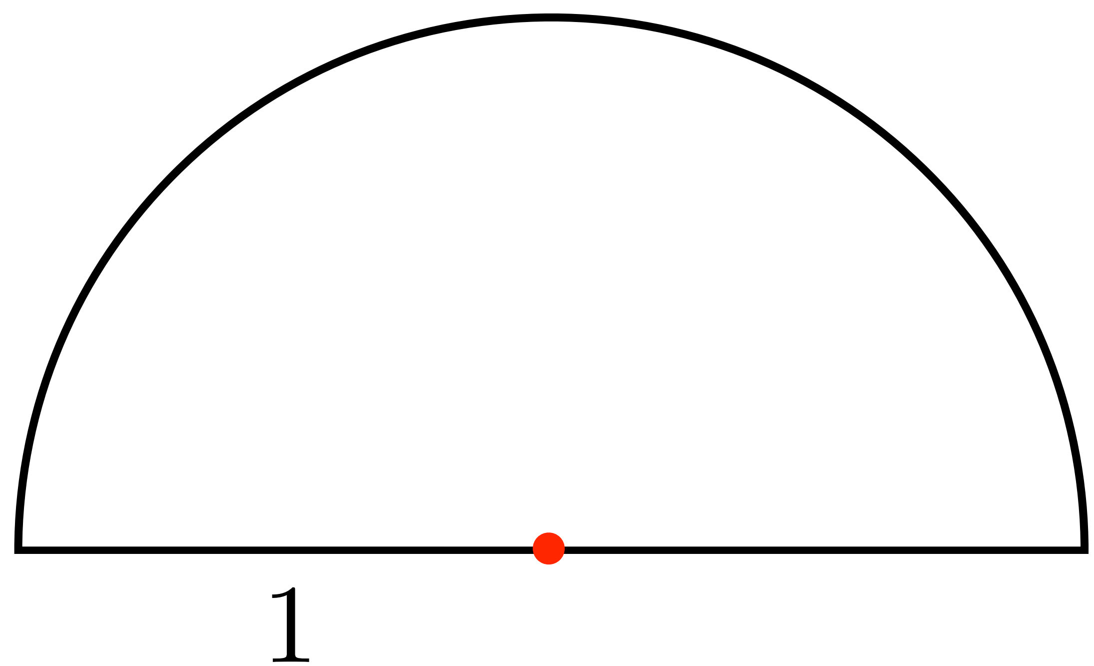

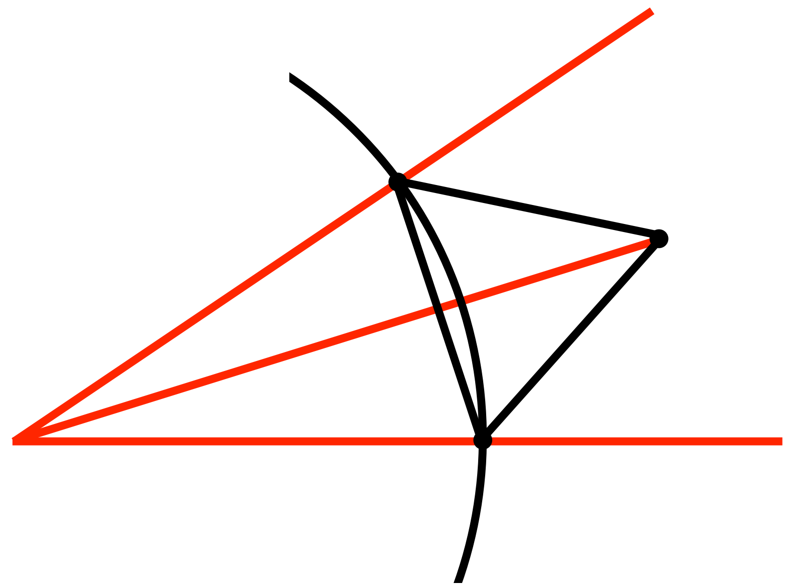

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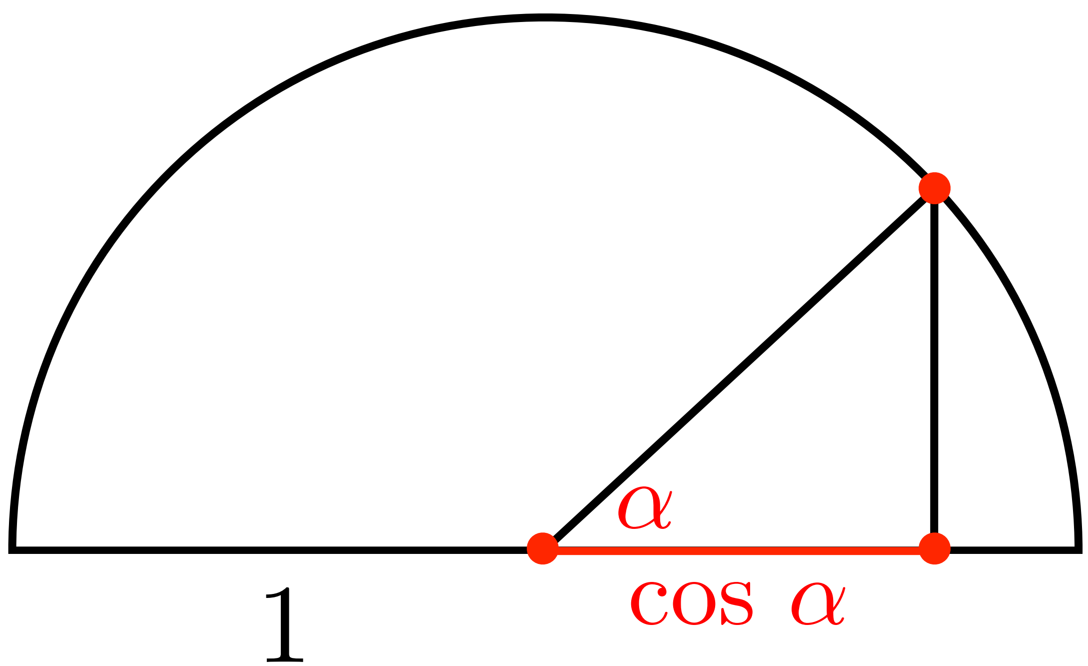

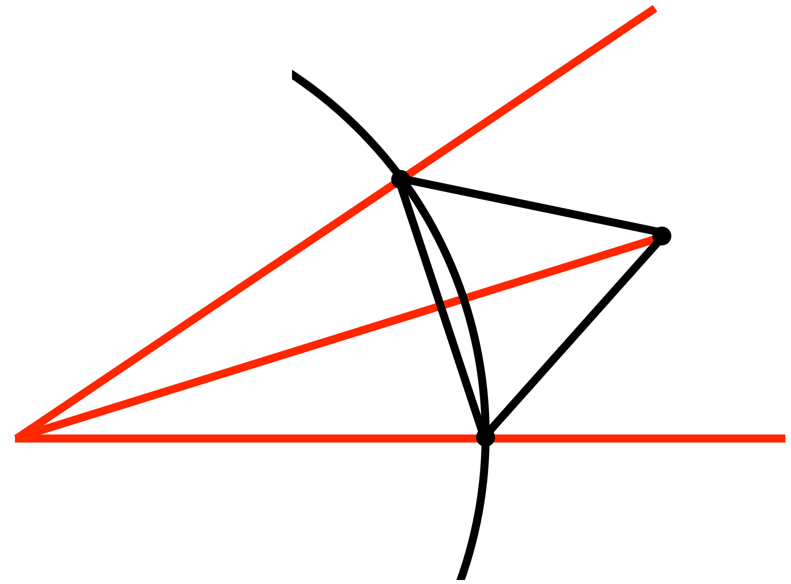

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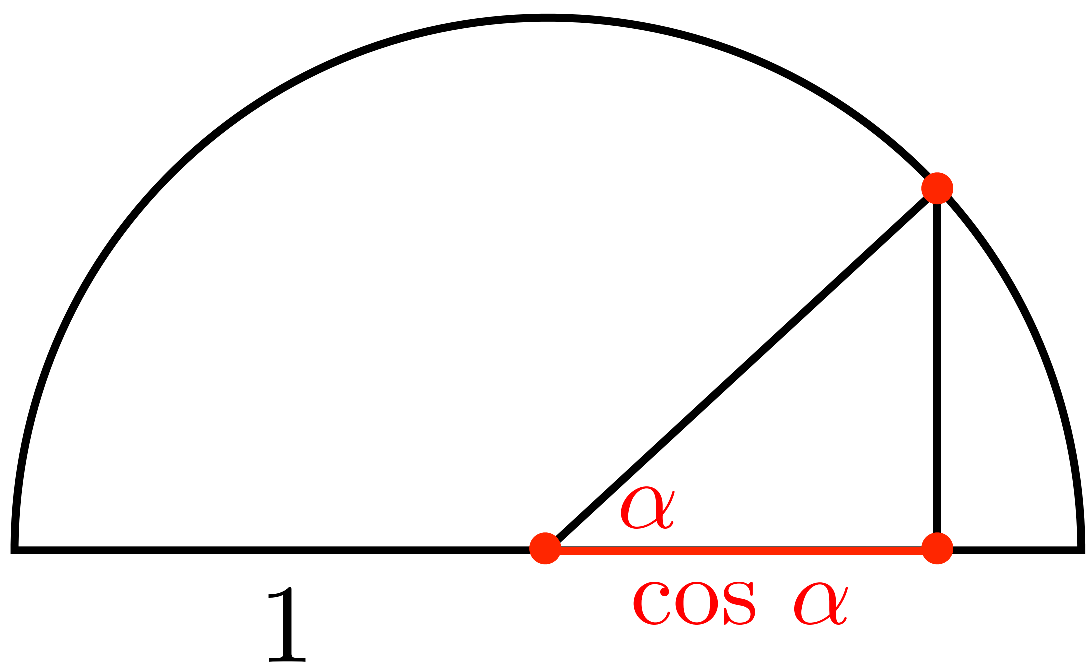
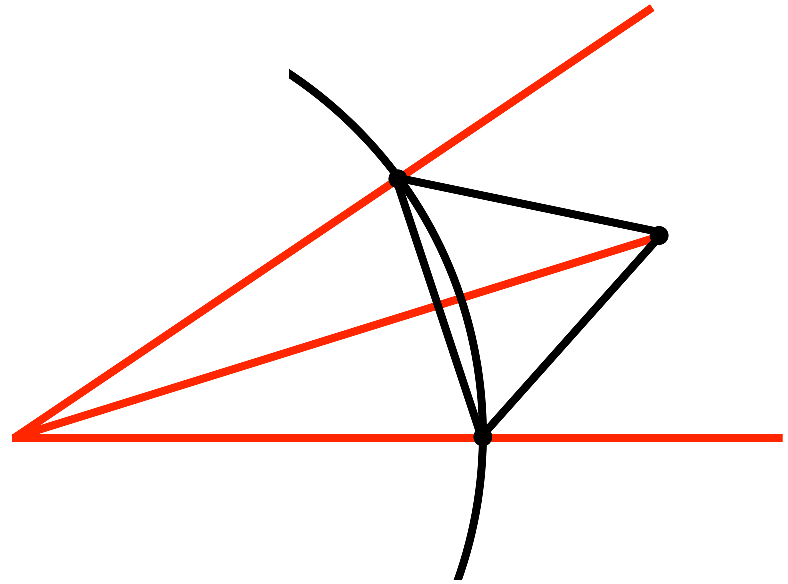


$$\cos \theta = 4(\cos \tfrac{\theta}{3})^3 - 3(\cos \tfrac{\theta}{3})$$



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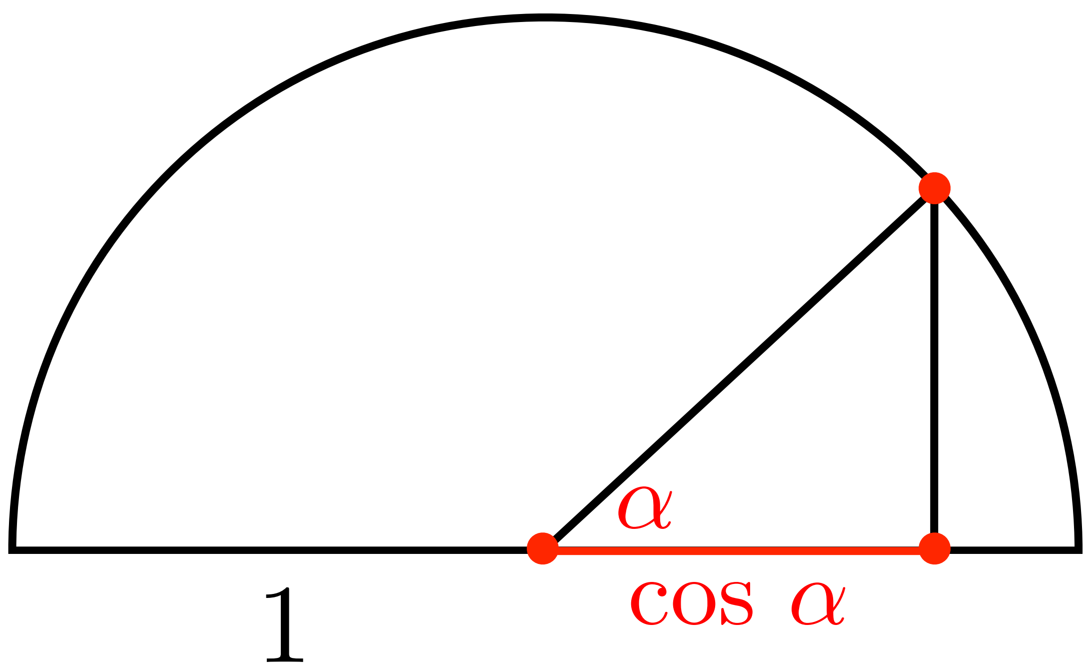
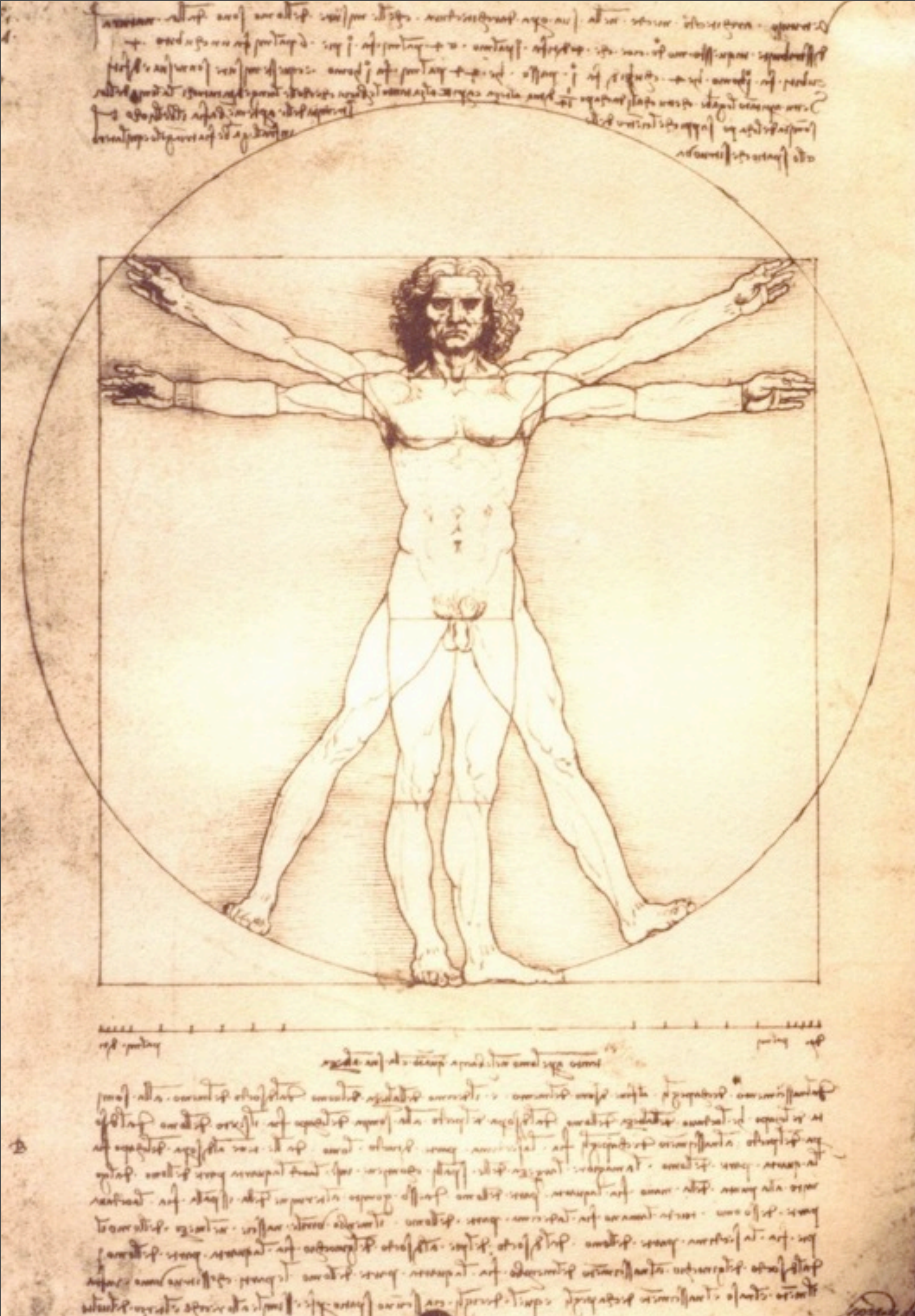
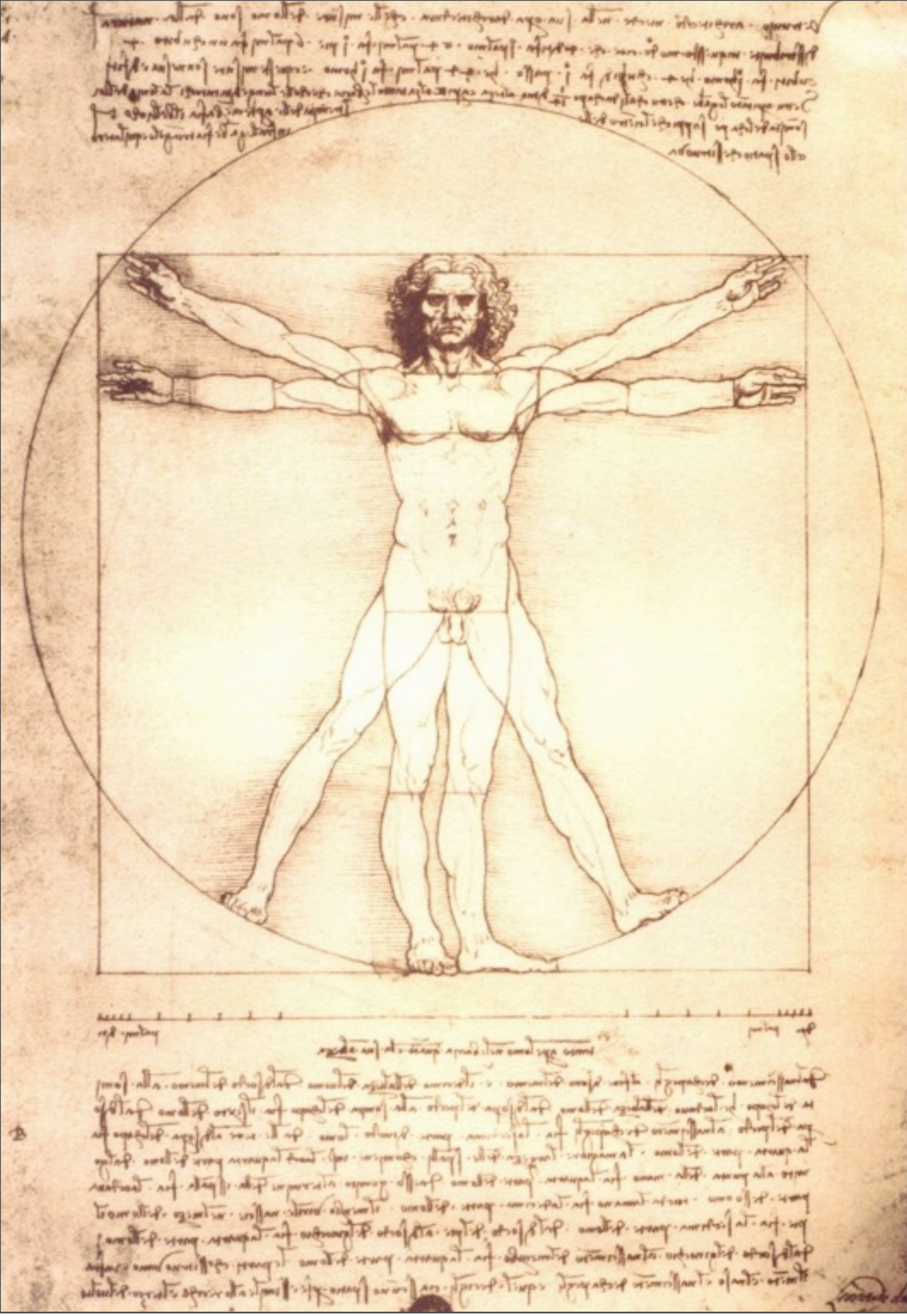
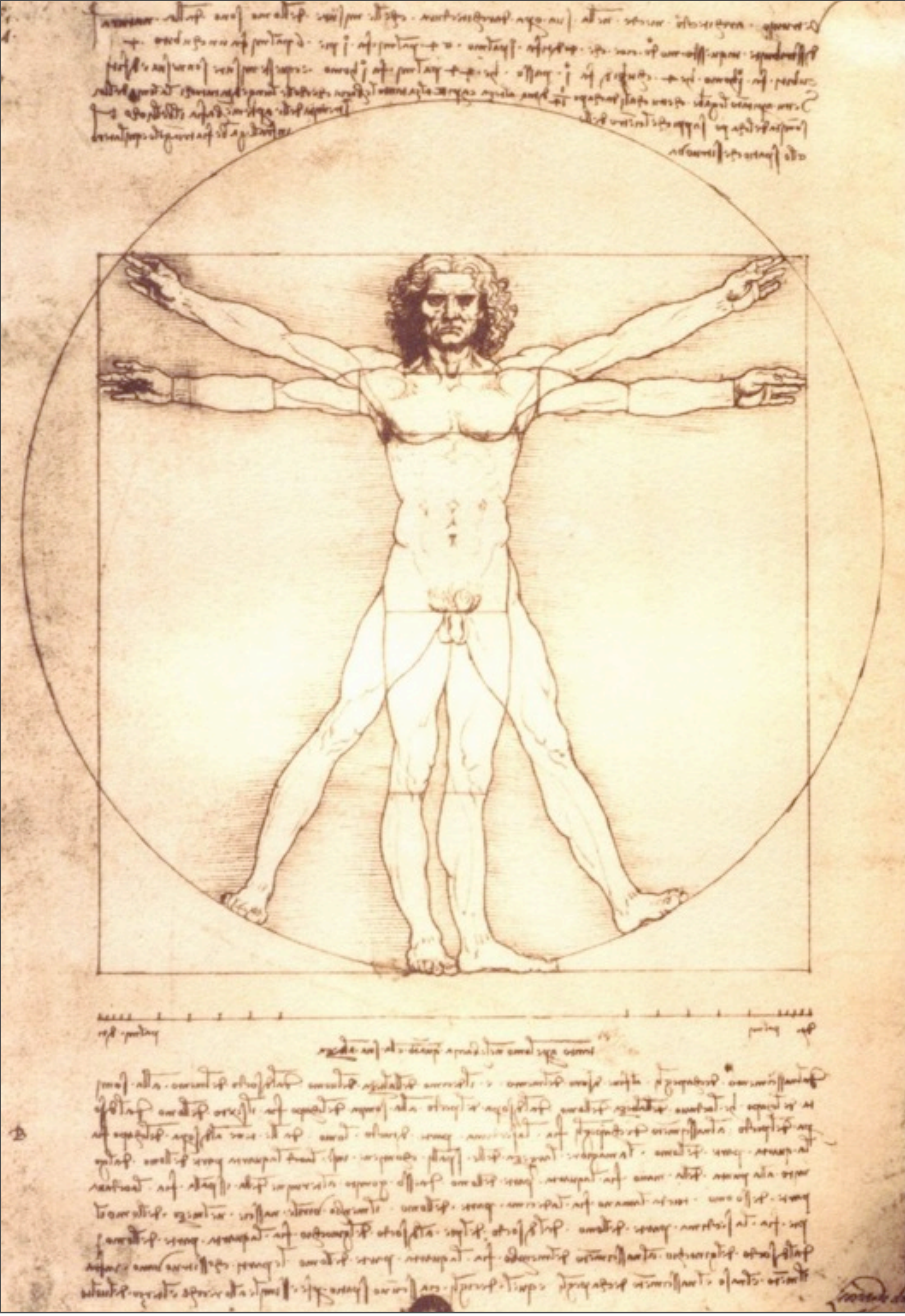


$$\cos\ \theta = 4(\cos\ \tfrac{\theta}{3})^3 - 3(\cos\ \tfrac{\theta}{3}) \qquad \Longrightarrow \qquad \tfrac{1}{2} = 4\,x^3 - 3\,x$$





Given a circle, build a square with the same area.



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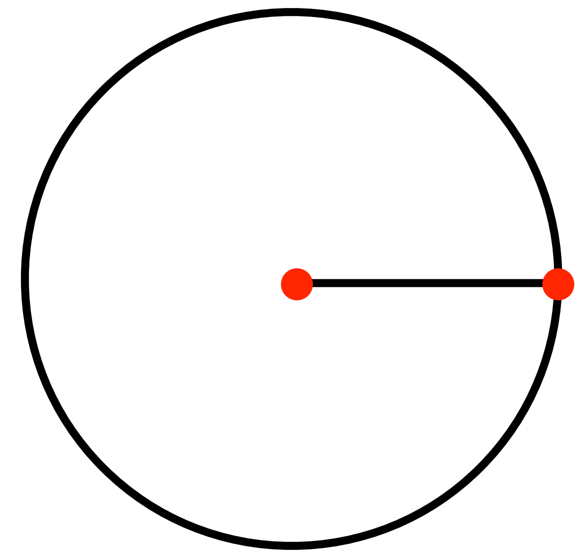

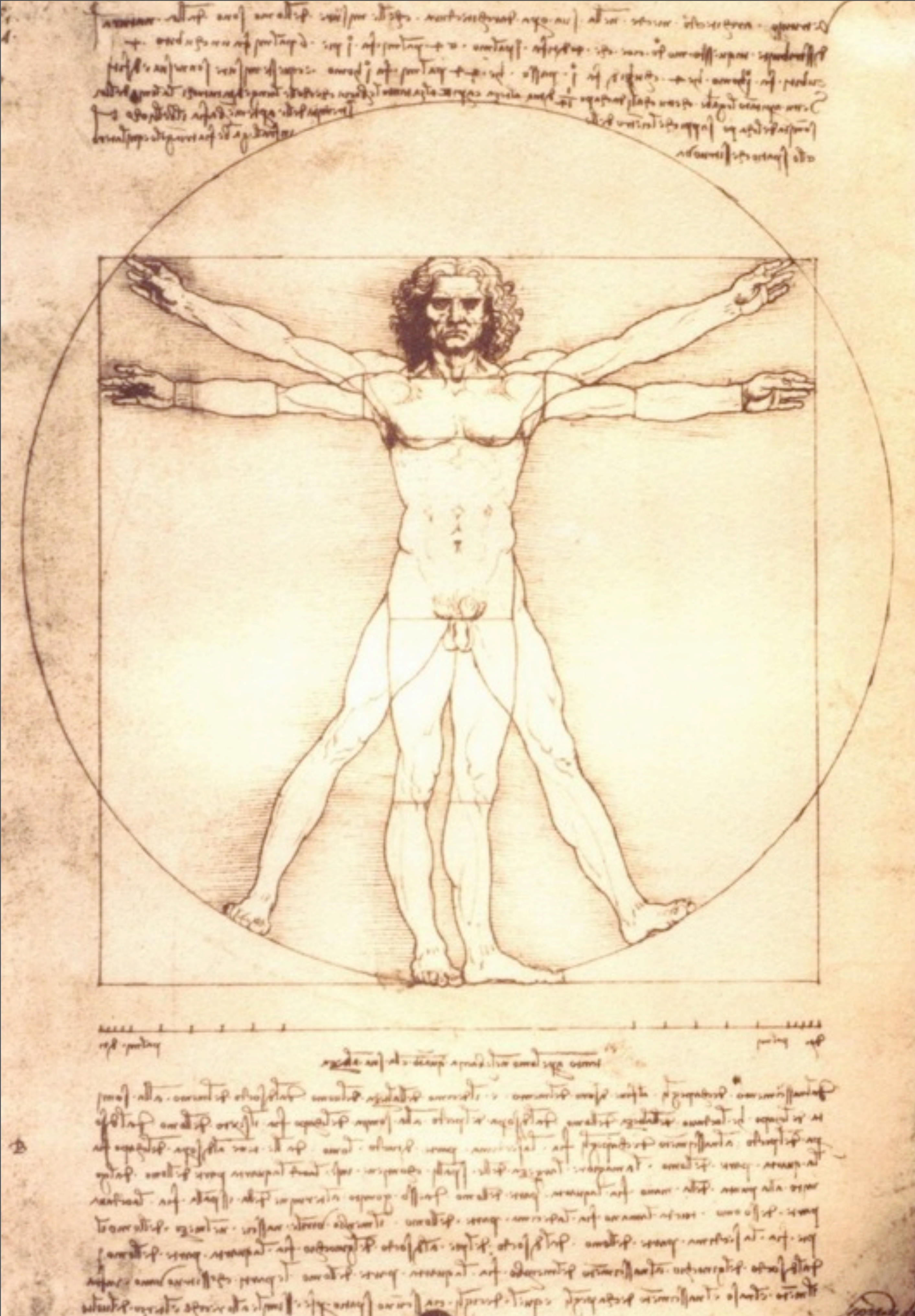

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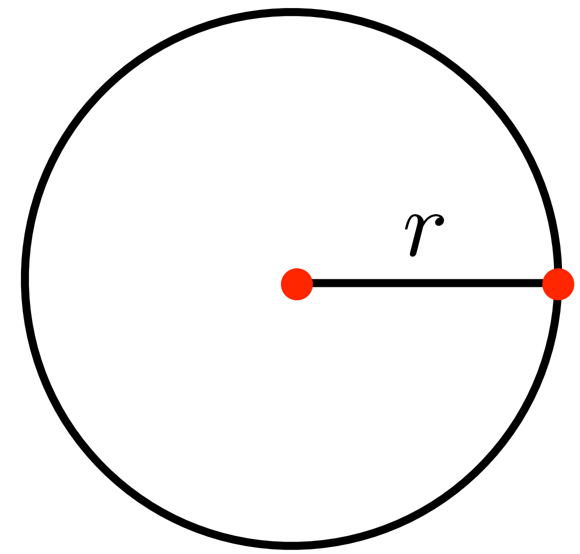
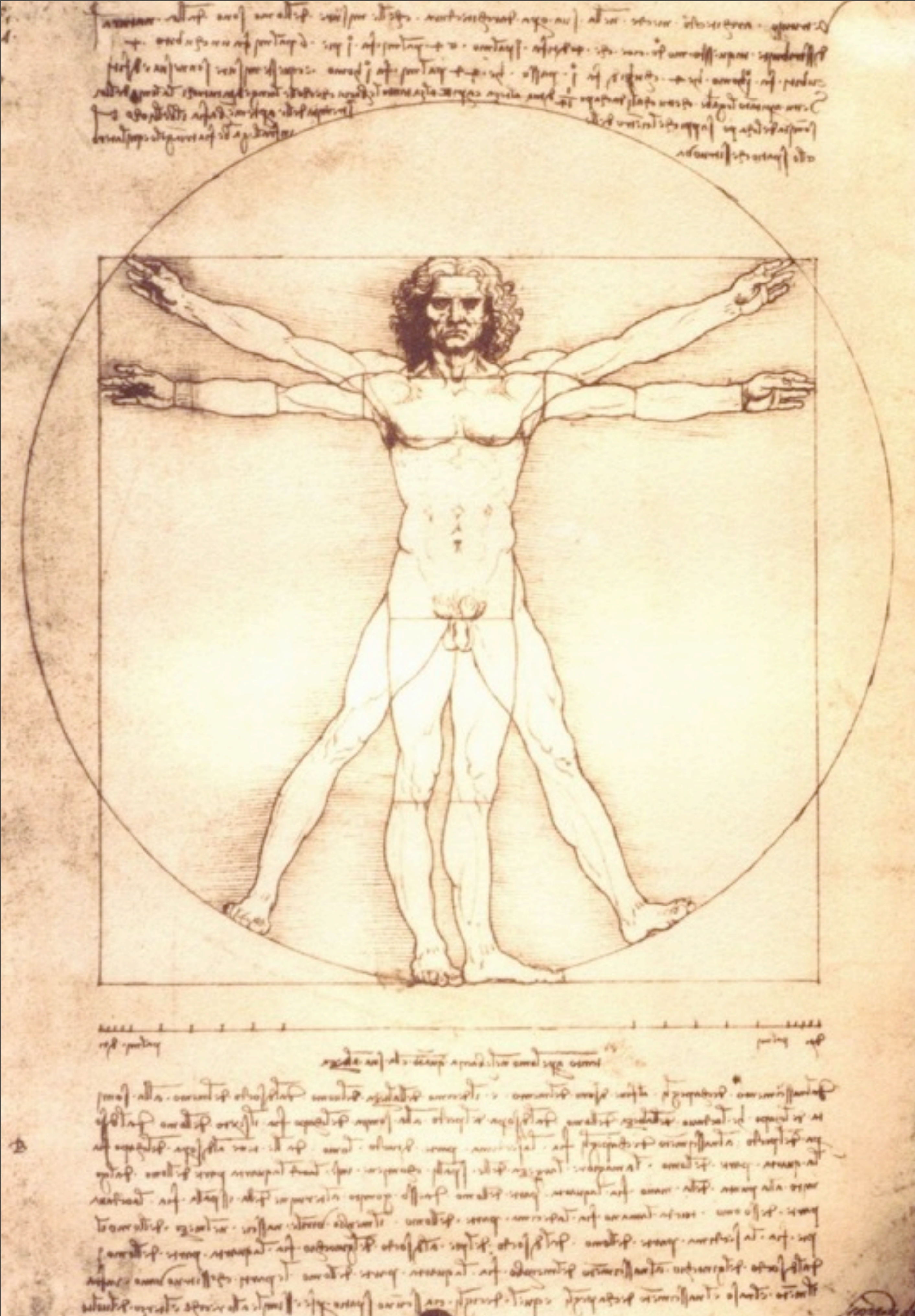


area of the circle $= \pi r^2$



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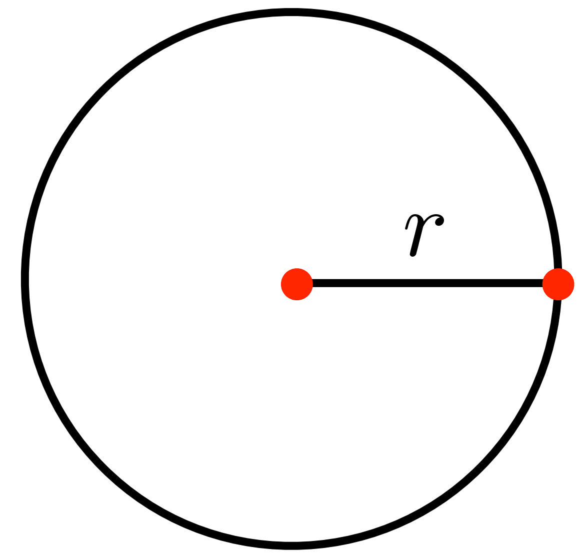
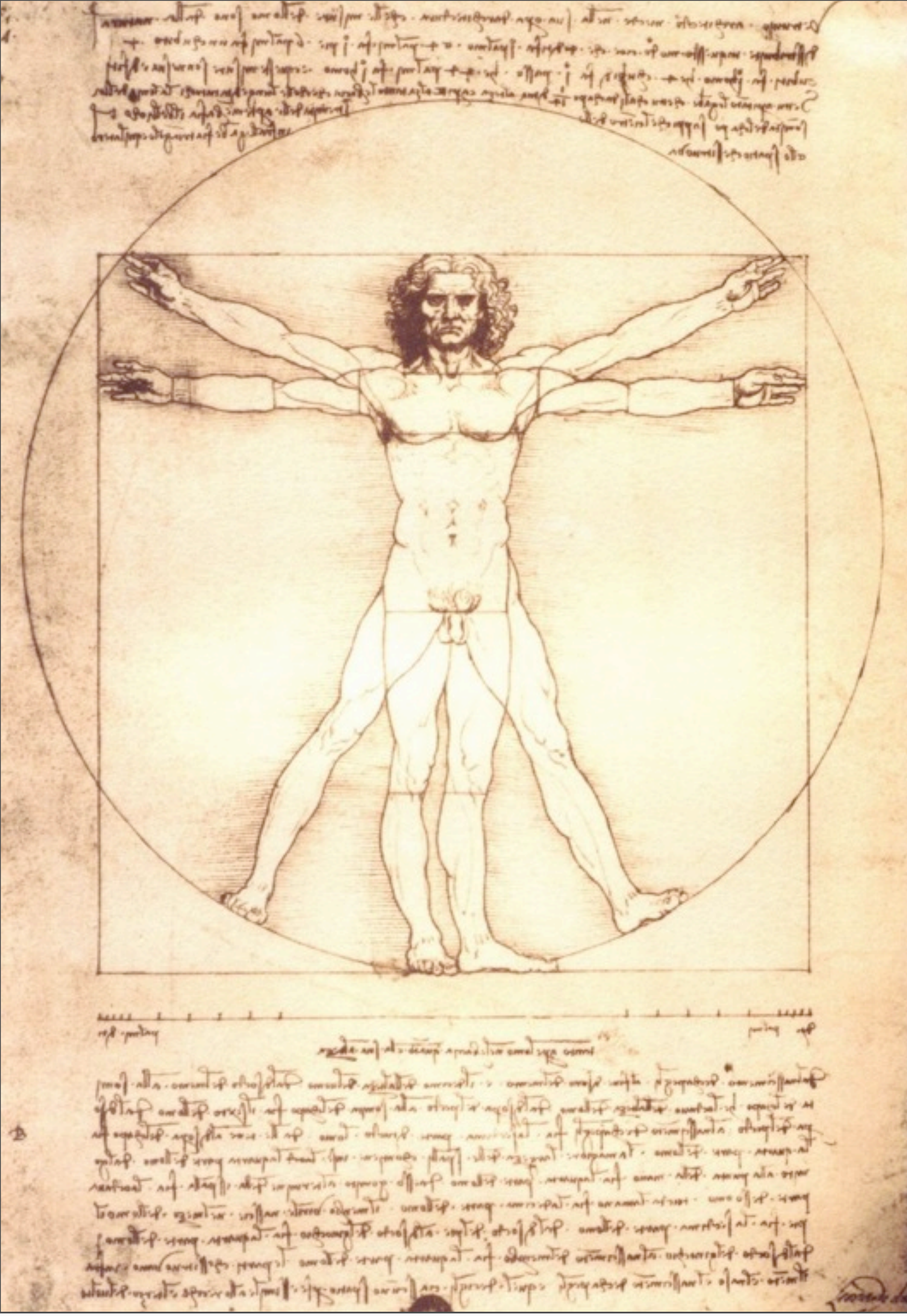


area of the circle $= \pi r^2$

area of the square $= (\text{edge})^2$



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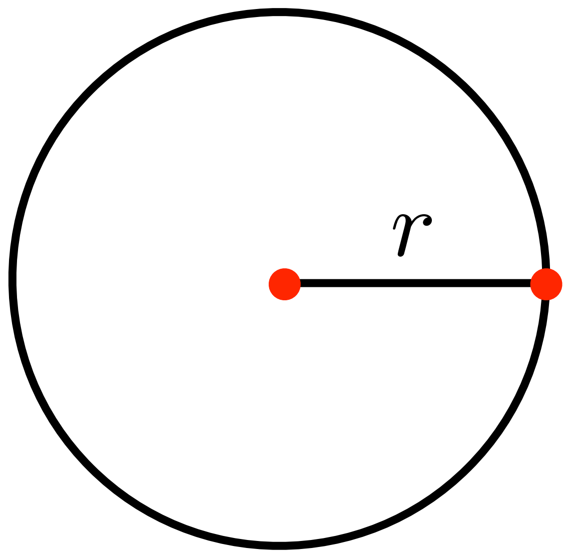
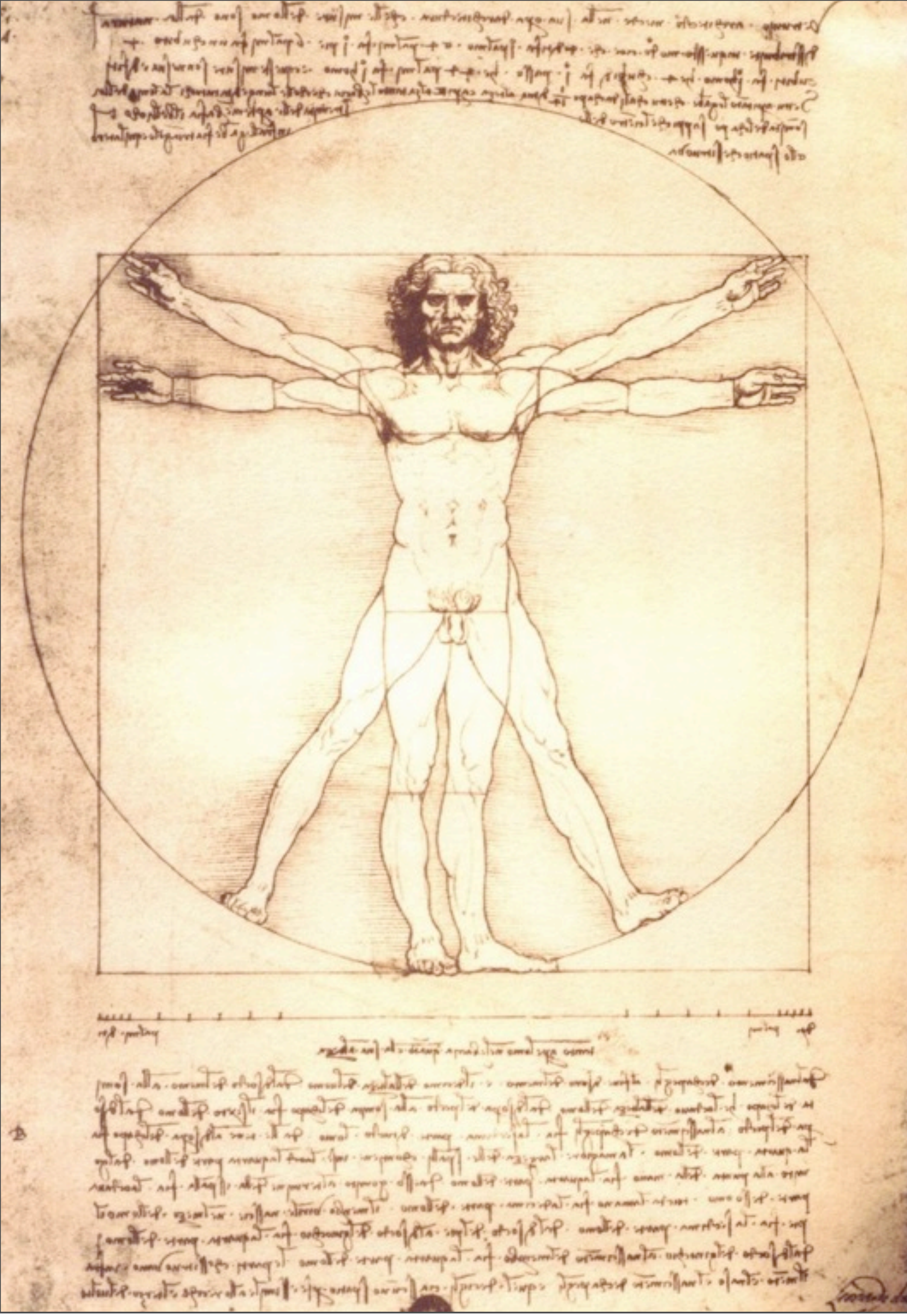


area of the circle $= \pi r^2$

area of the square $= (\text{edge})^2$

edge $= \sqrt{\pi} r$



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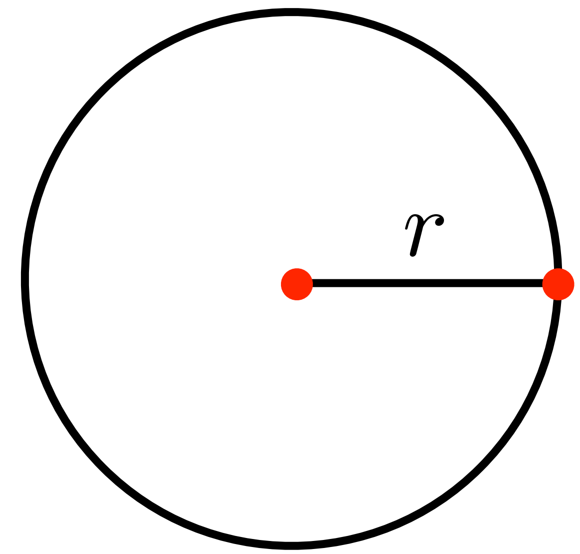
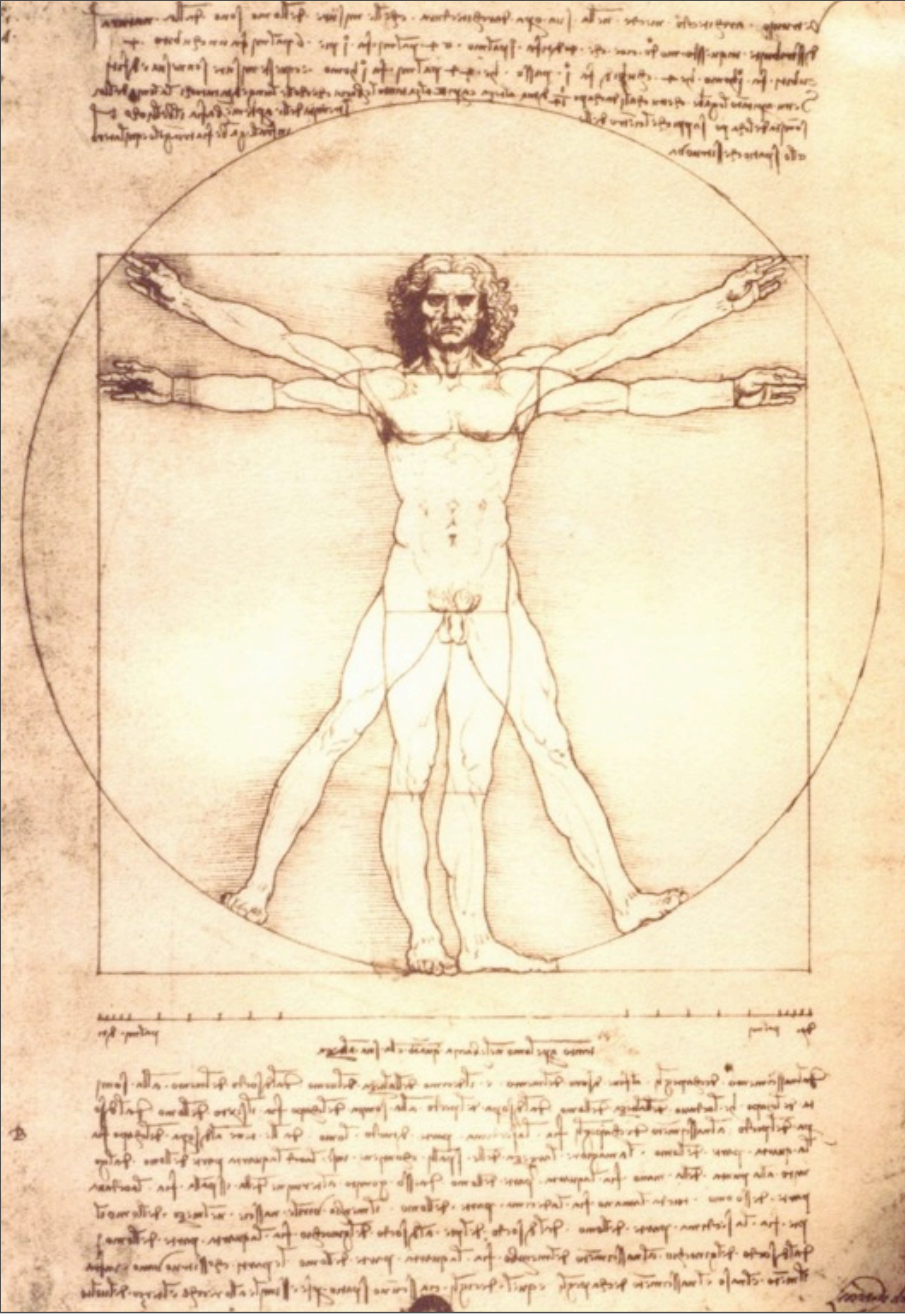


area of the circle $= \pi r^2$

area of the square $= (\text{edge})^2$

edge $= \sqrt{\pi} r$

Is $\sqrt{\pi}$ constructible with ruler and compass?



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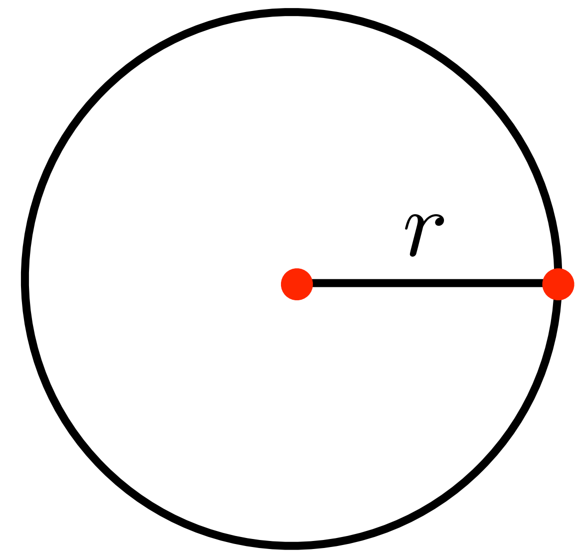

area of the circle $= \pi r^2$

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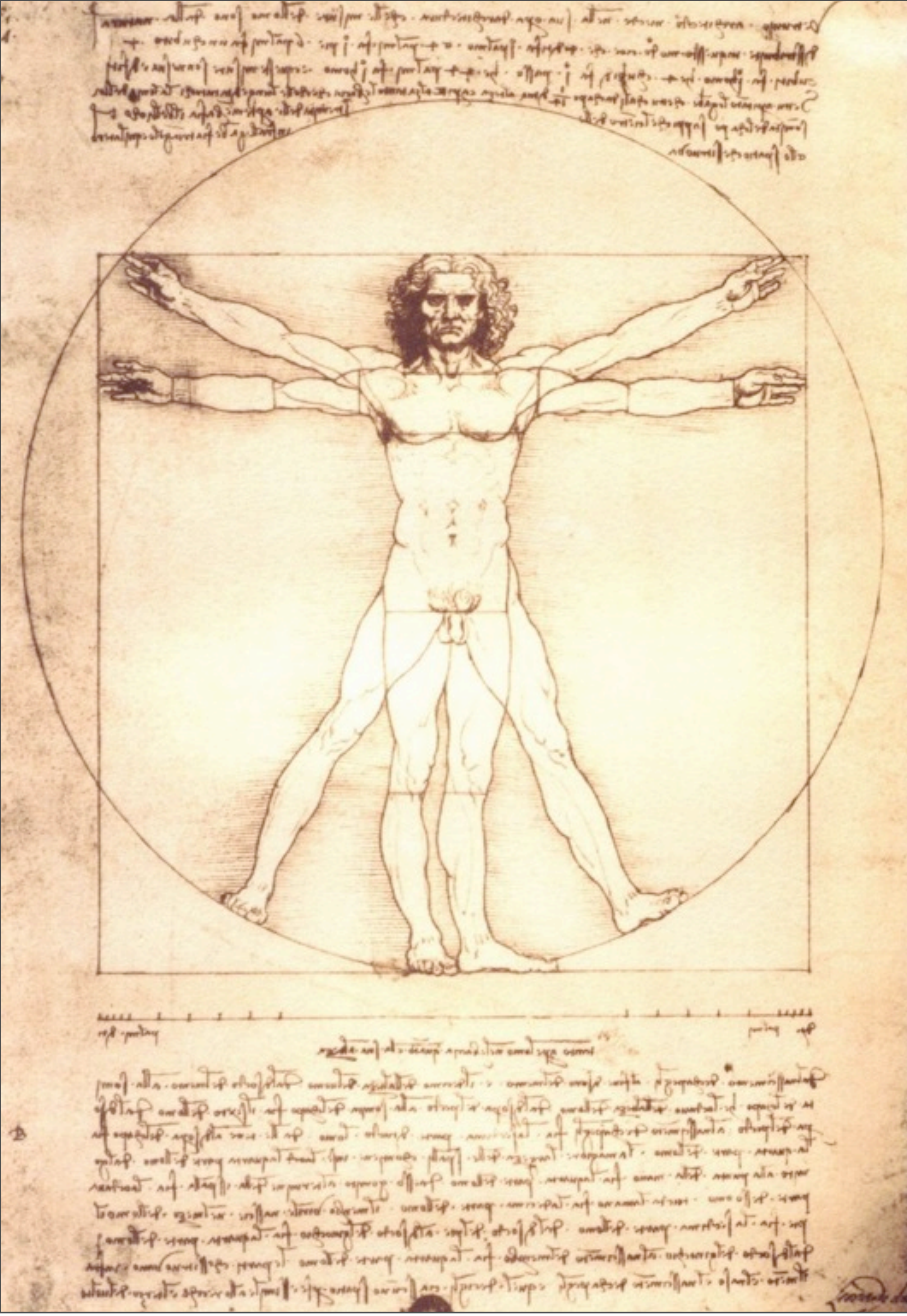
Is π constructible with ruler and compass?



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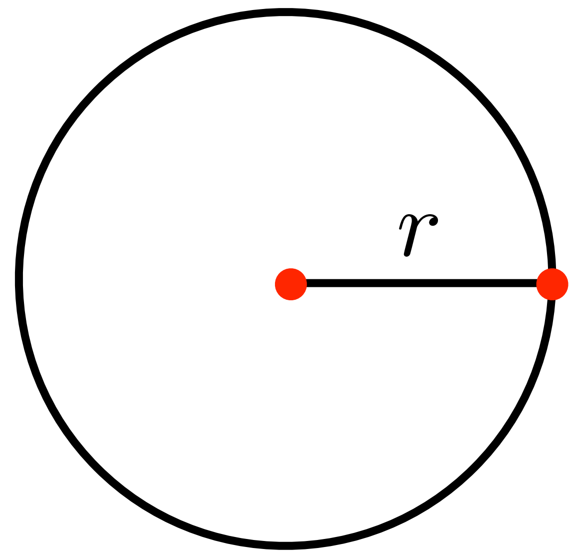

area of the circle $= \pi r^2$

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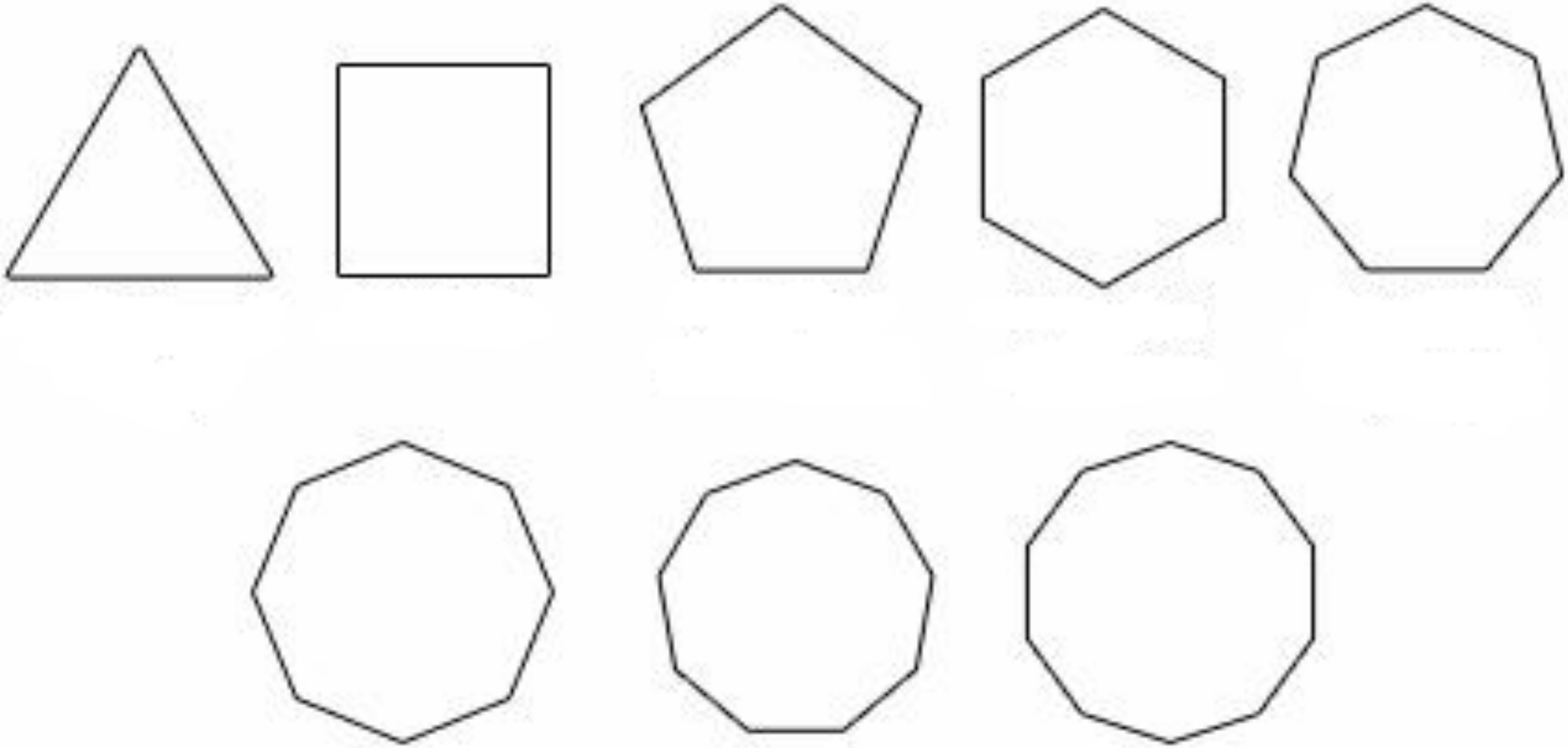
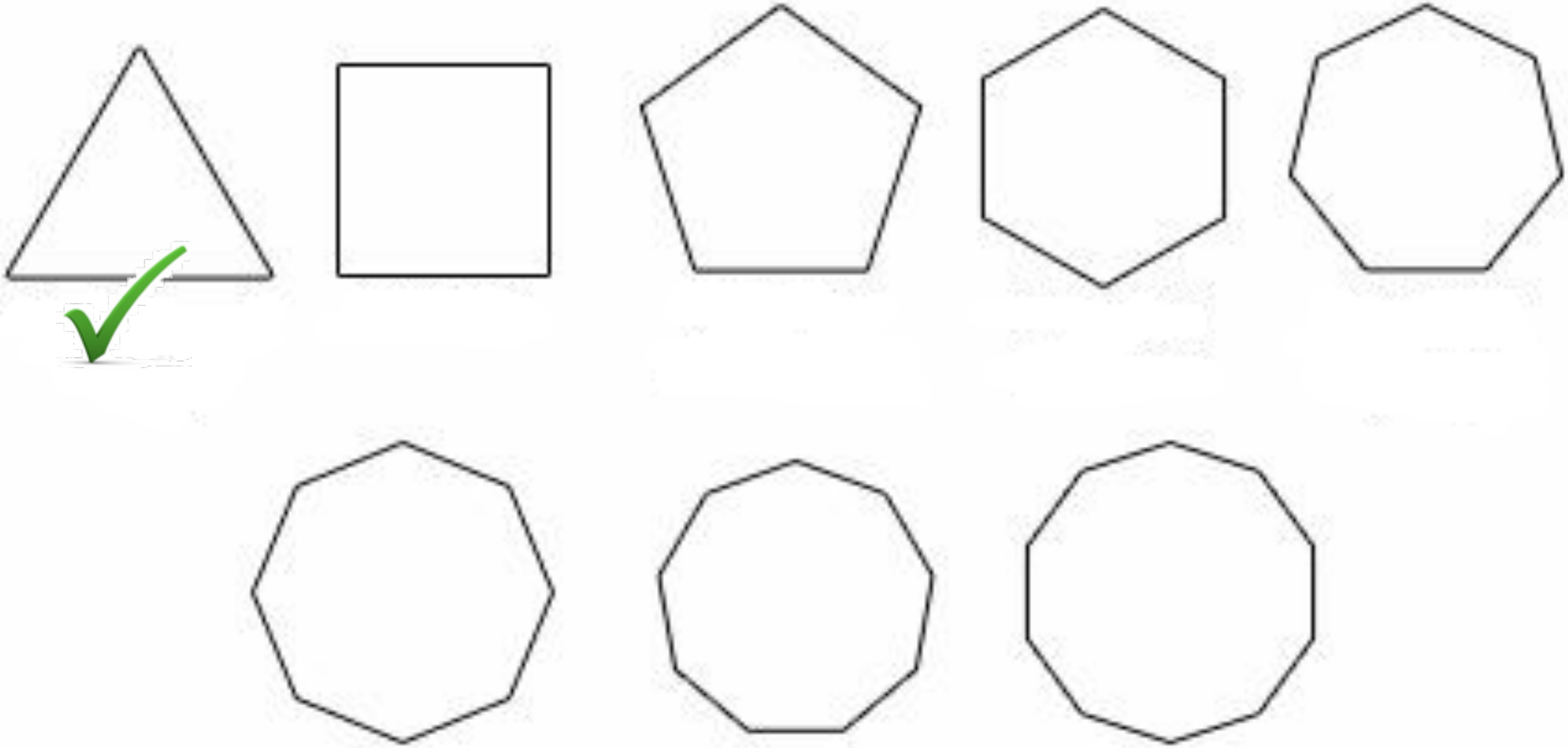
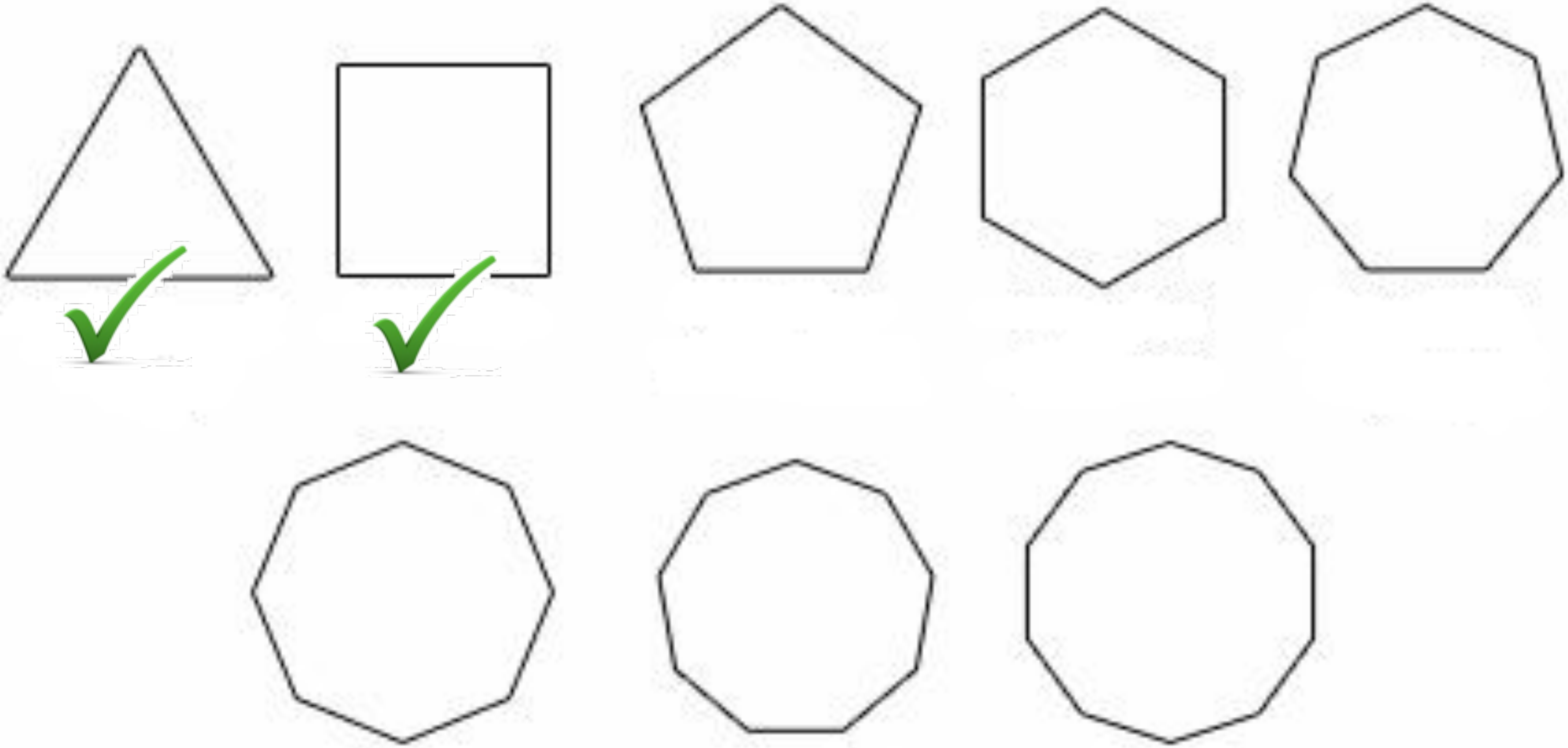
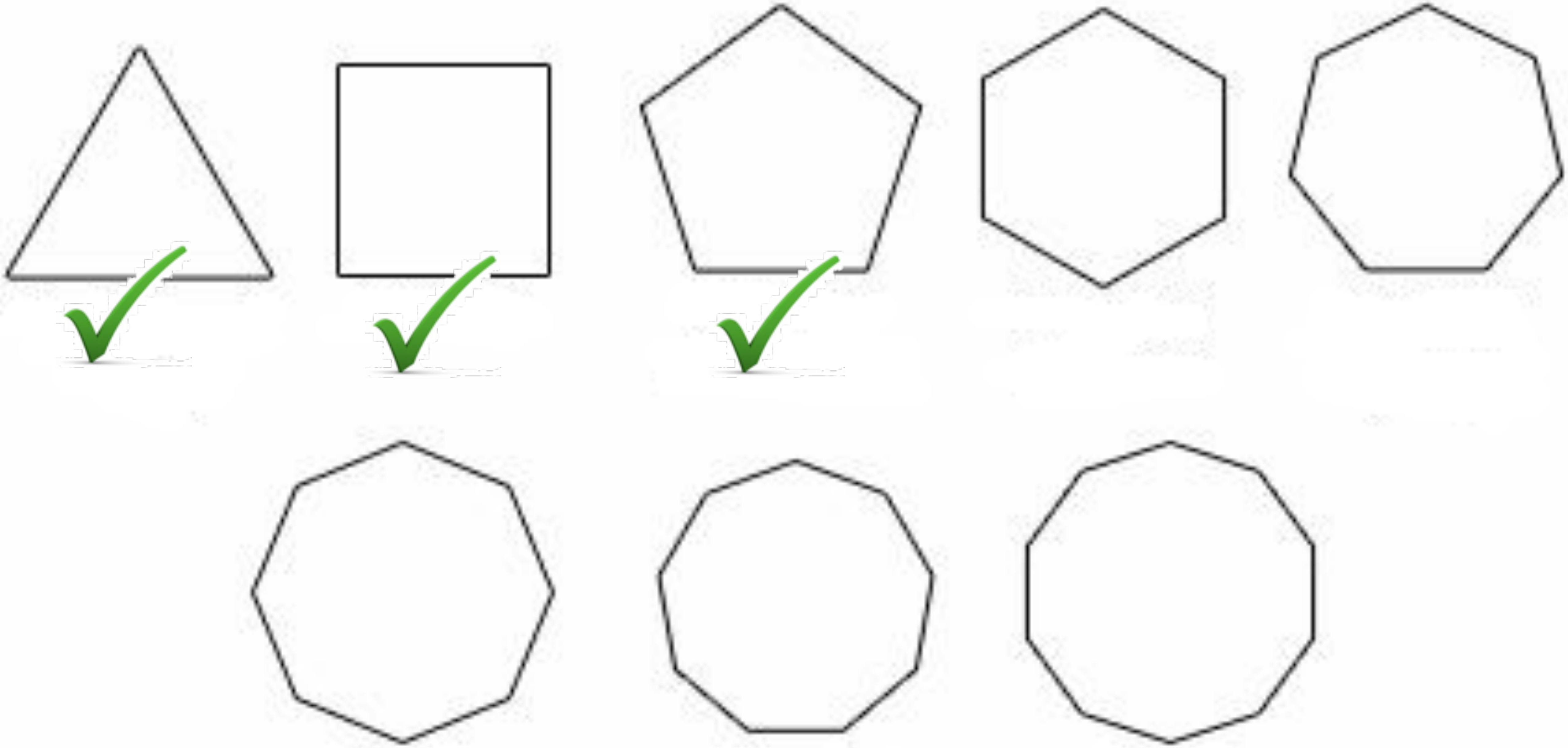
edge $= \sqrt{\pi} r$

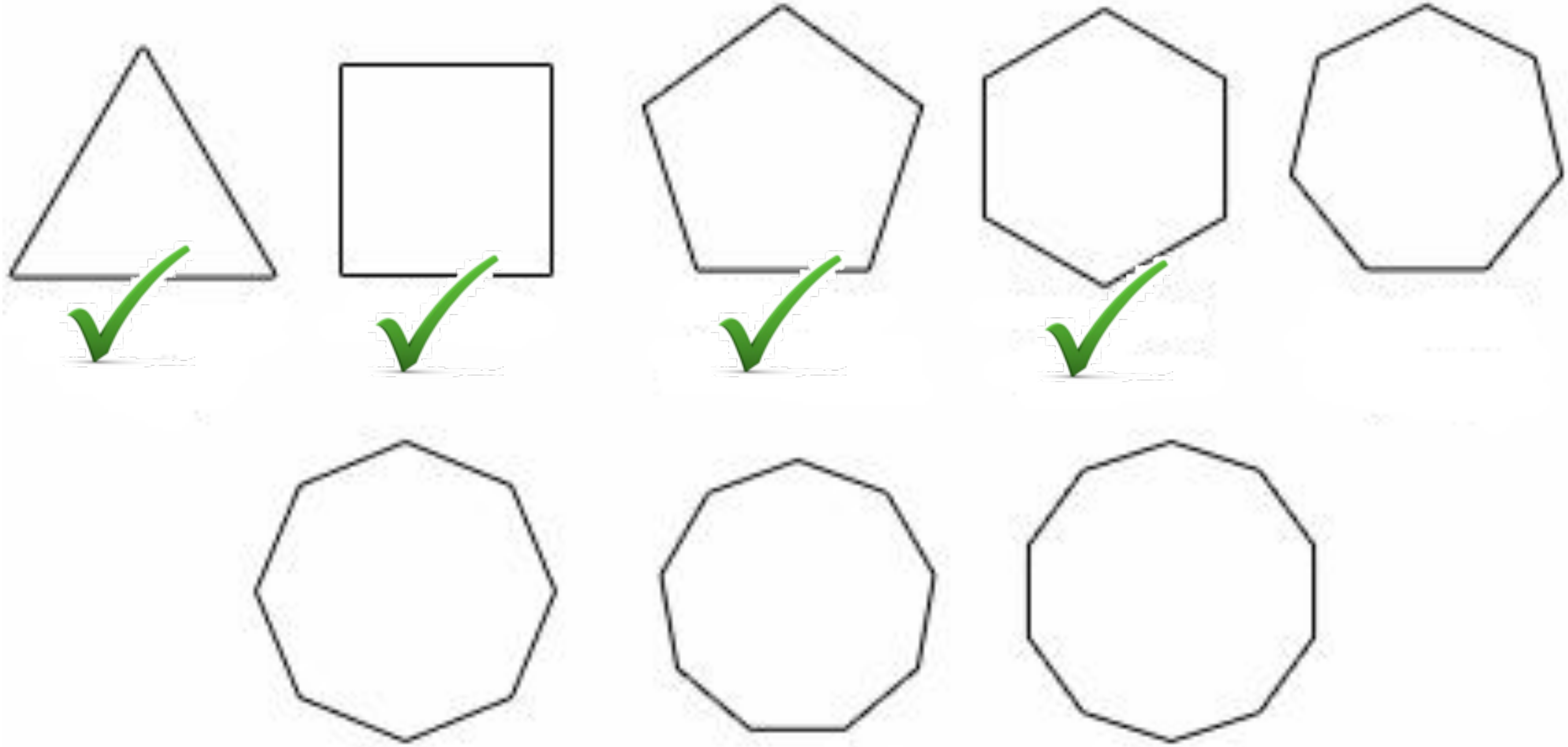
Is π constructible with ruler and compass? No.

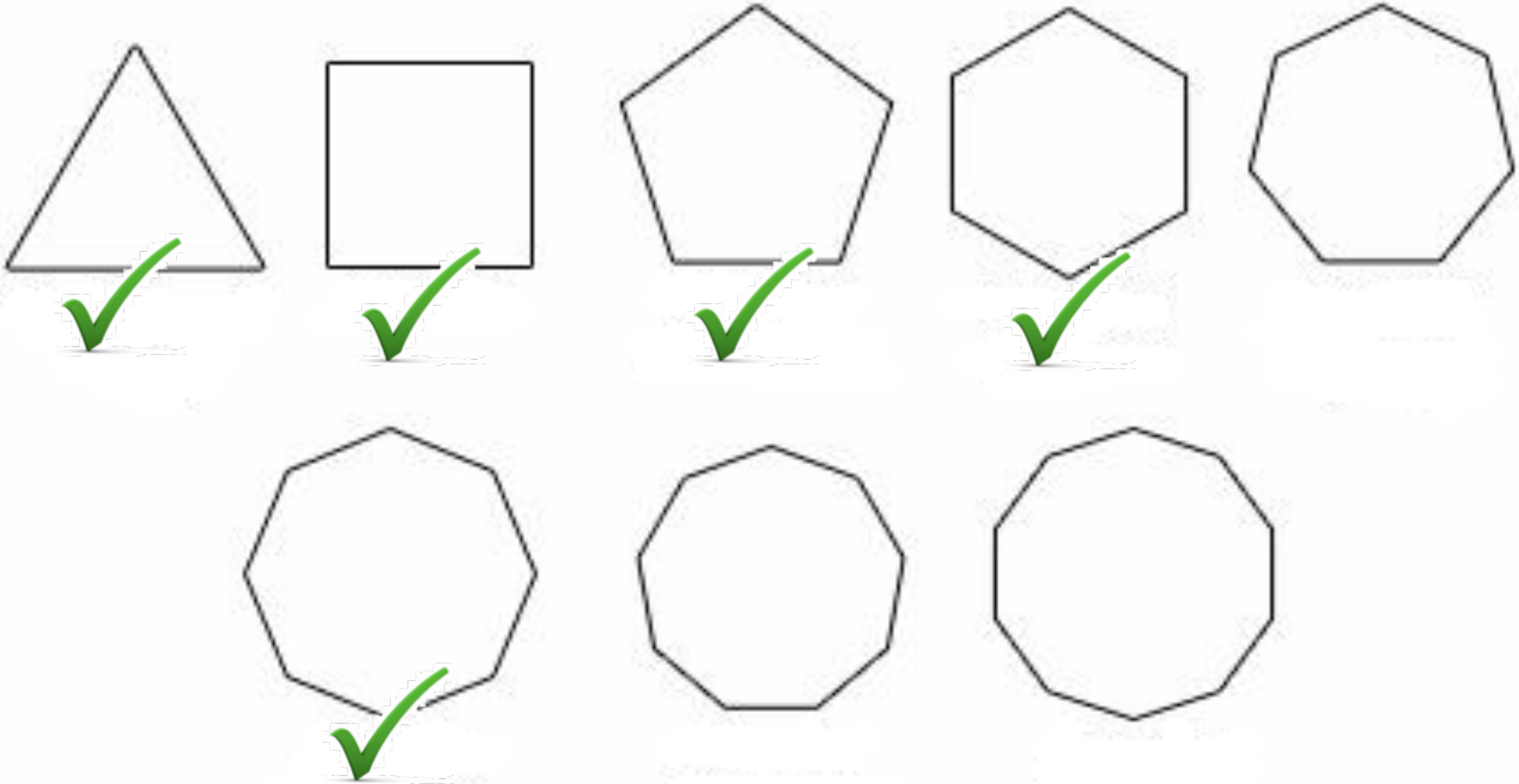


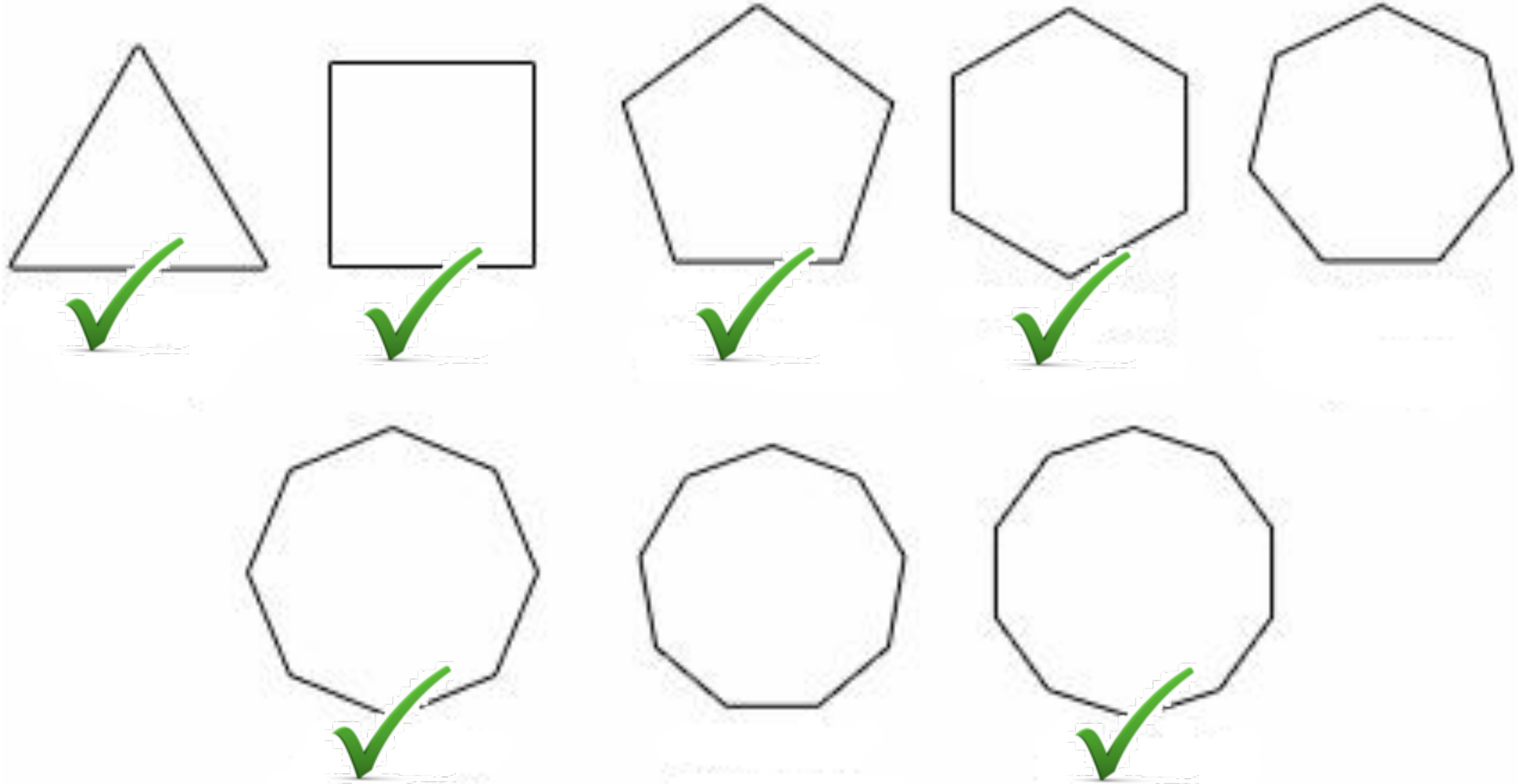


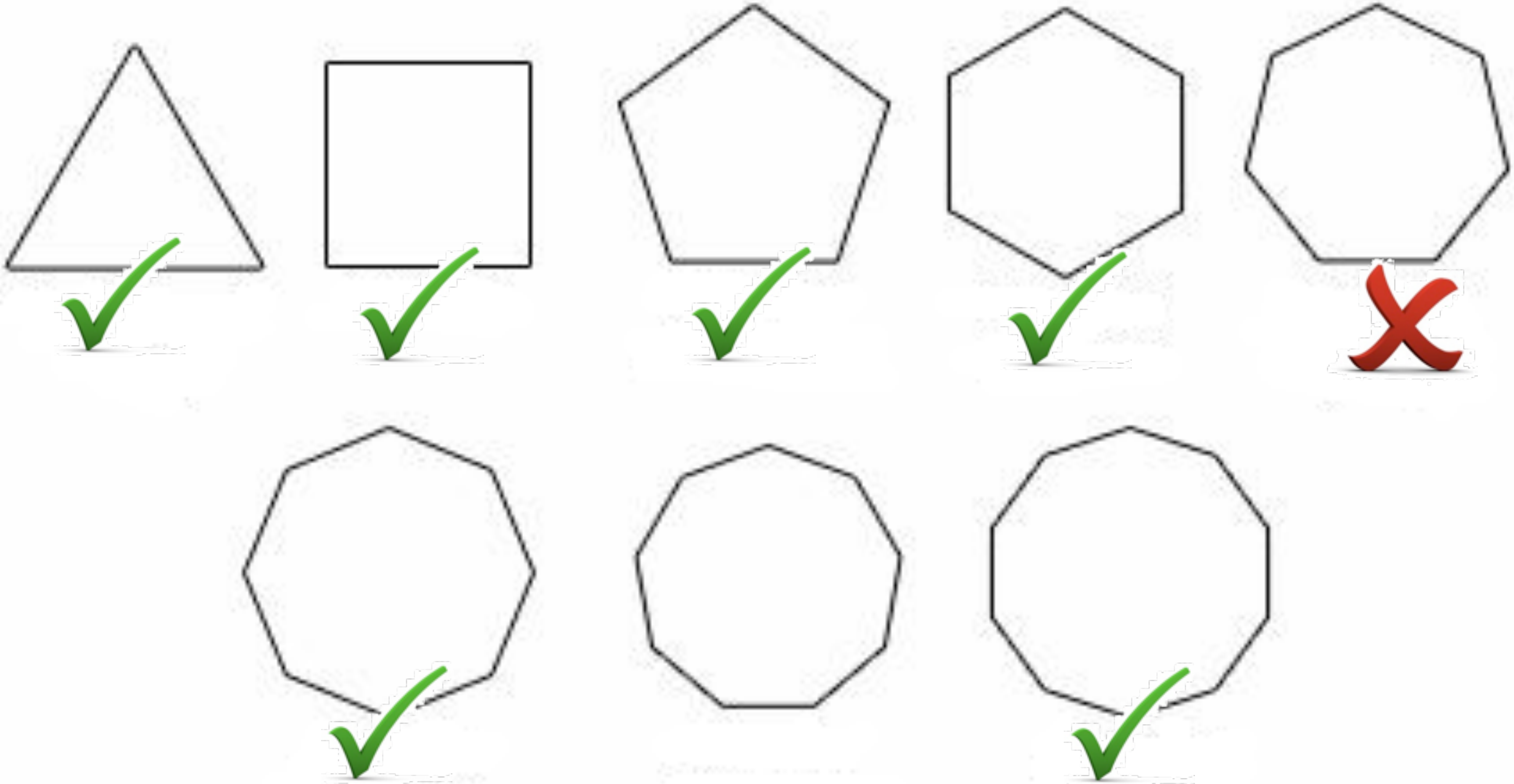


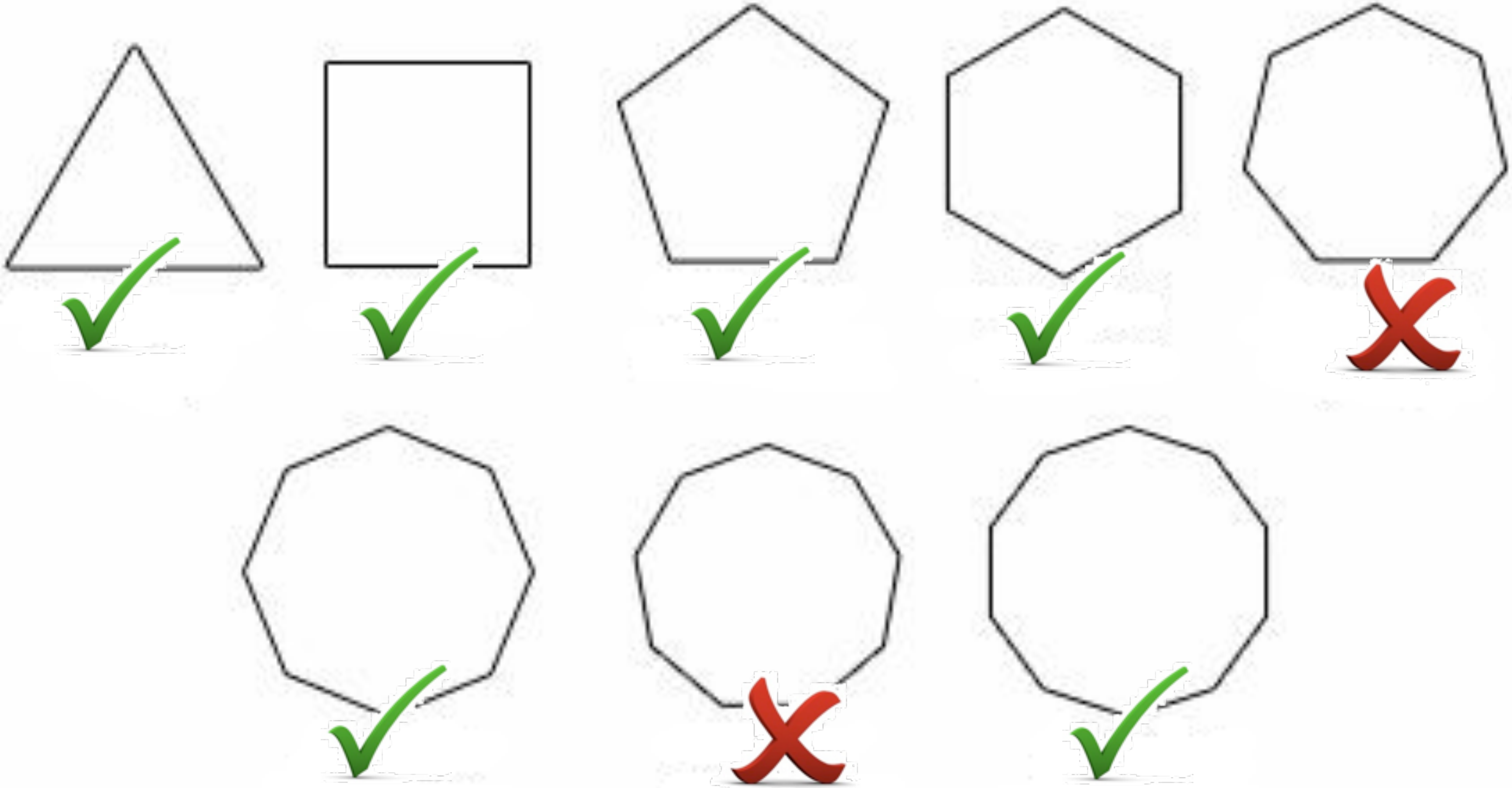


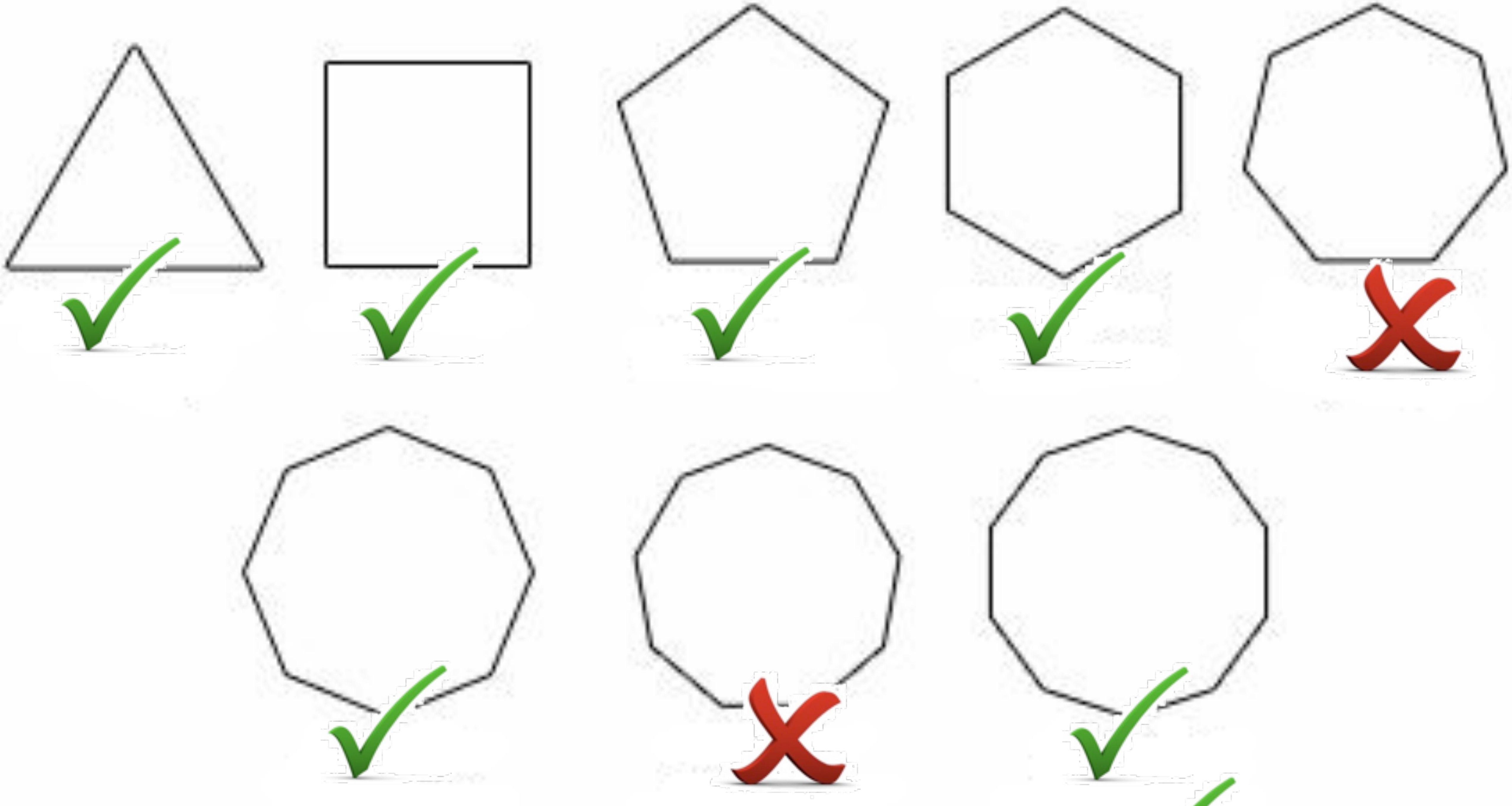




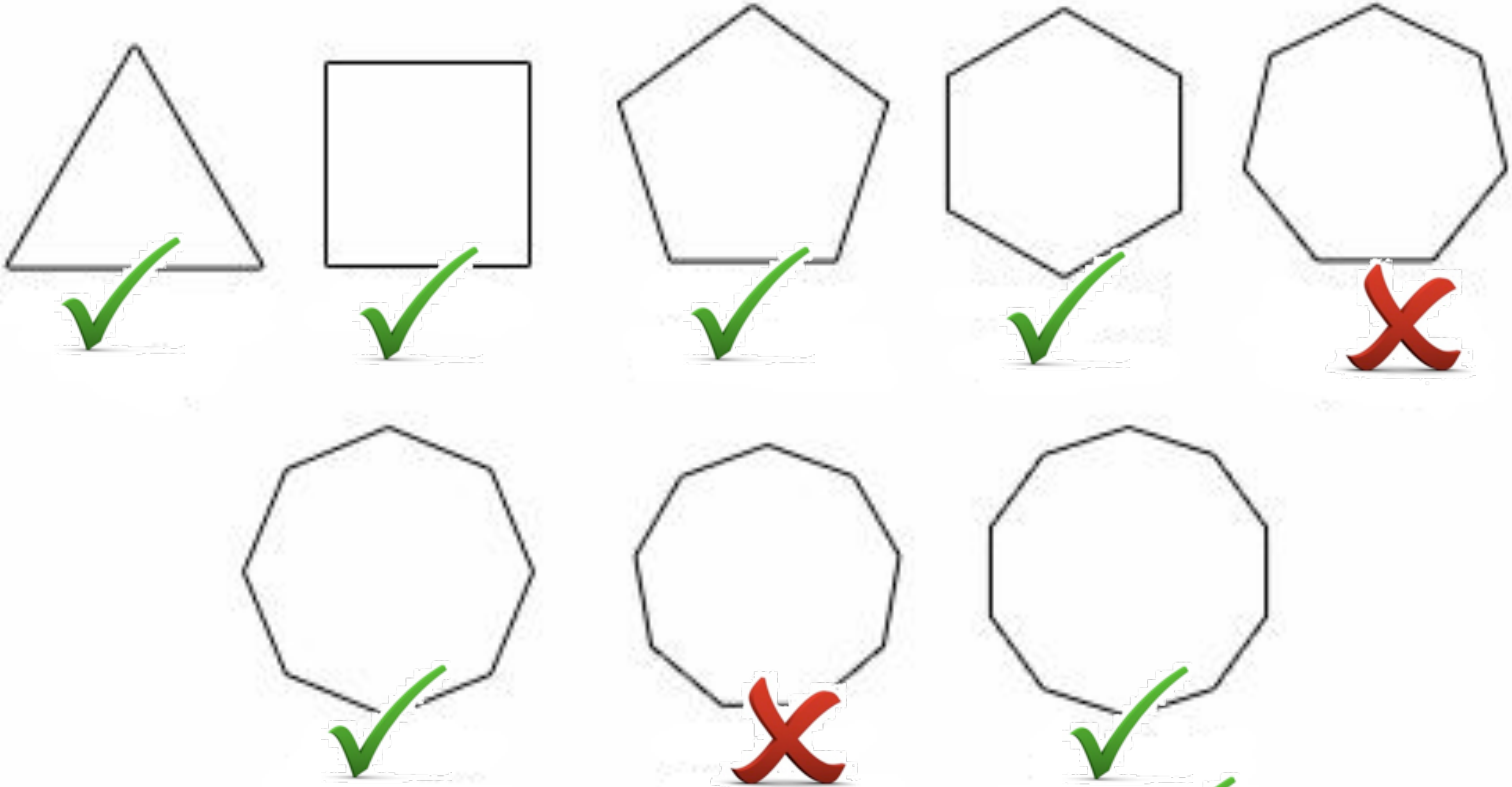






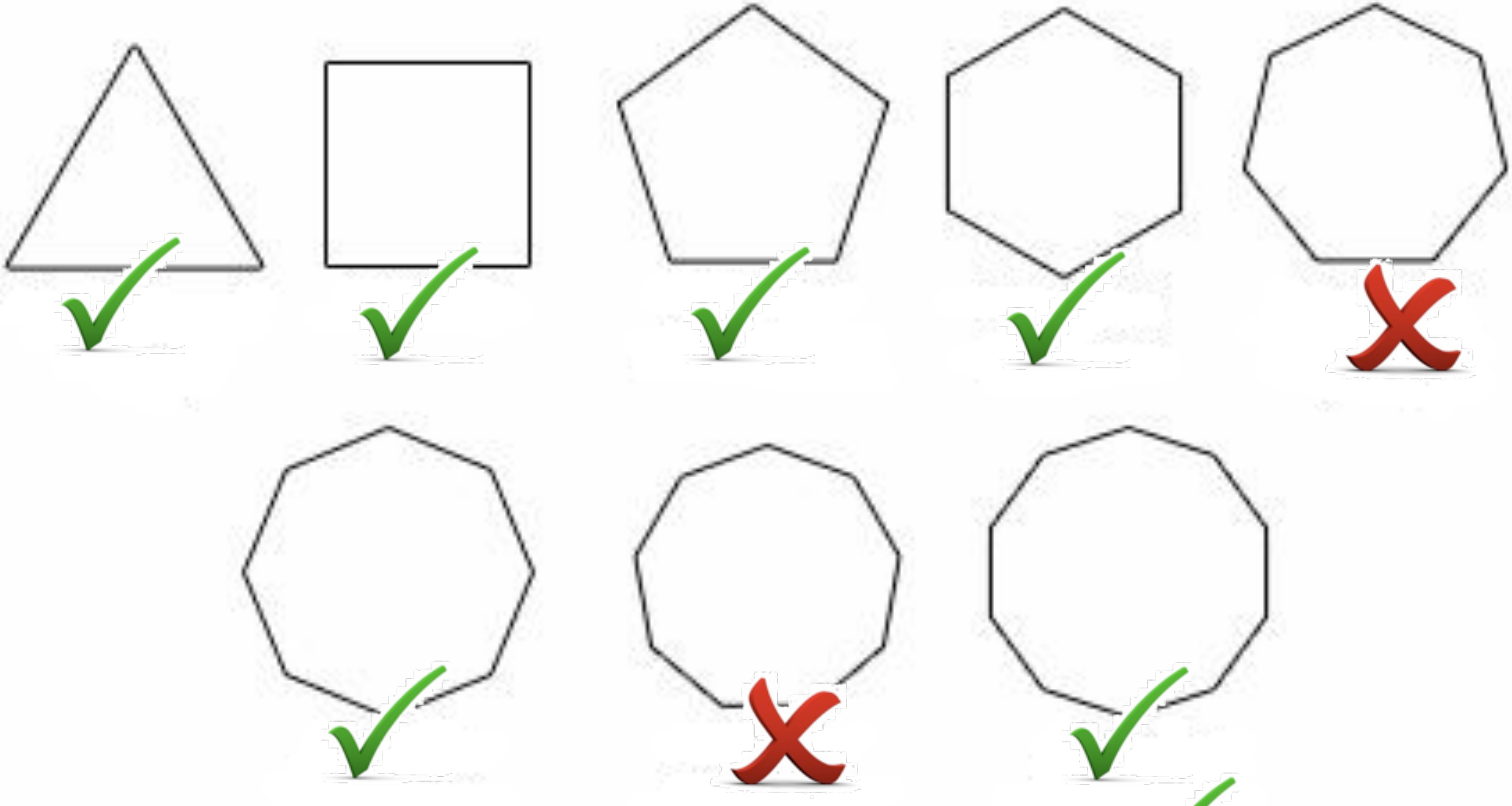


$n = 2^k \times$ product of distinct Fermat primes ✓



$n = 2^k \times$ product of distinct Fermat primes

$$F_n = 2^{2^n} + 1$$



$n = 2^k \times$ product of distinct Fermat primes ✓

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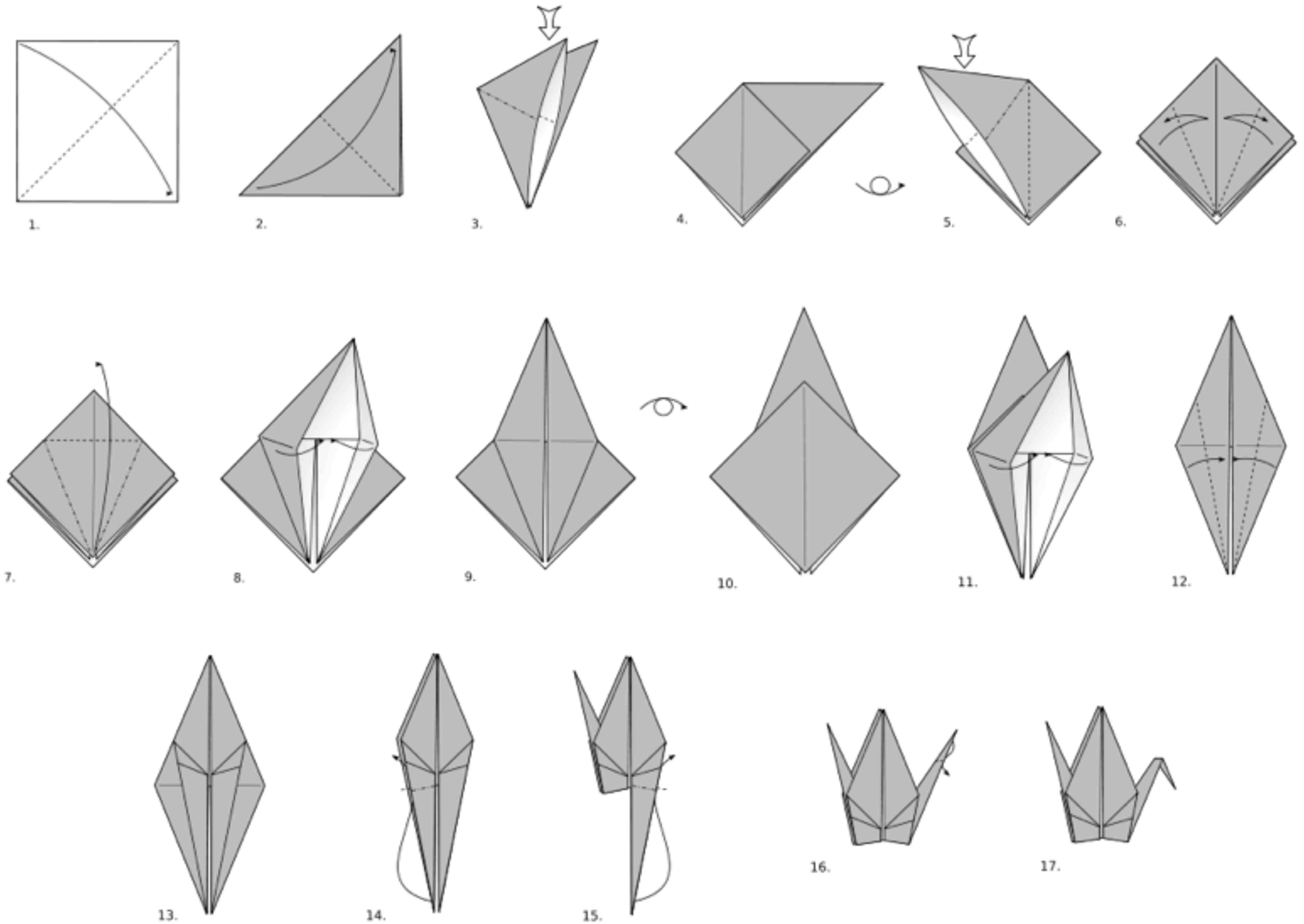
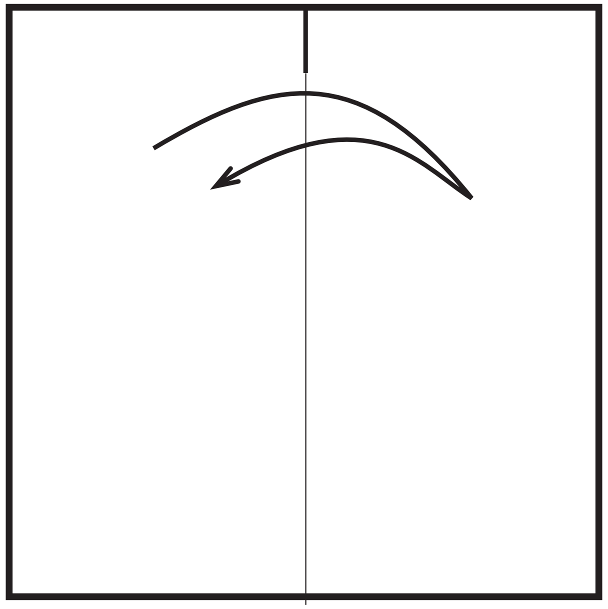
$F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537, F_5$ is not prime, ...

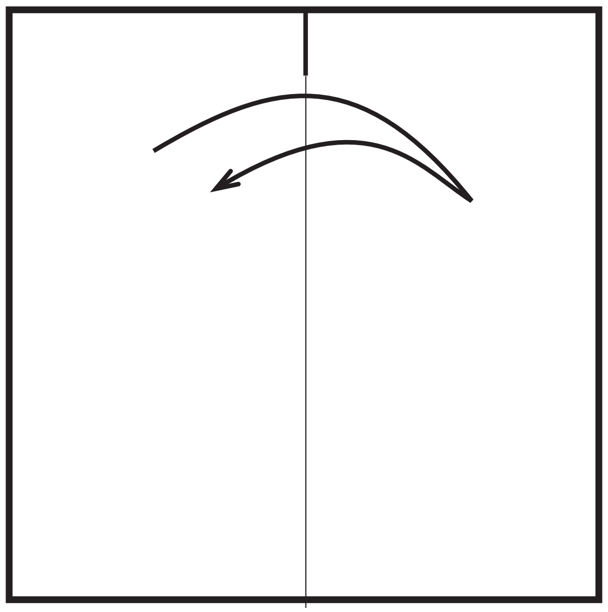
Orizuru

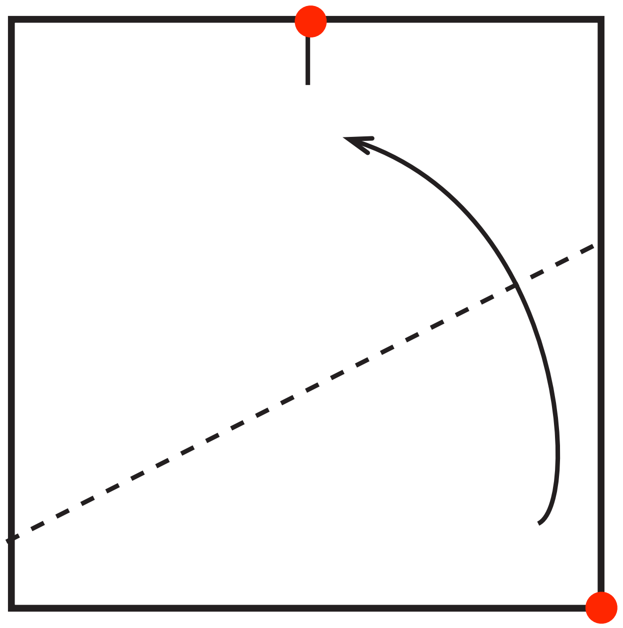
Traditional Japanese Model
Diagram by Andrew Hudson

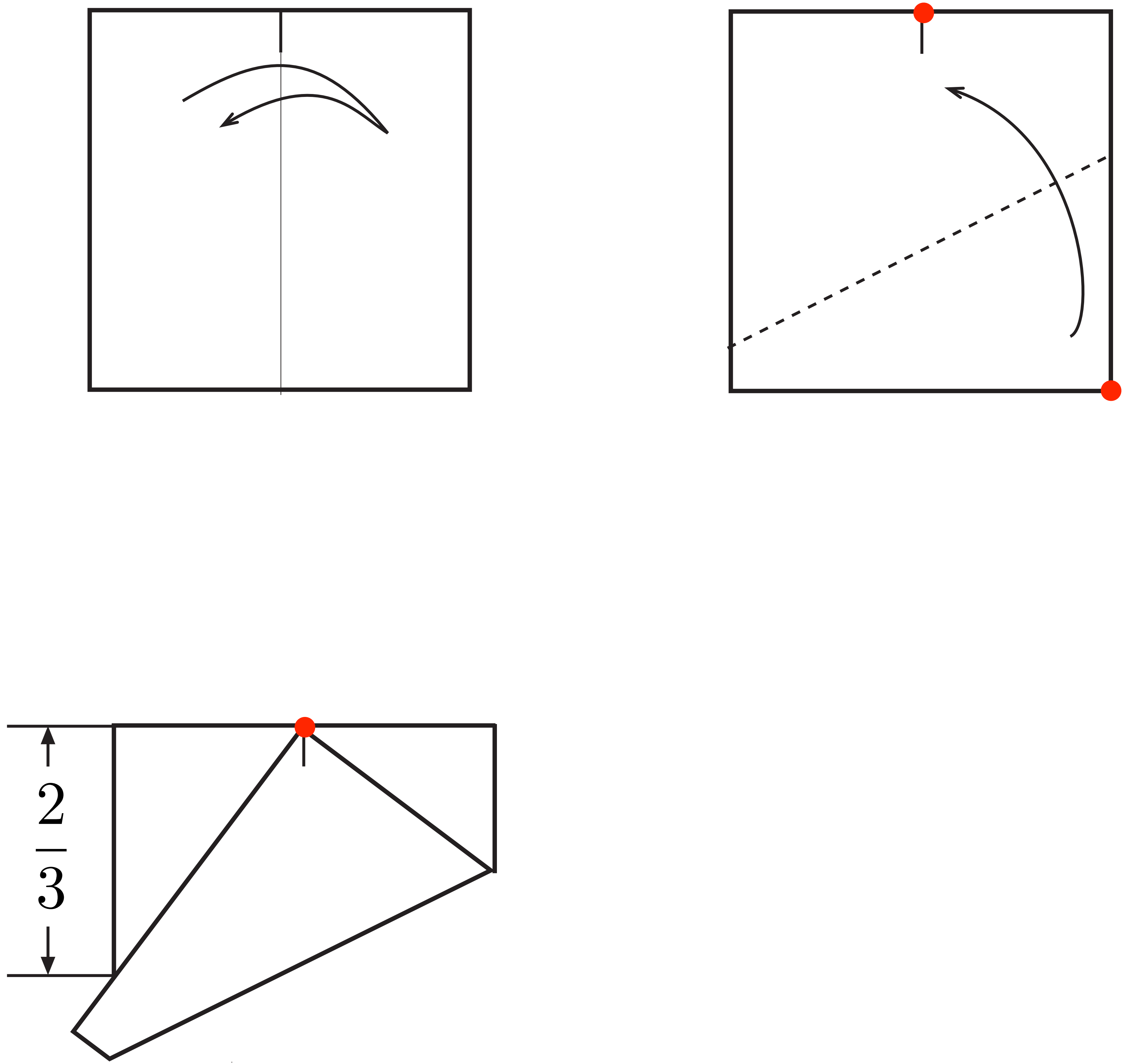
Public Diagram Project







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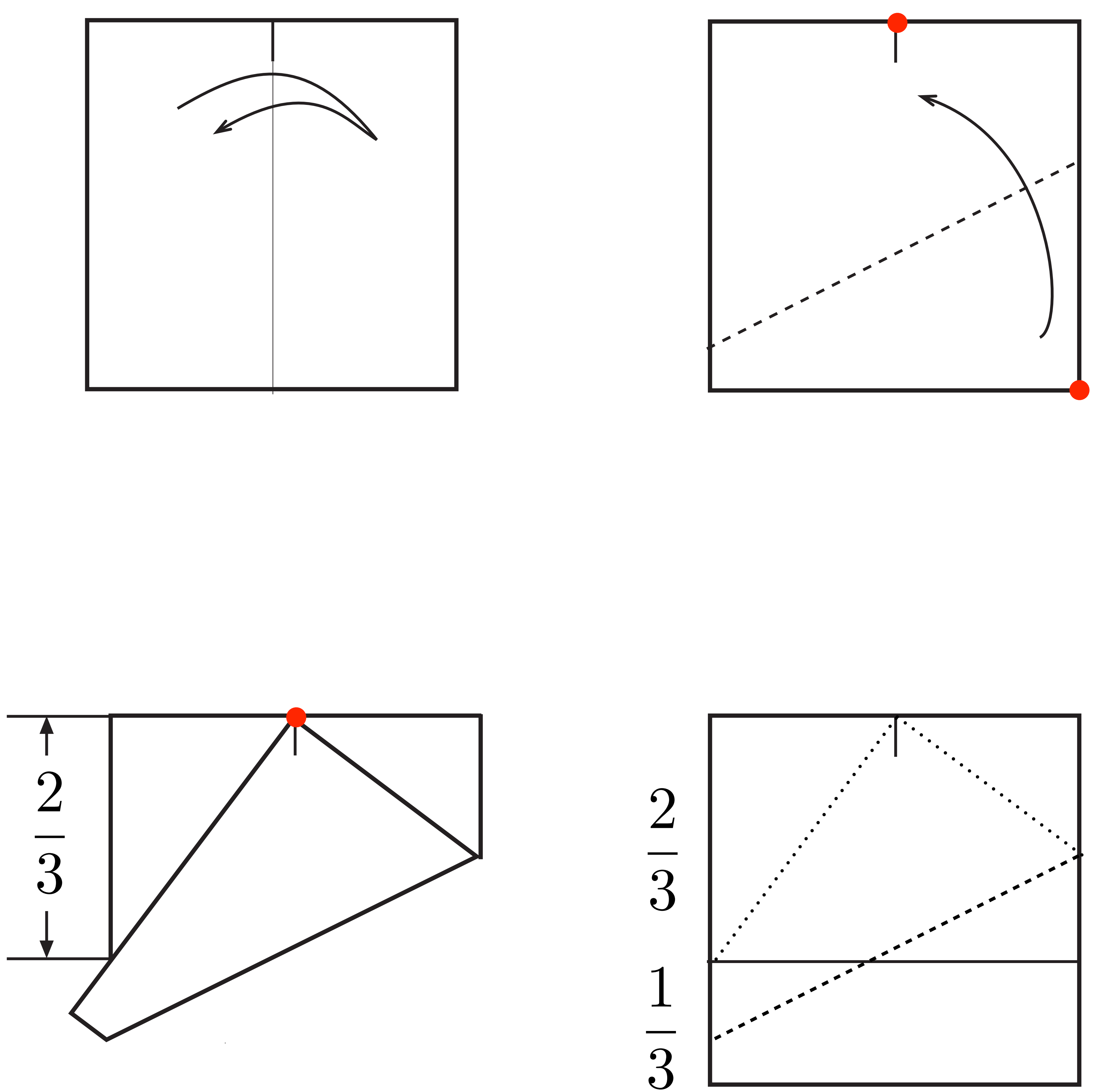
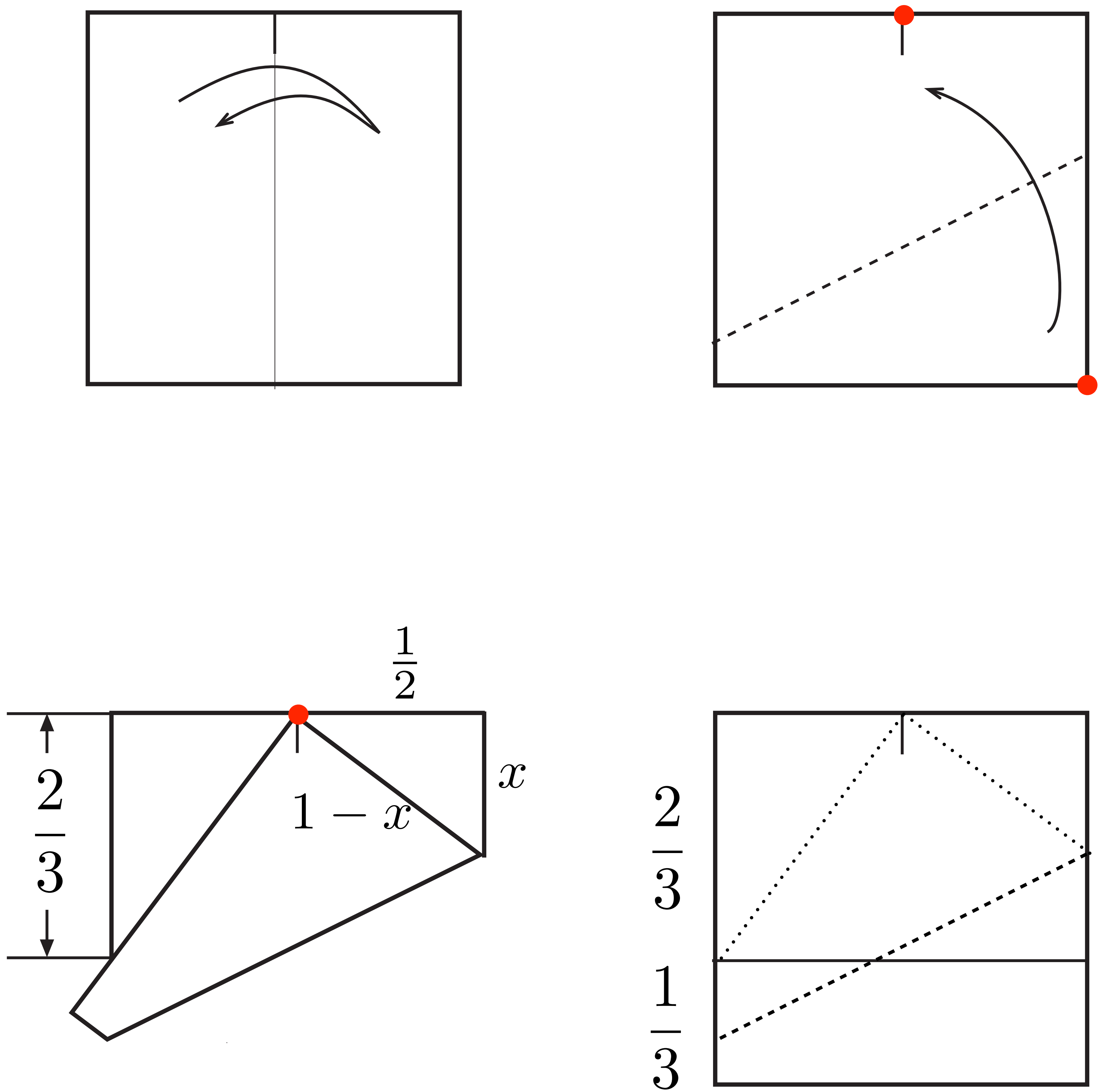
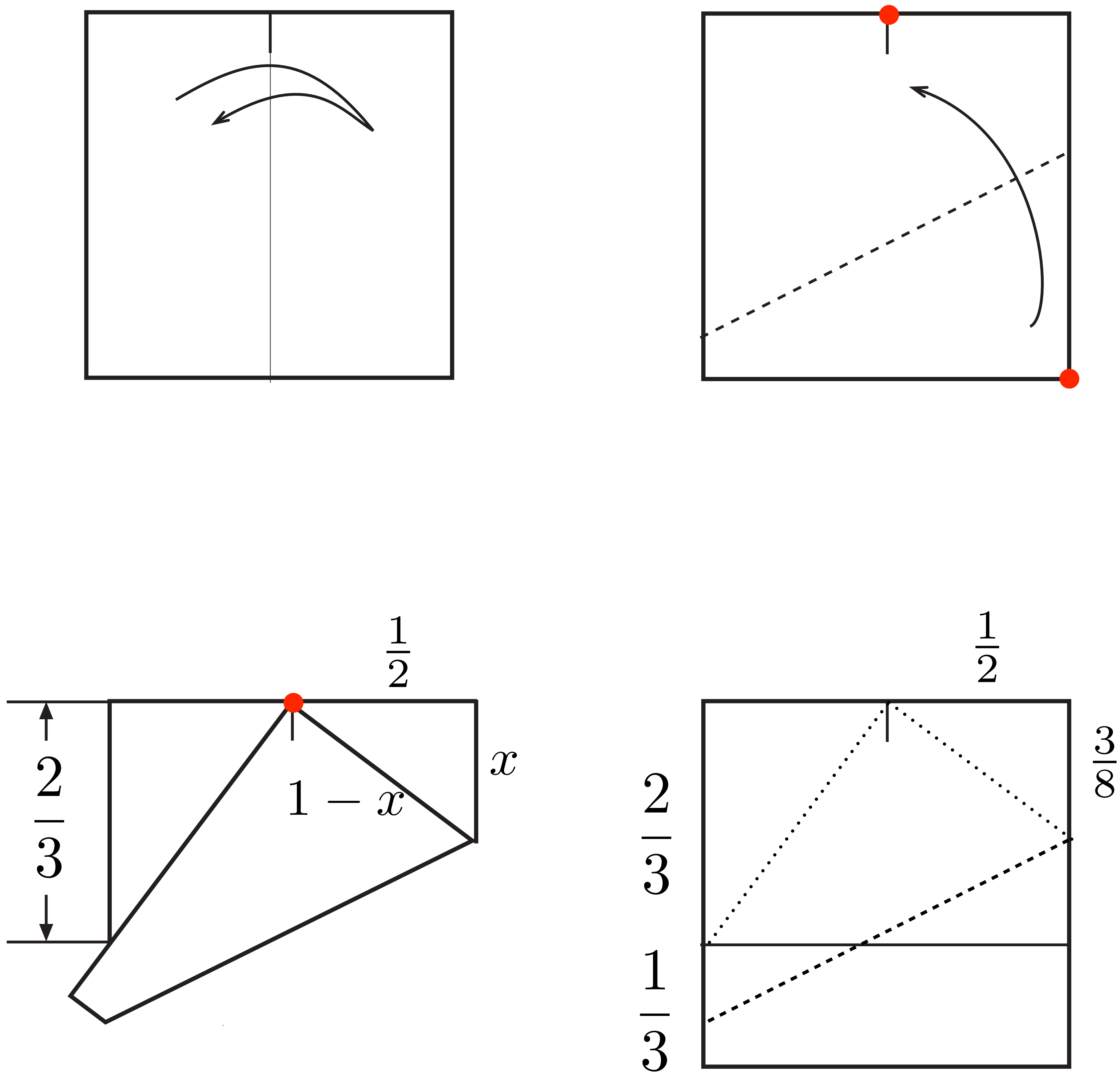
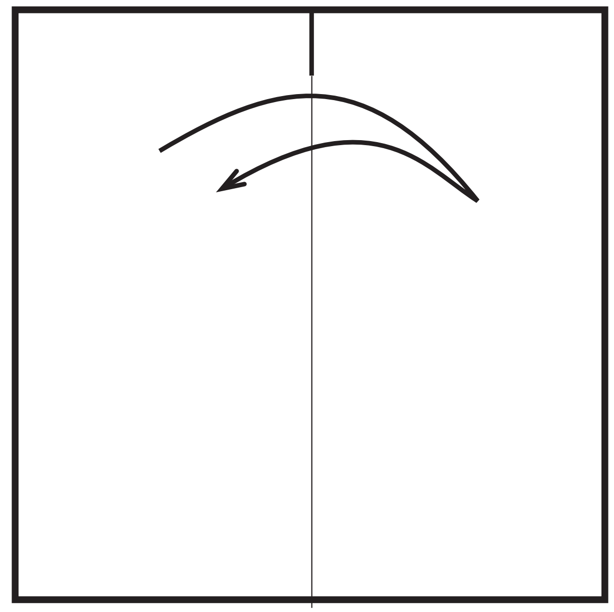
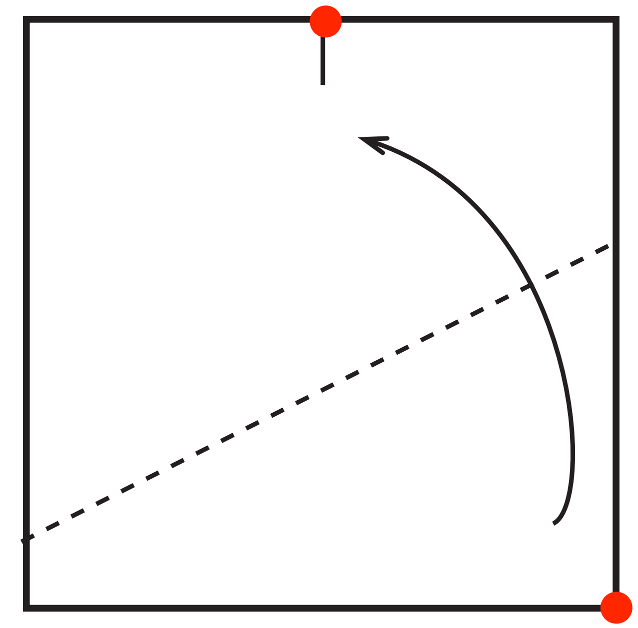

2/3
2/3
1/3

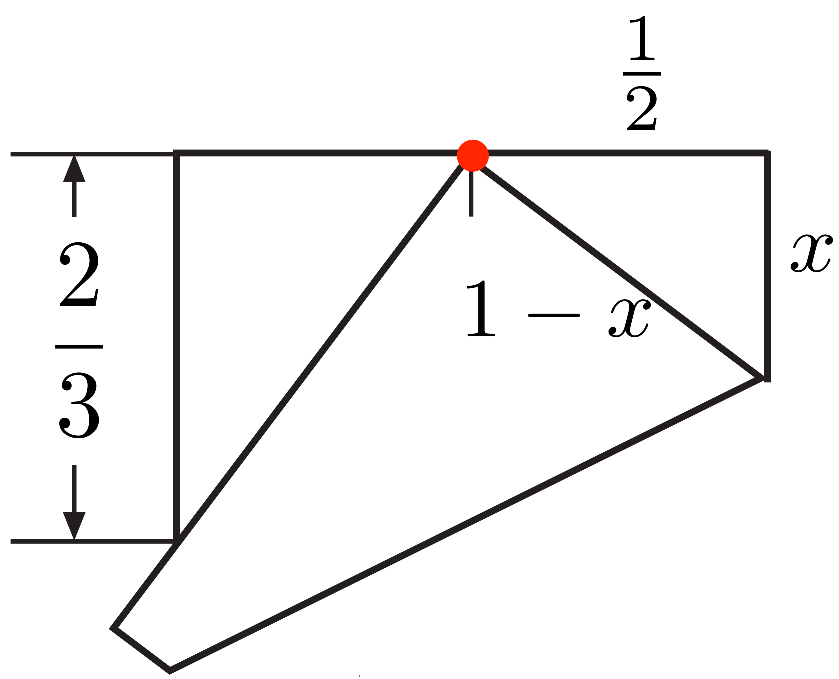


$\frac{1}{2}$
$\frac{2}{3}$
x
$1 - x$
$\frac{2}{3}$
$\frac{1}{3}$

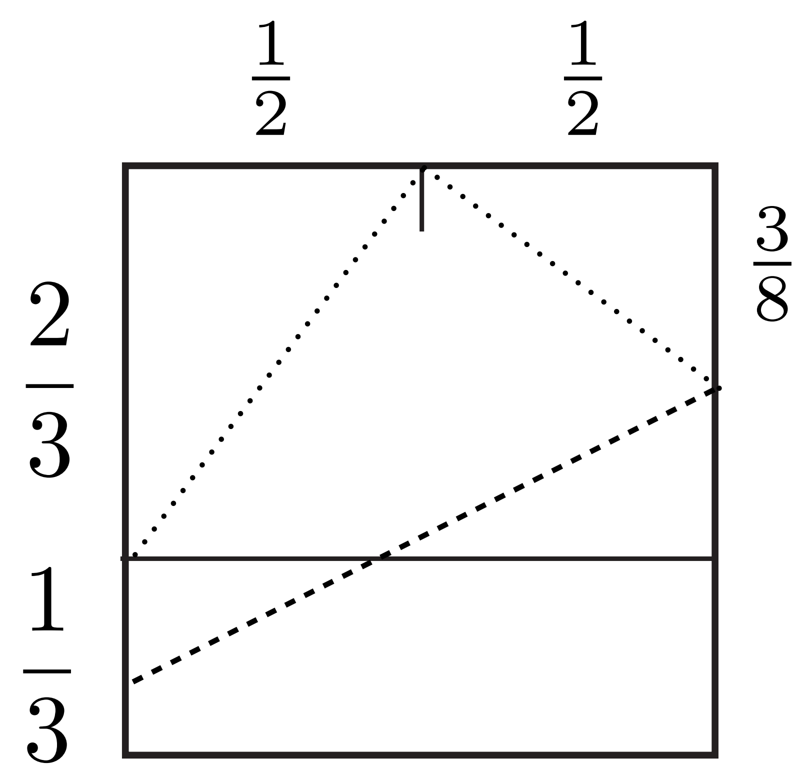

1/2
2/3
x
1 − x
1/2
3/8
2/3
1/3



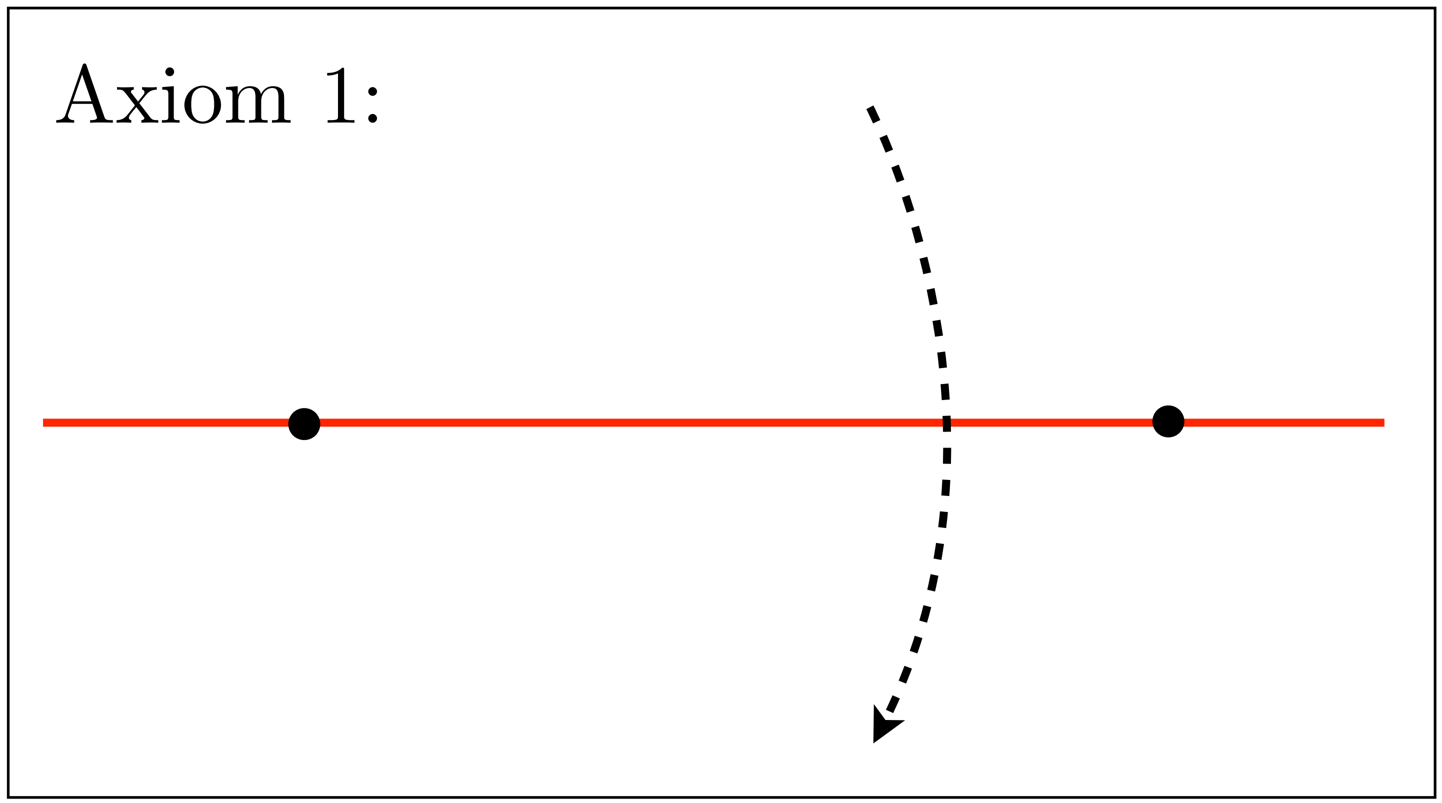




$\frac{1}{2}$
$\frac{2}{3}$
x
$1 - x$



$\frac{1}{2}$
$\frac{1}{2}$
$\frac{3}{8}$
$\frac{2}{3}$
$\frac{1}{3}$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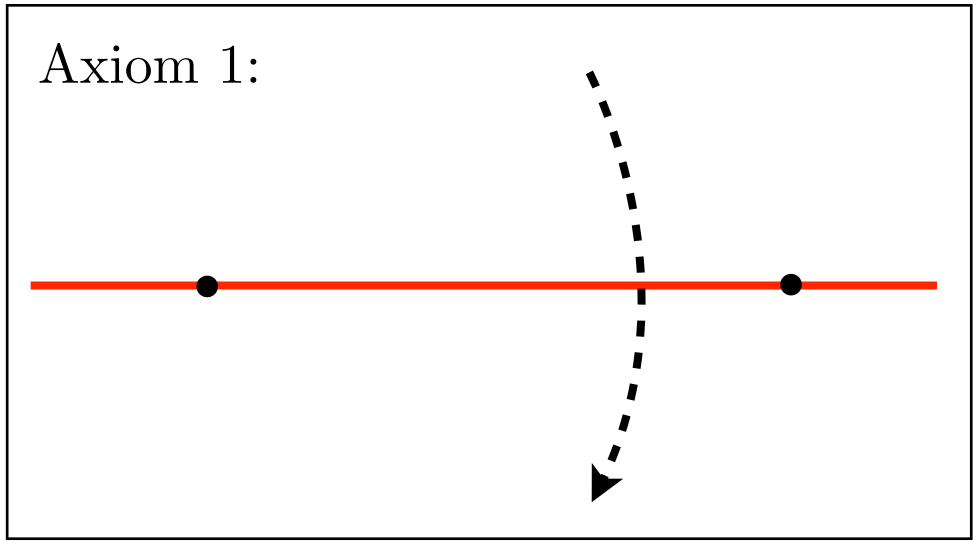
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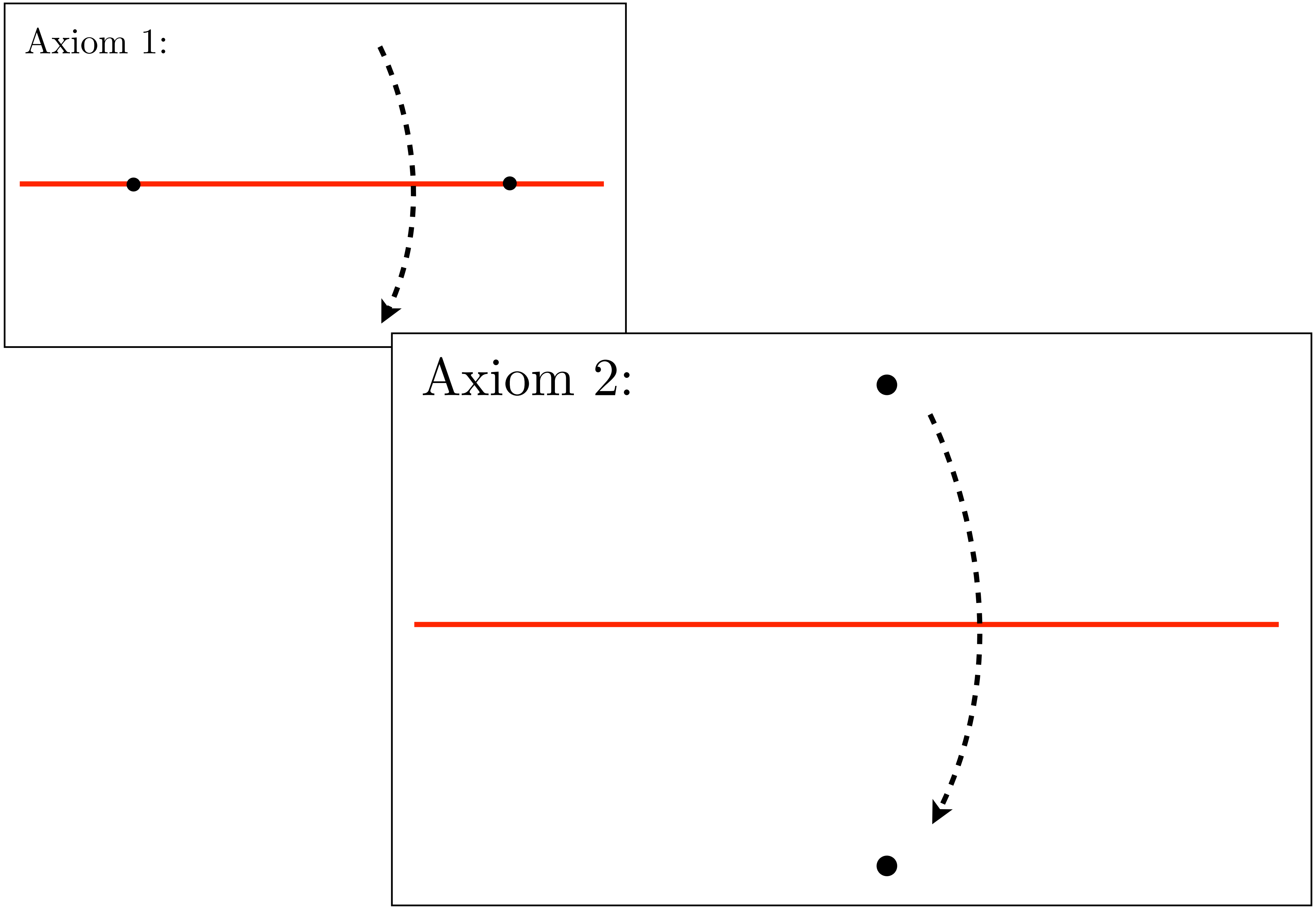
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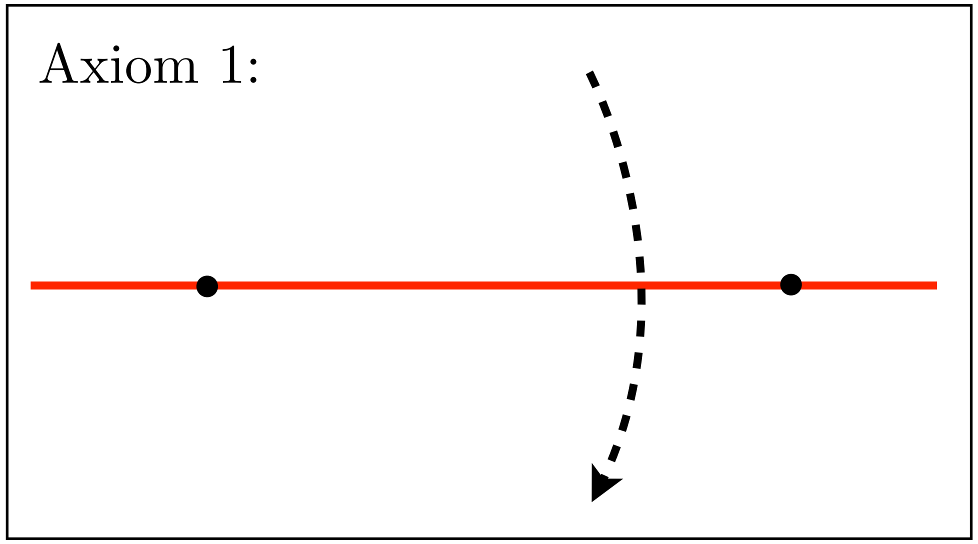
Axiom 2: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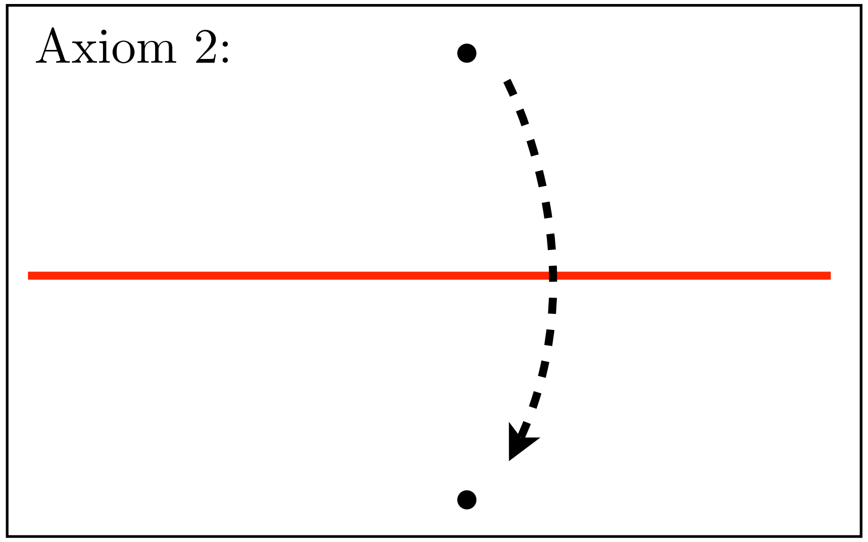
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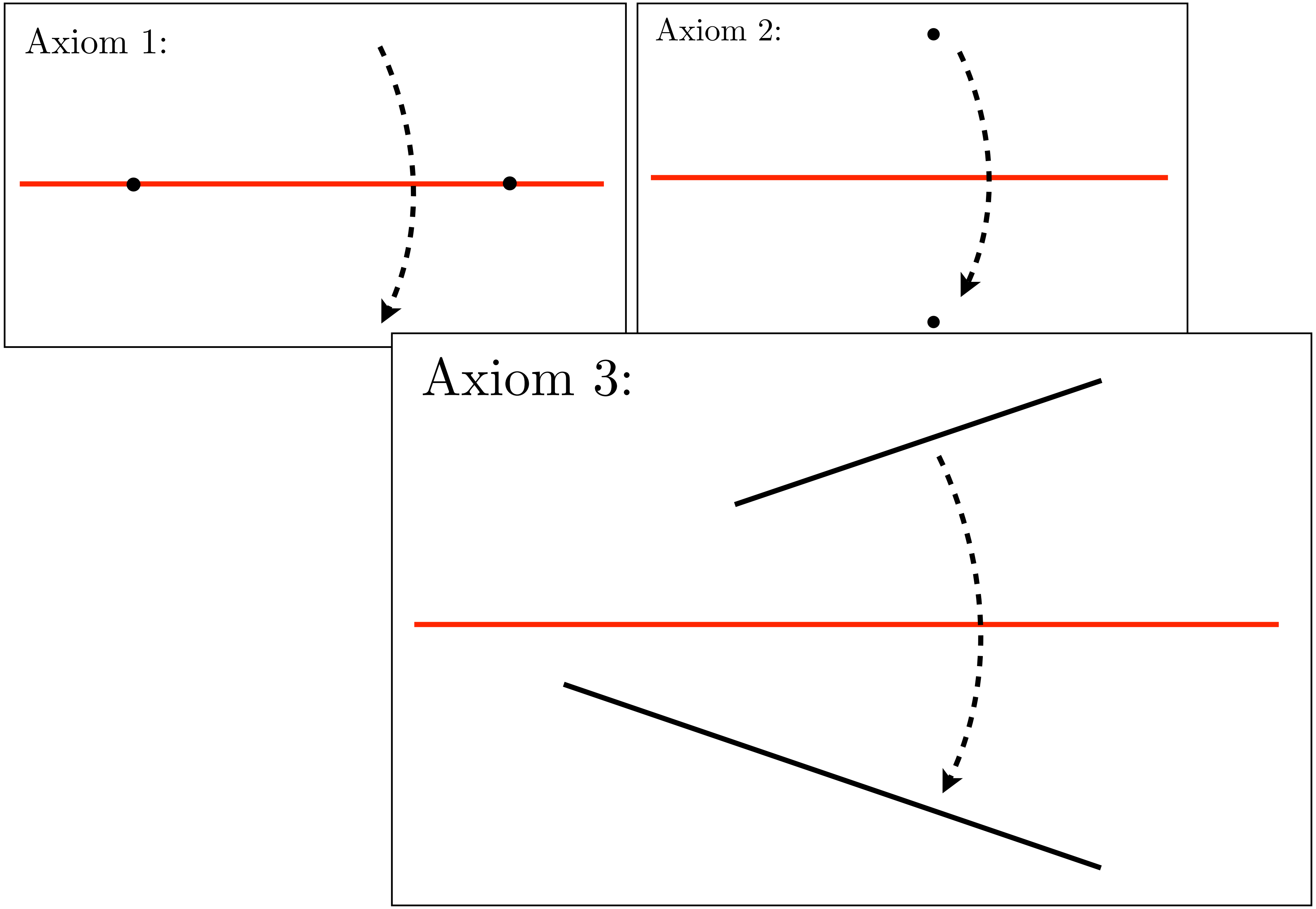
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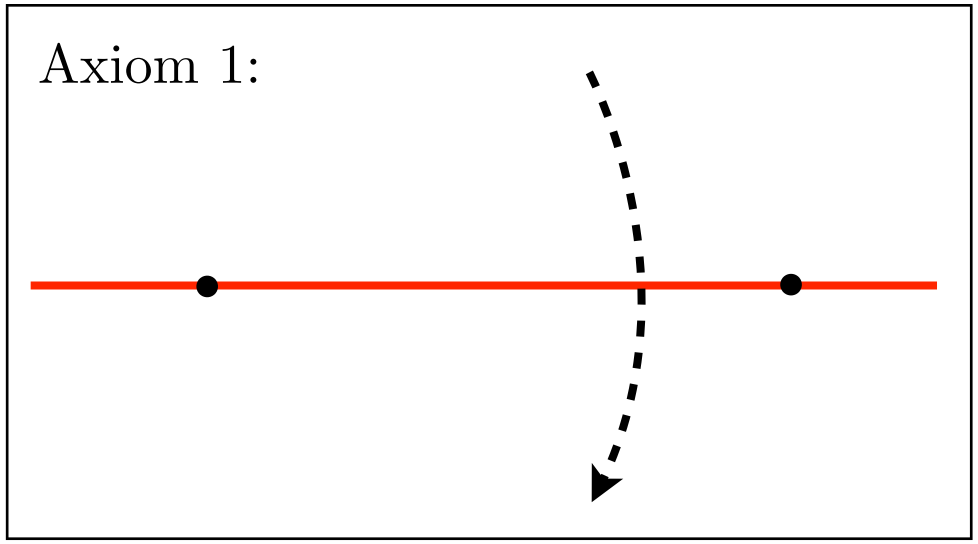
Axiom 2:
Axiom 3: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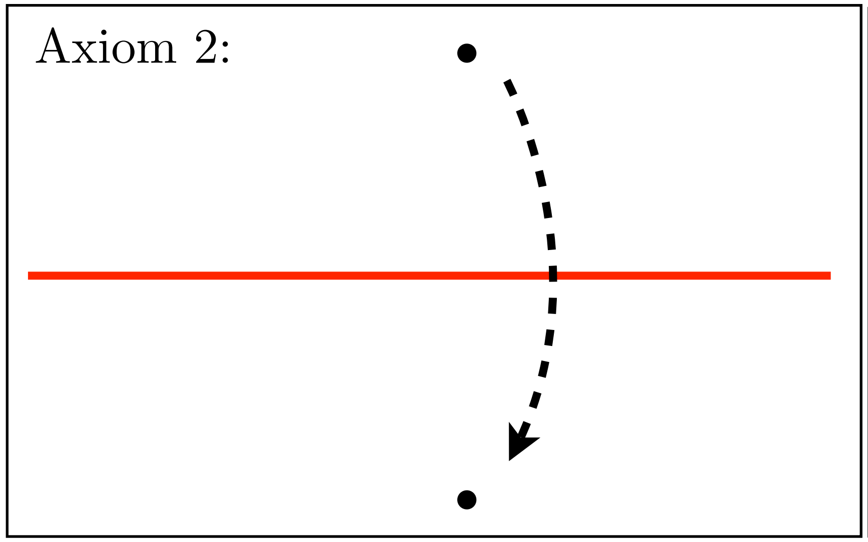
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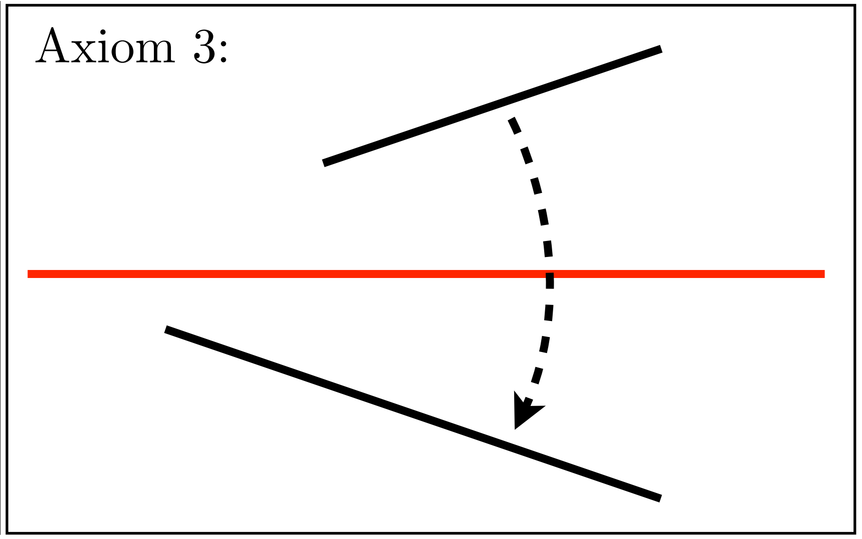
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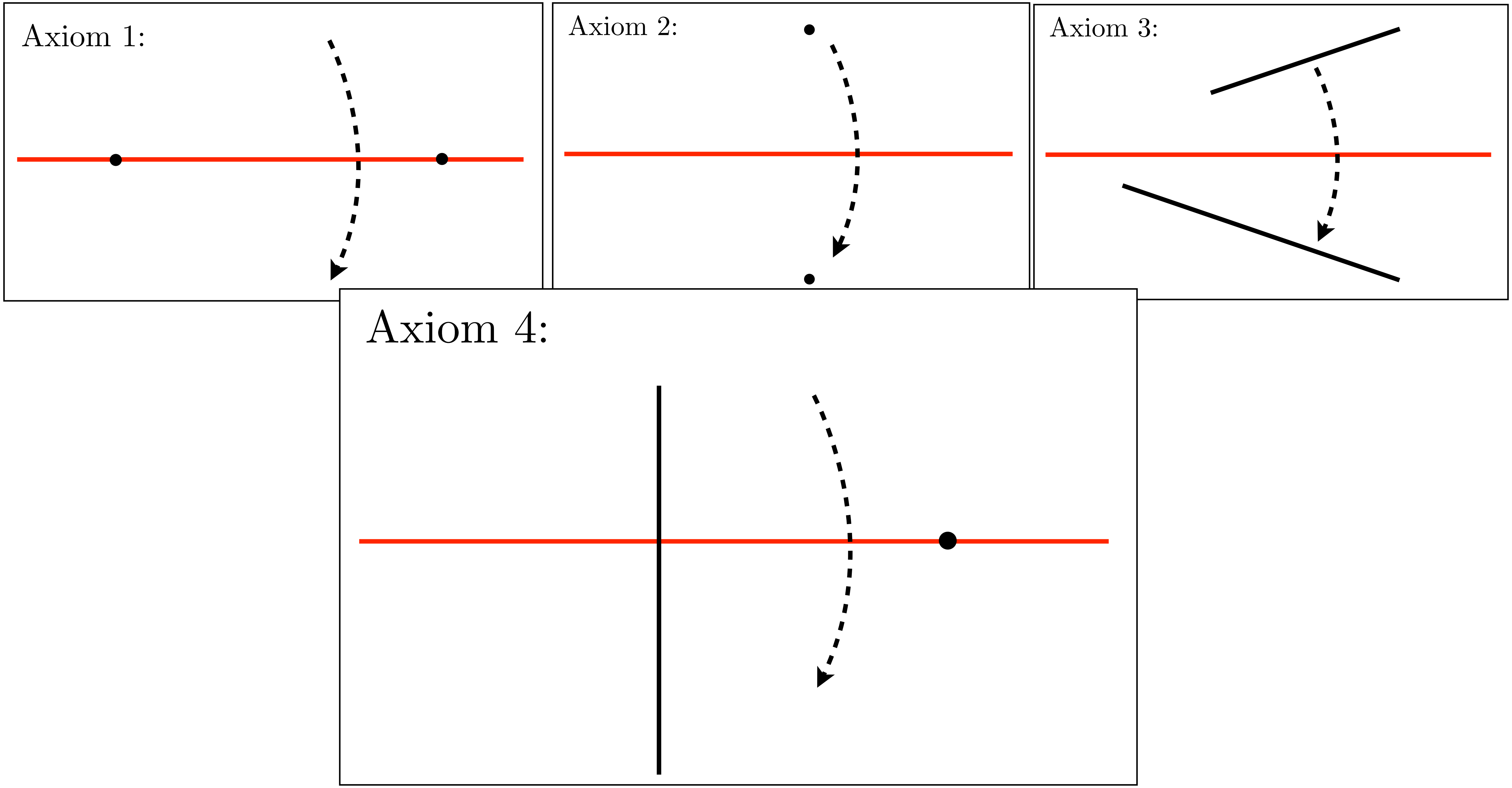
Axiom 1:
Axiom 2:
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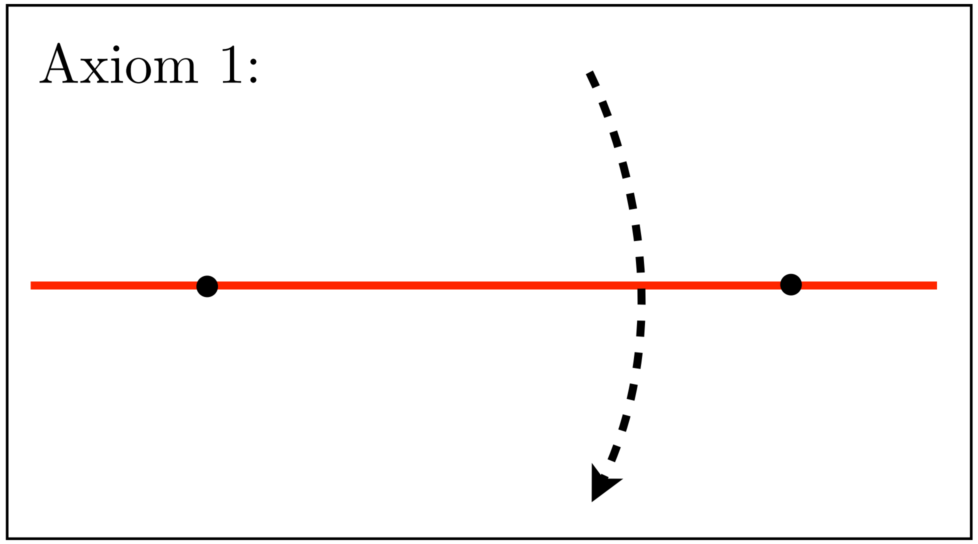
Axiom 4: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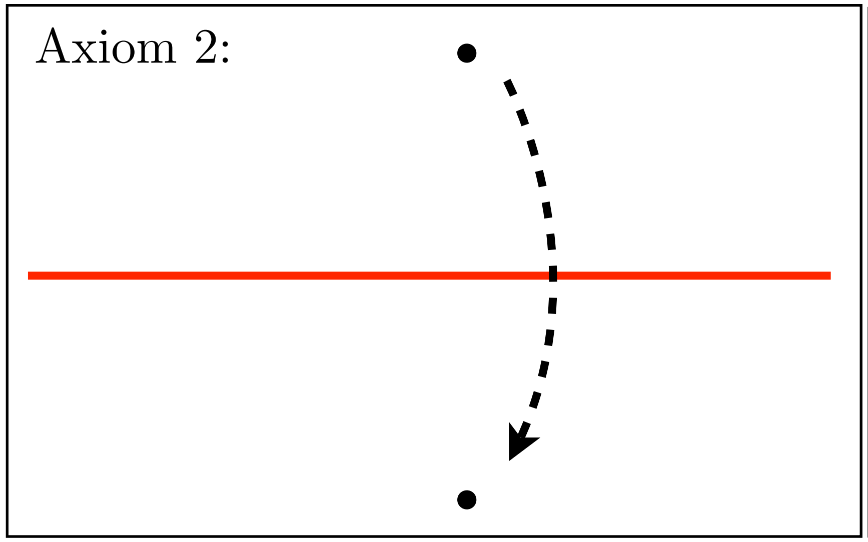
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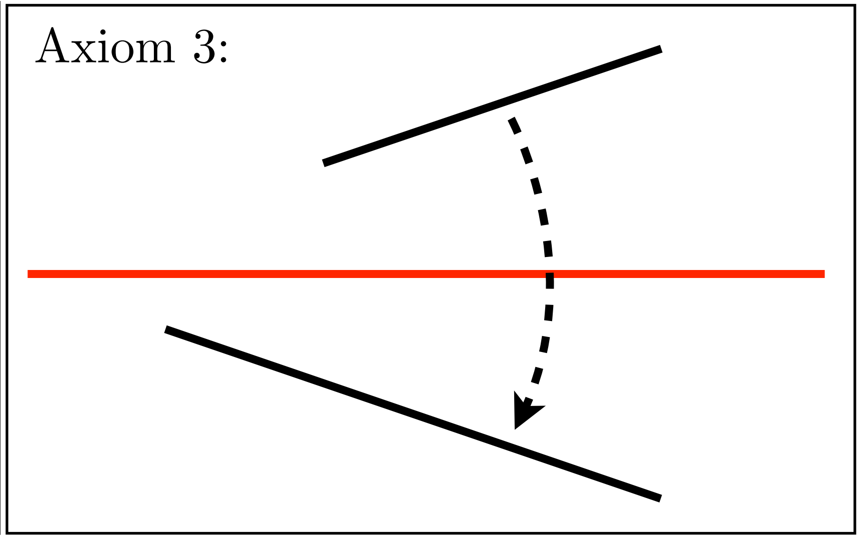
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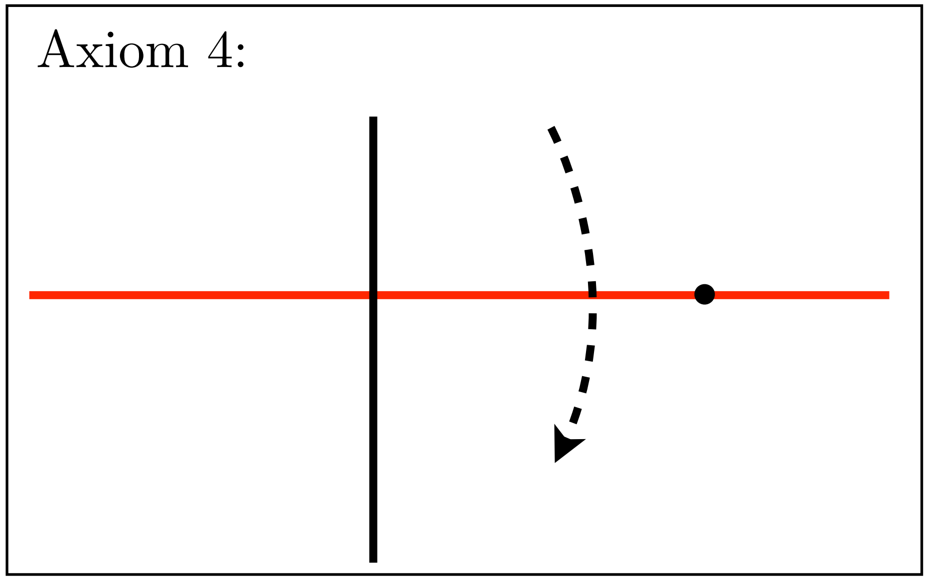
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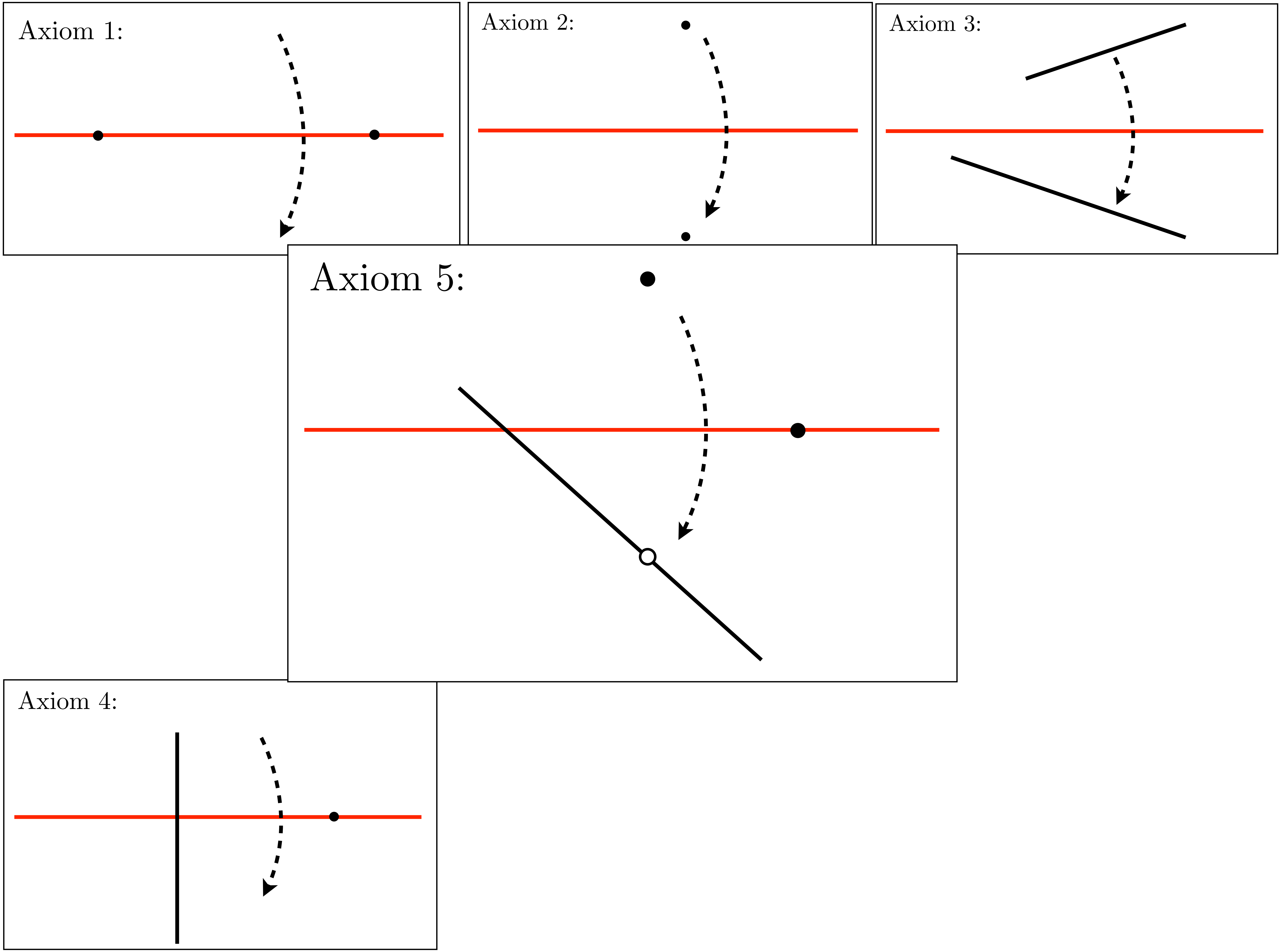
Axiom 1:
Axiom 2:
Axiom 3:
Axiom 5:
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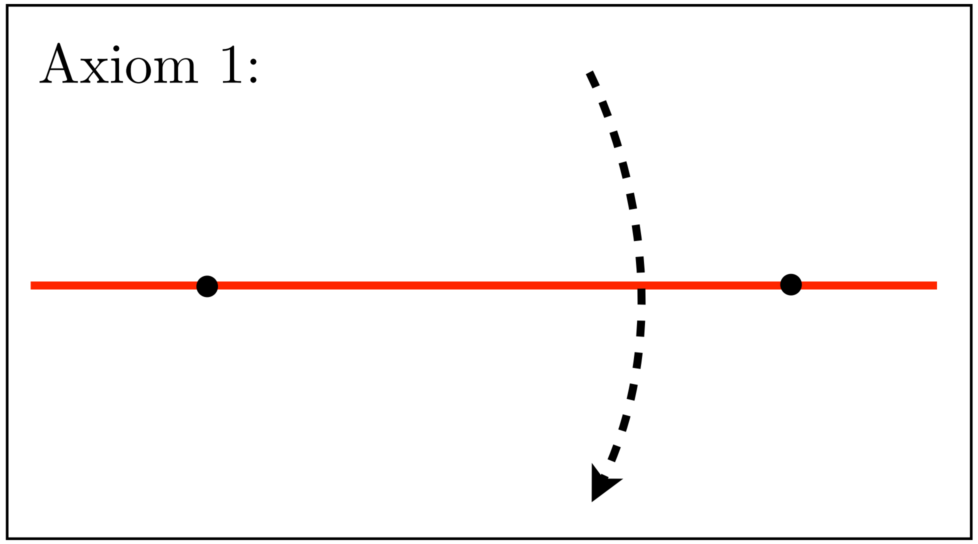
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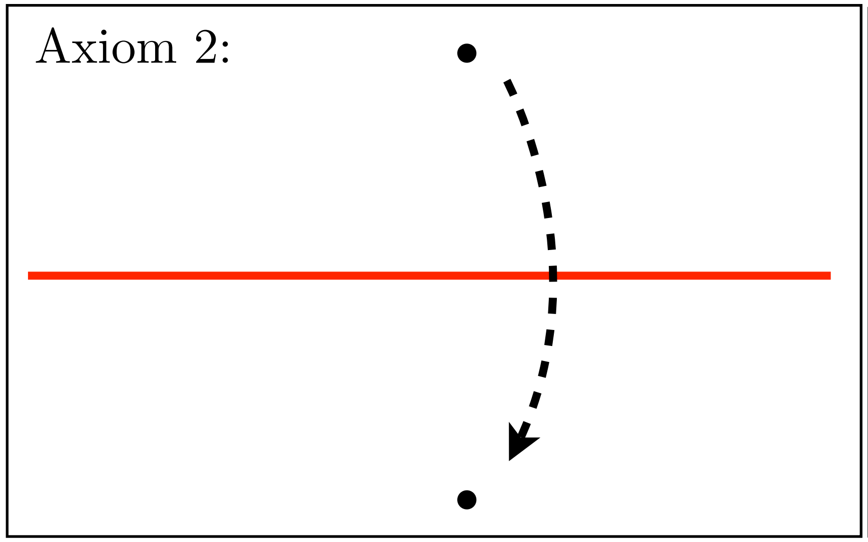
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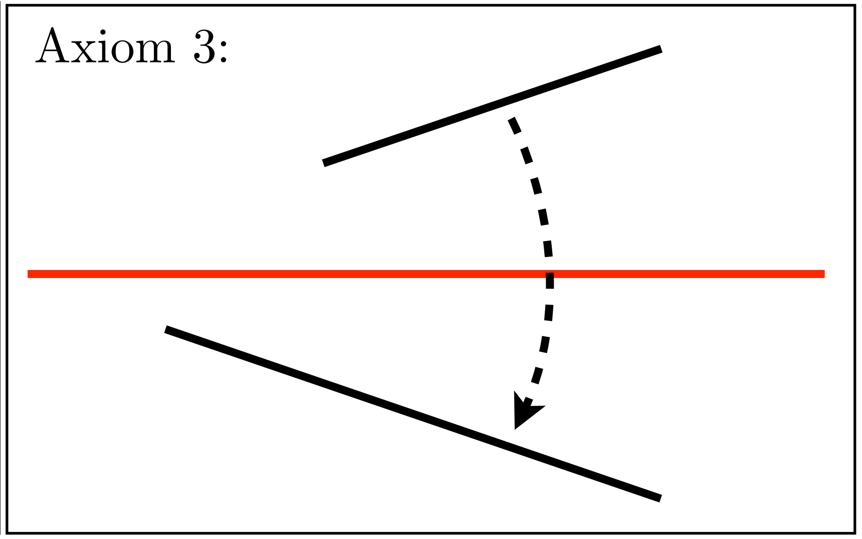
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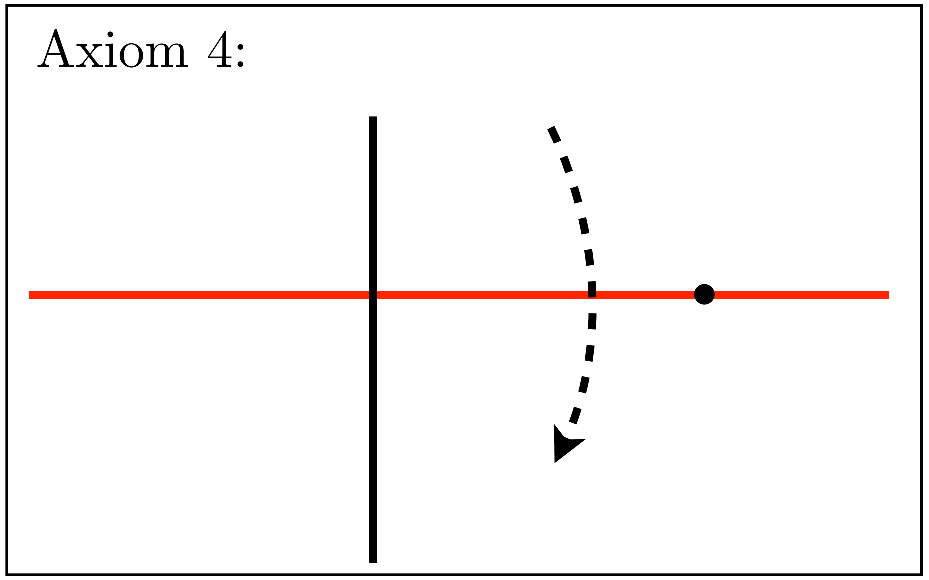
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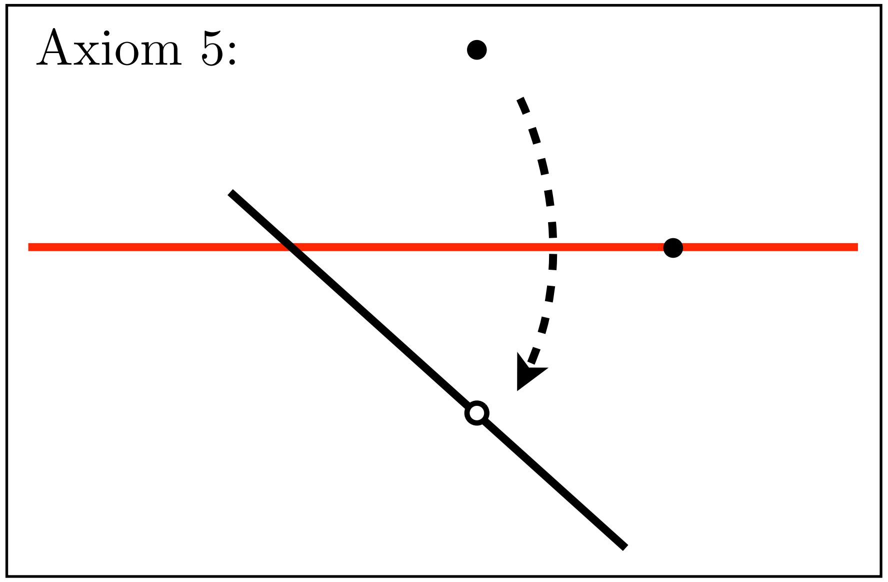
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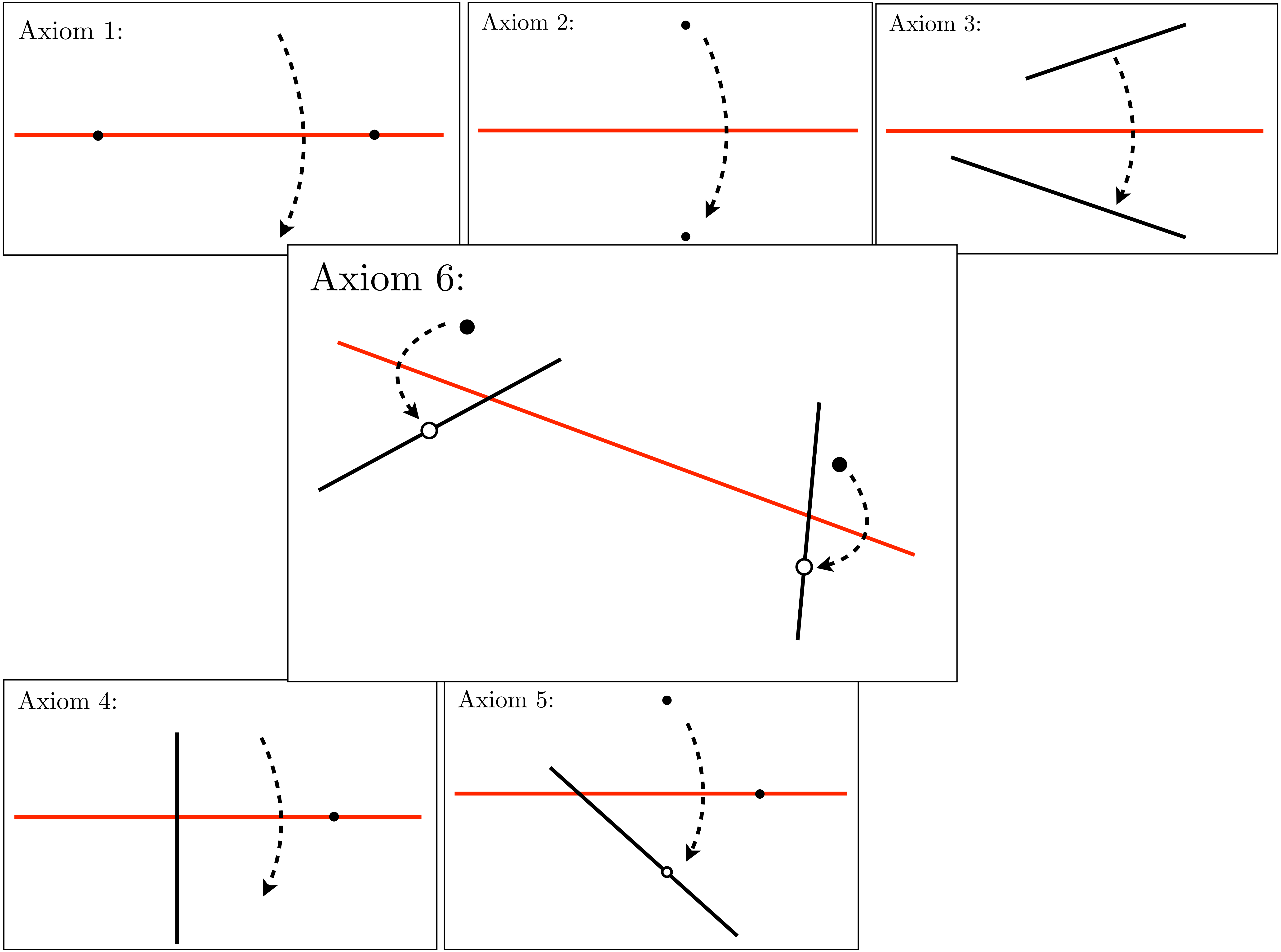
Axiom 1:
Axiom 2:
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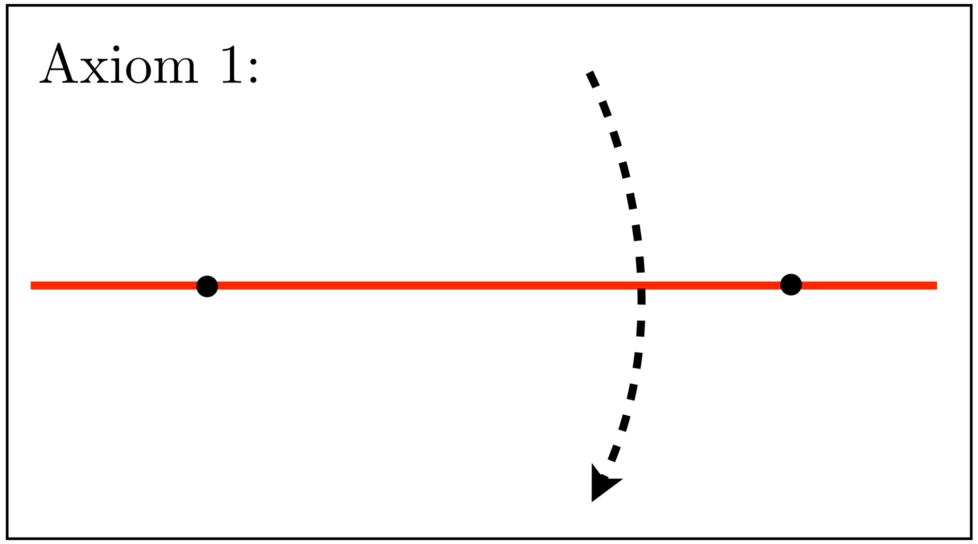
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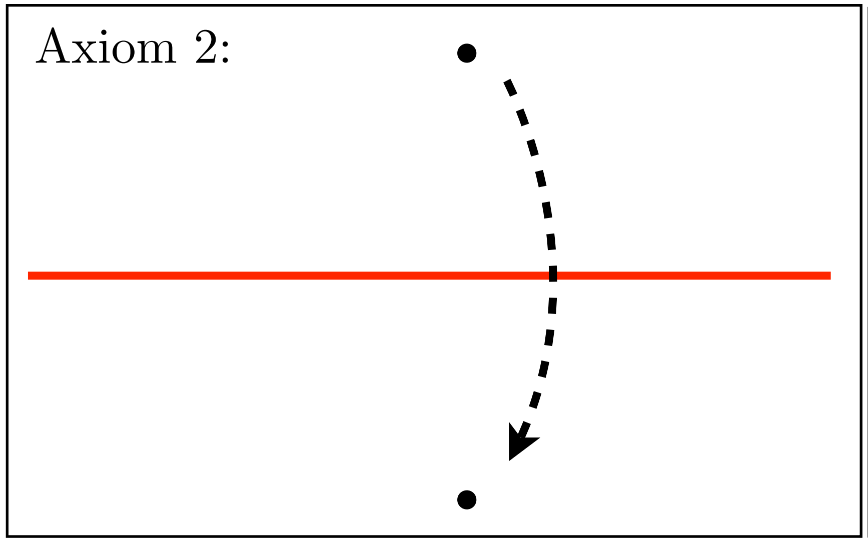
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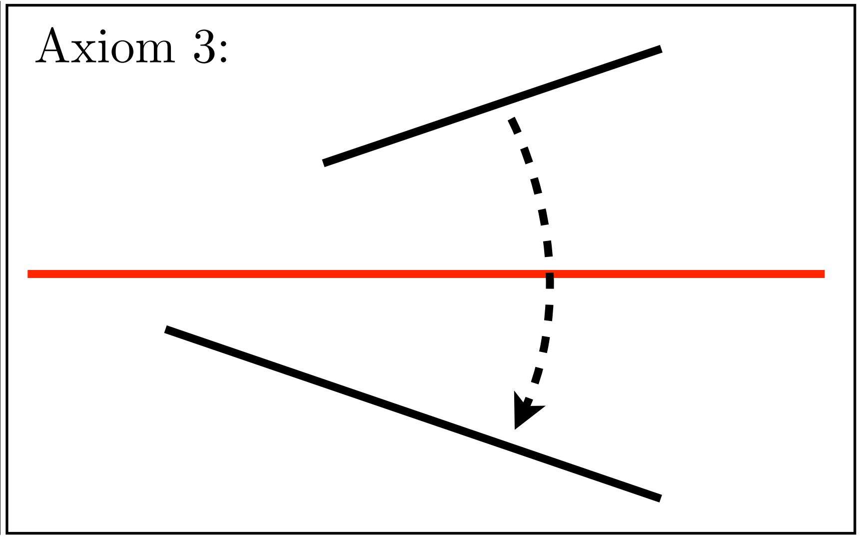
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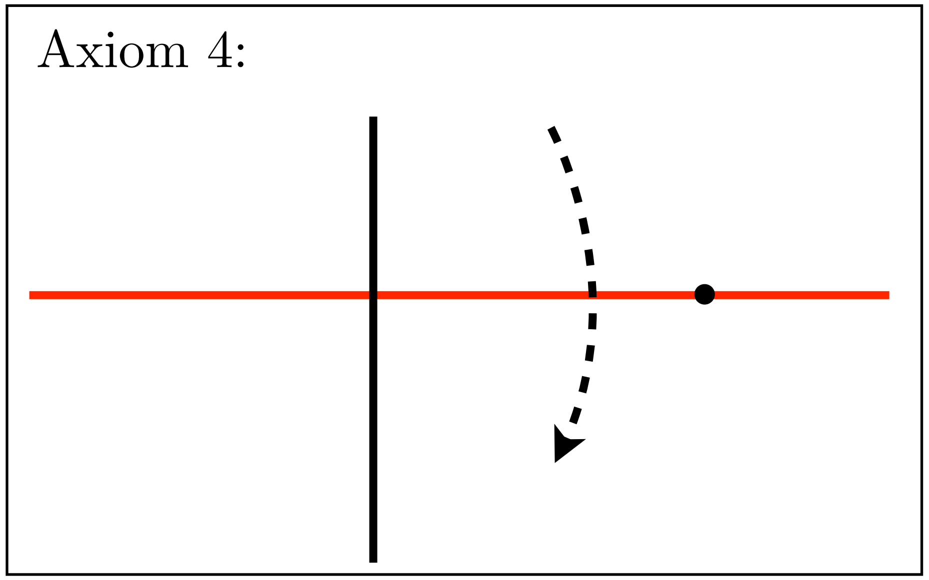
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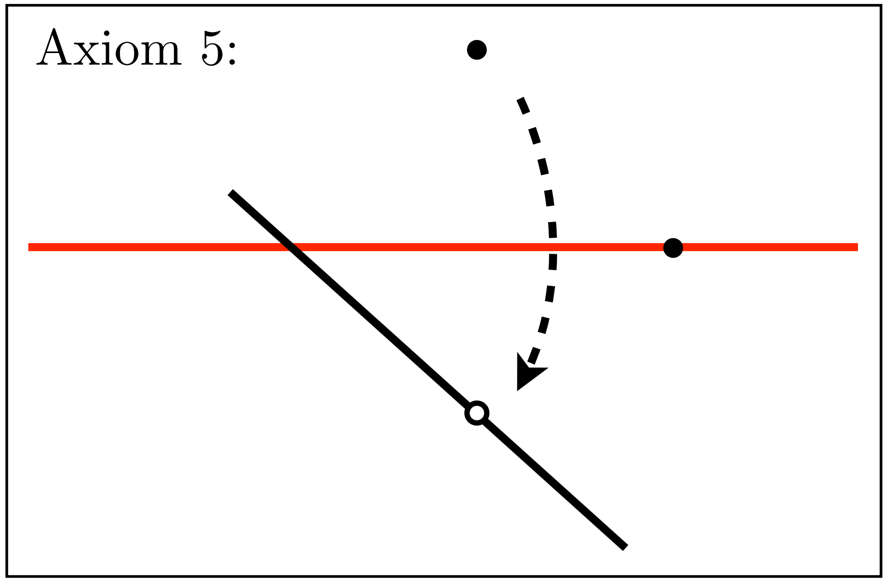
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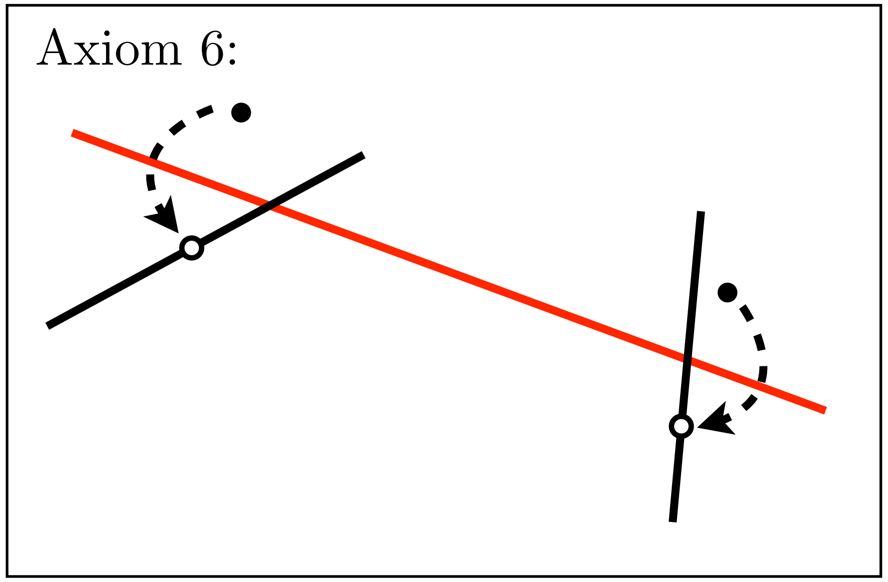
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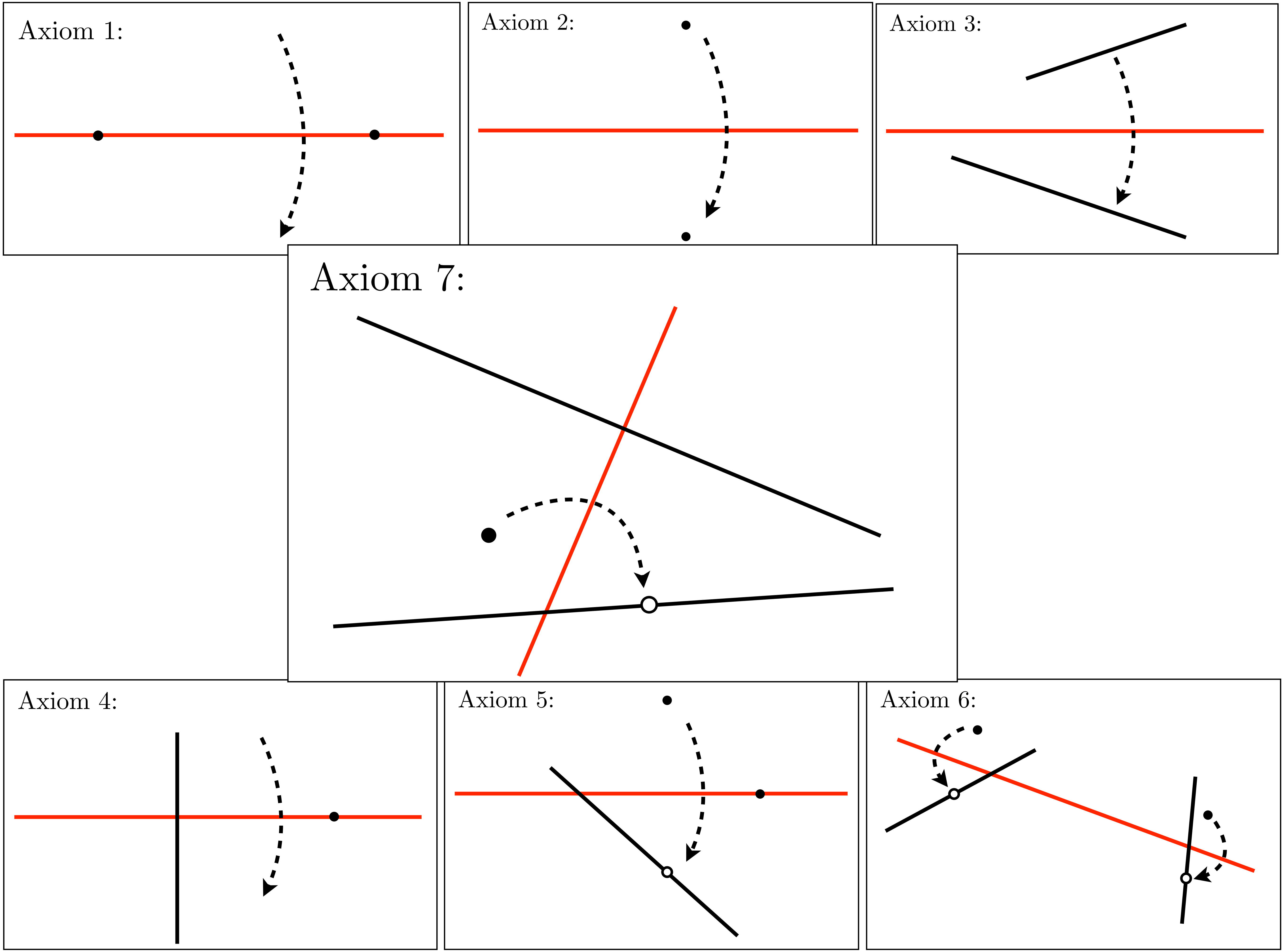
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Axiom 2:
Axiom 3:
Axiom 7:
Axiom 4:
Axiom 5:
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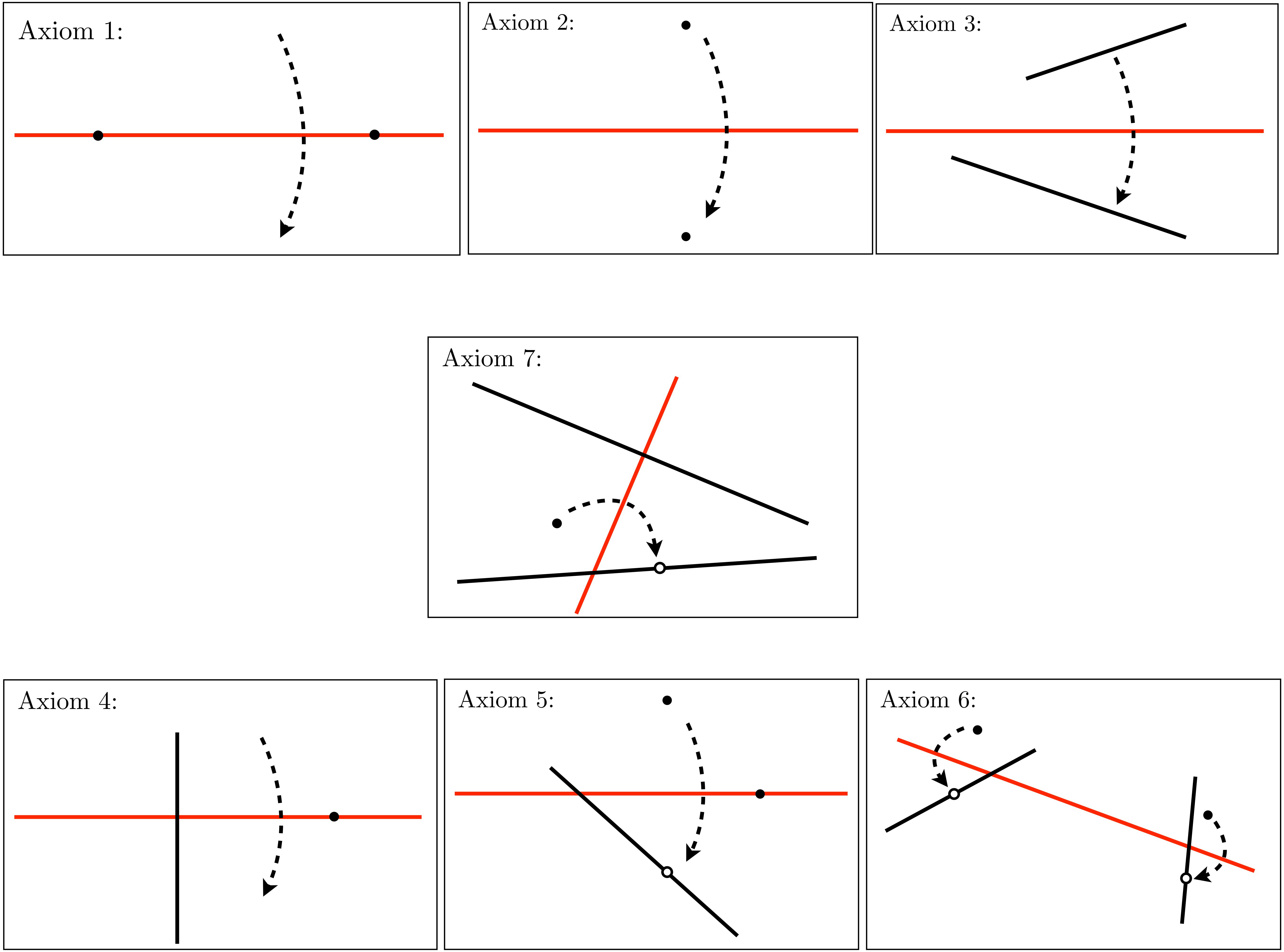
Axiom 1:
Axiom 2:
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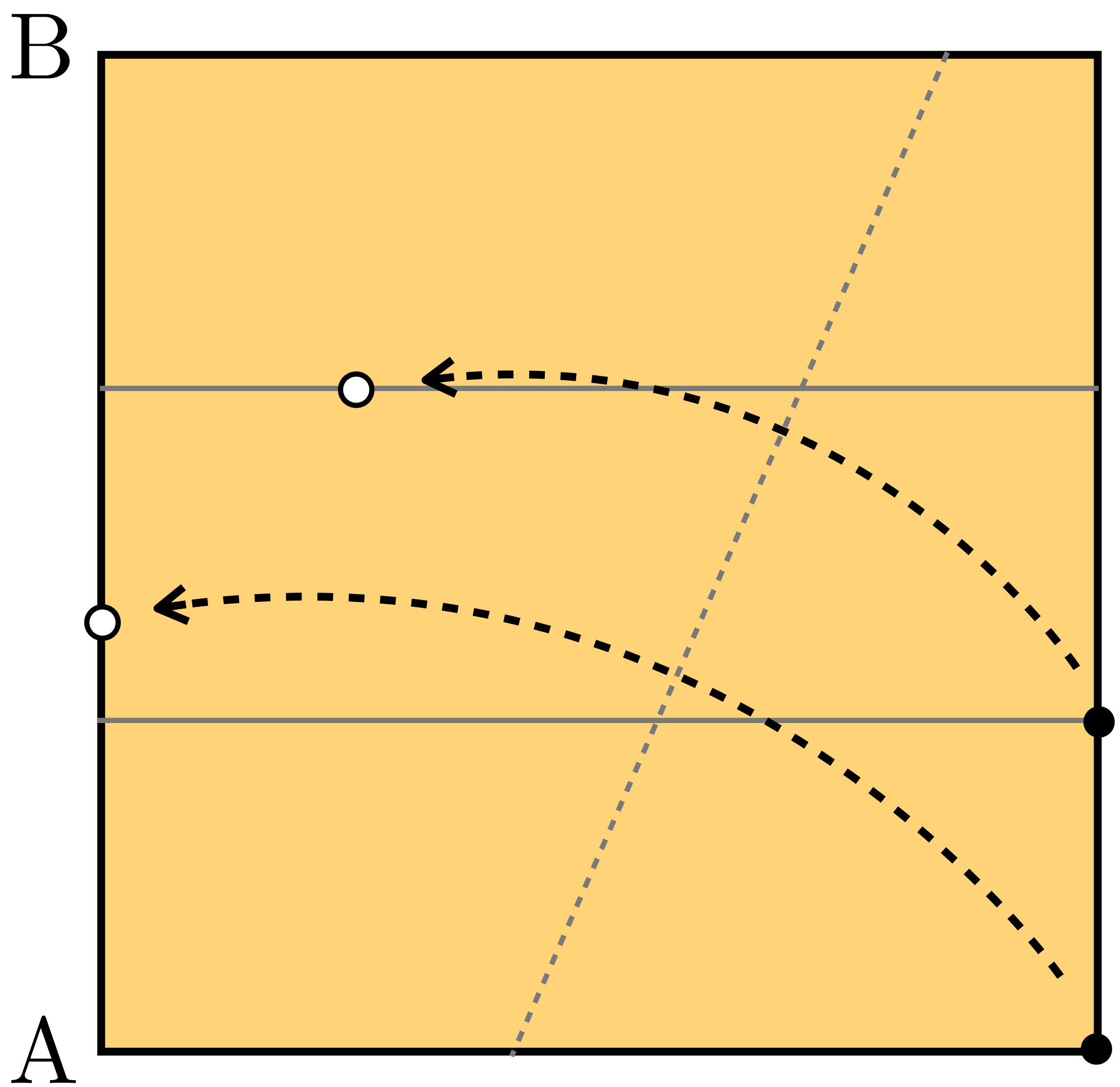
Peter Messer's construction of $\sqrt[3]{2}$:



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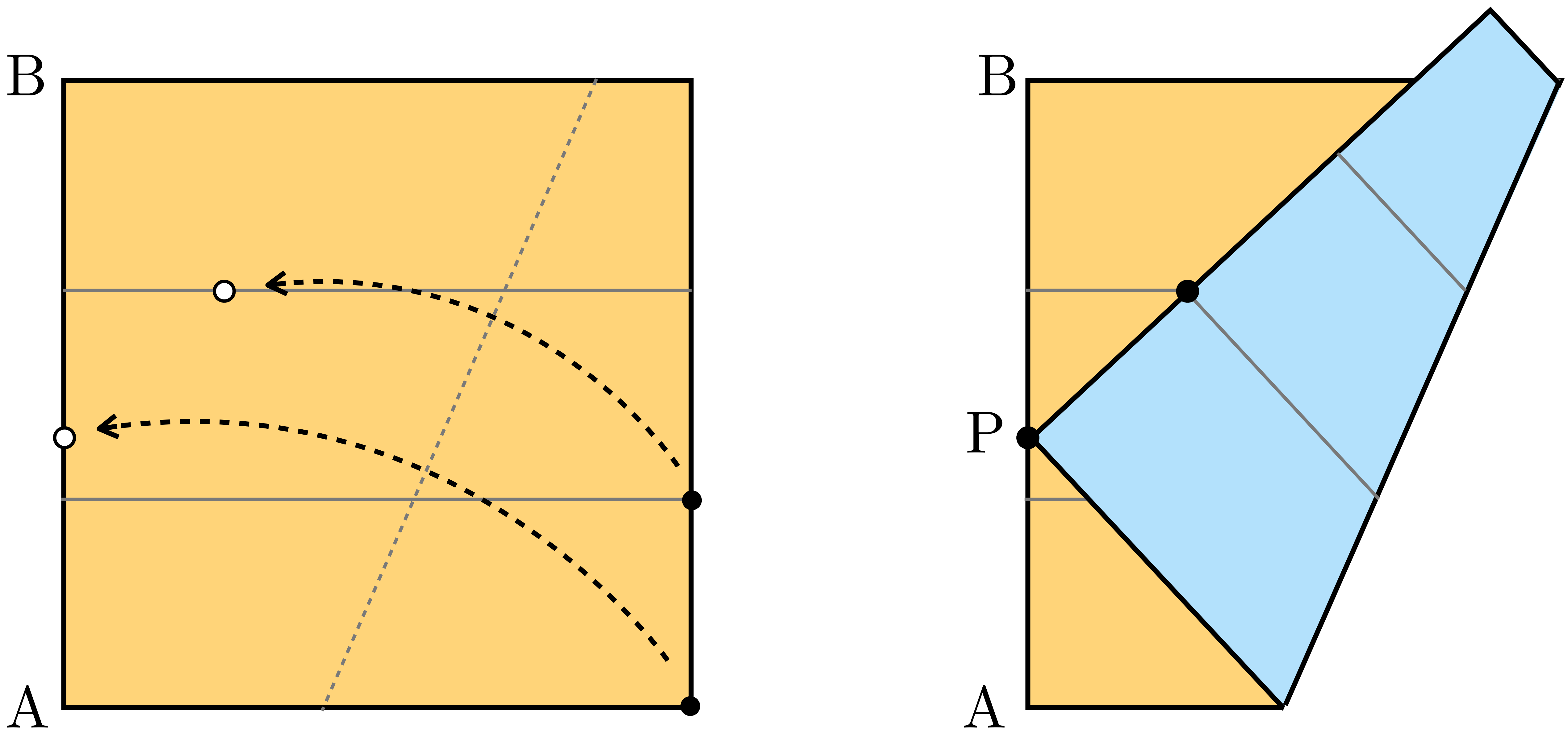

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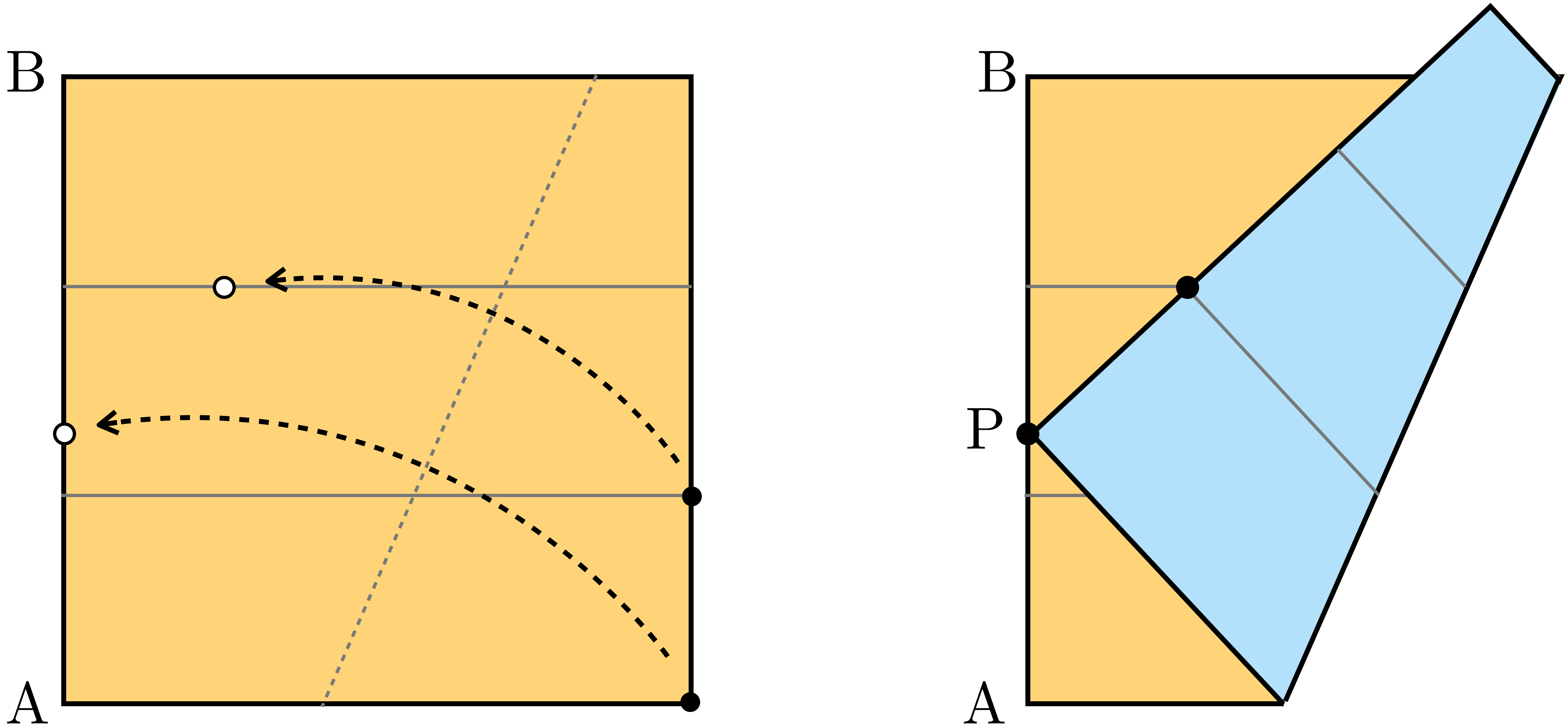
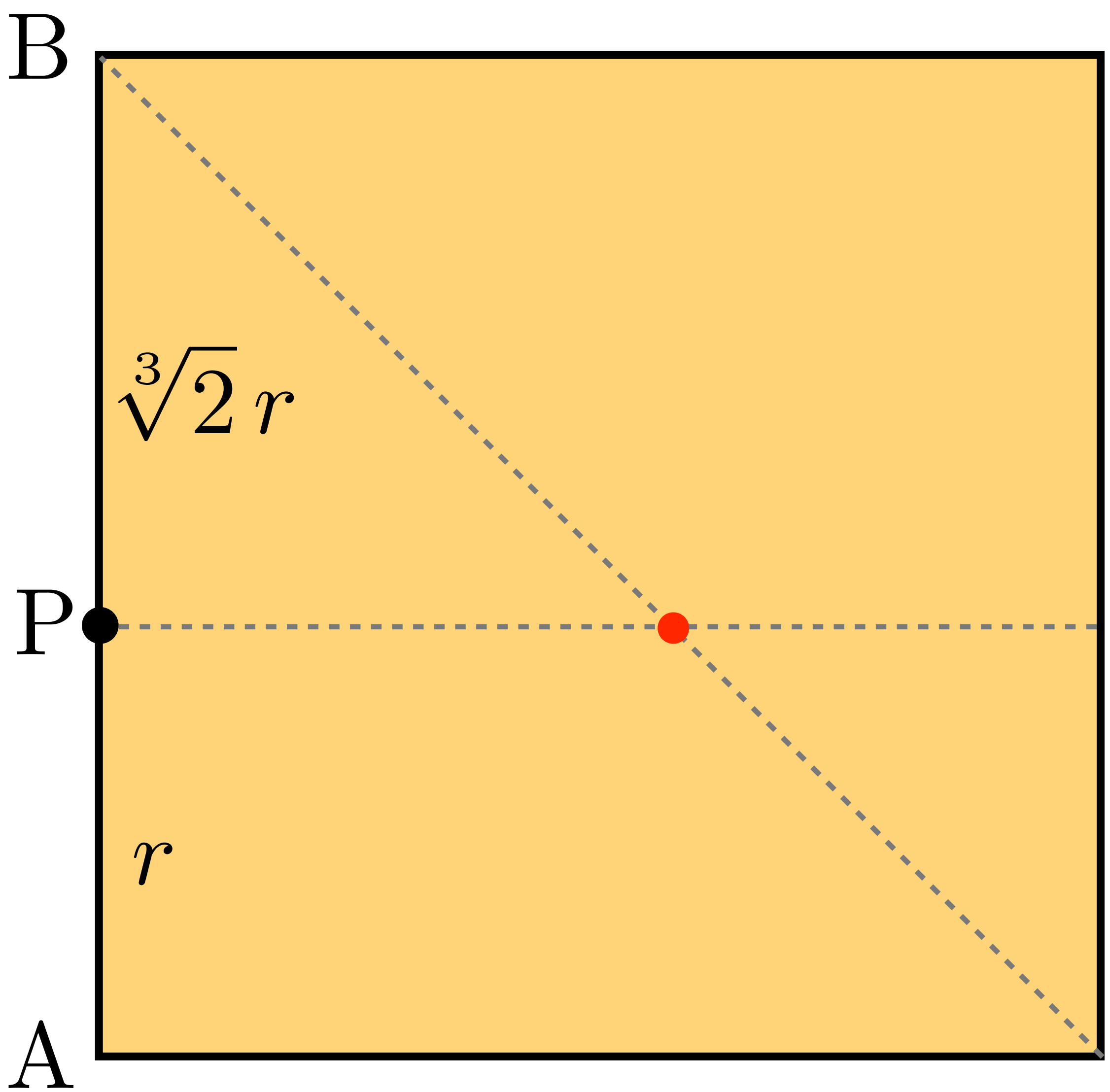
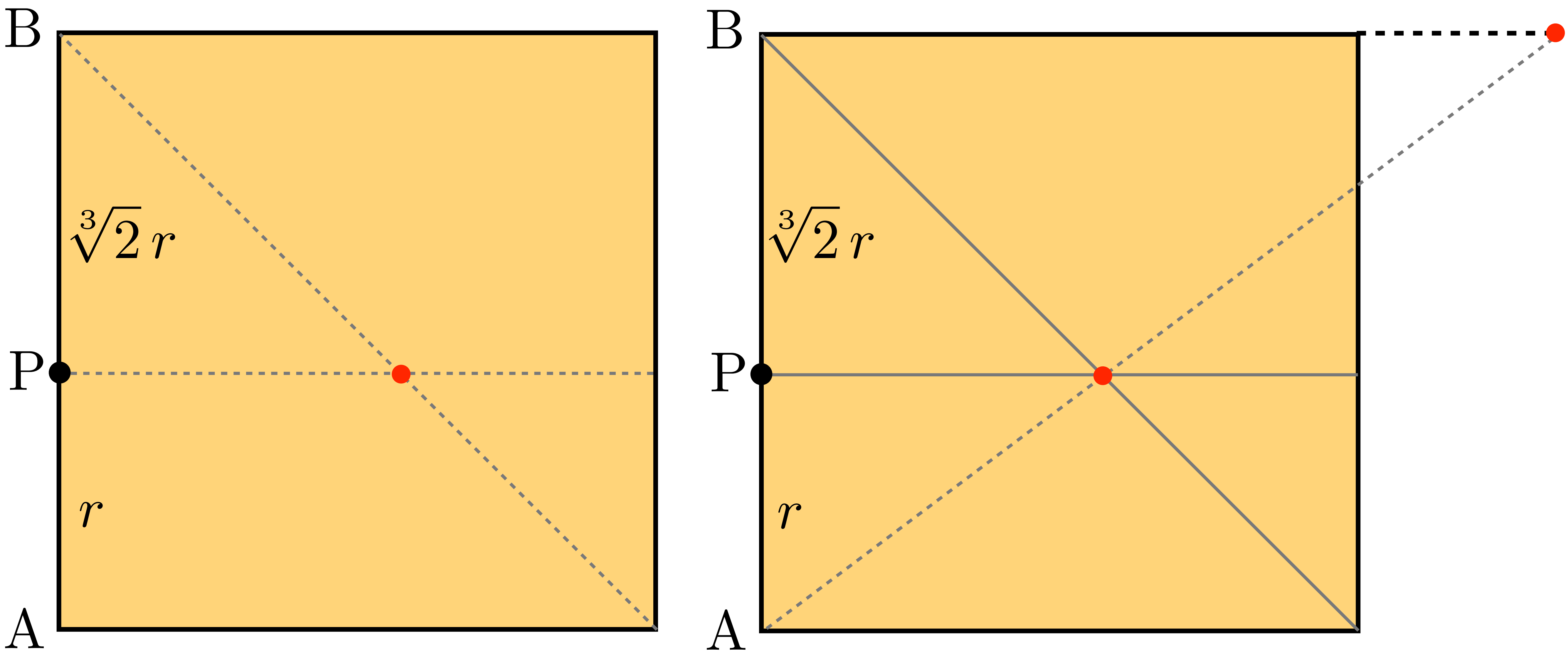
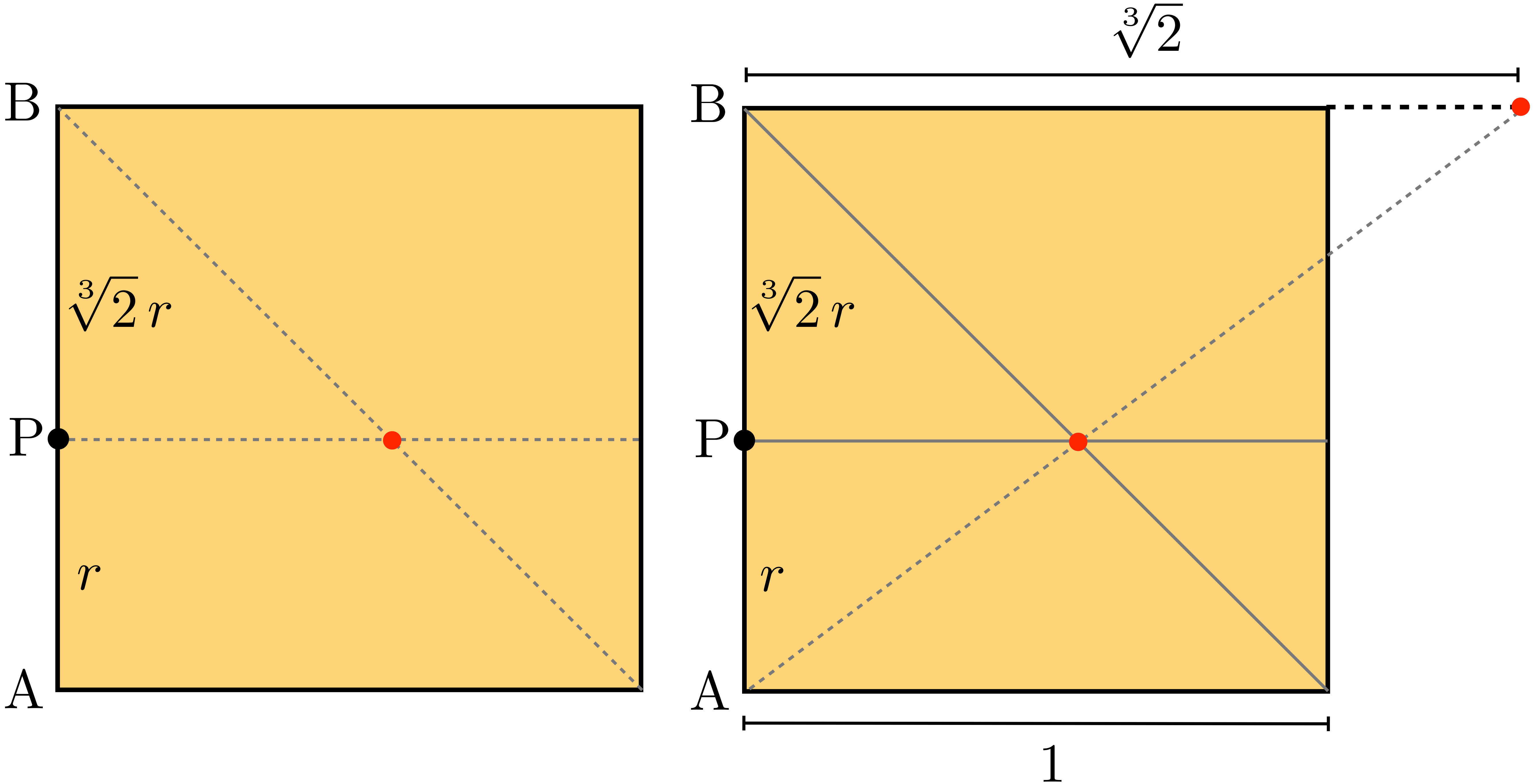
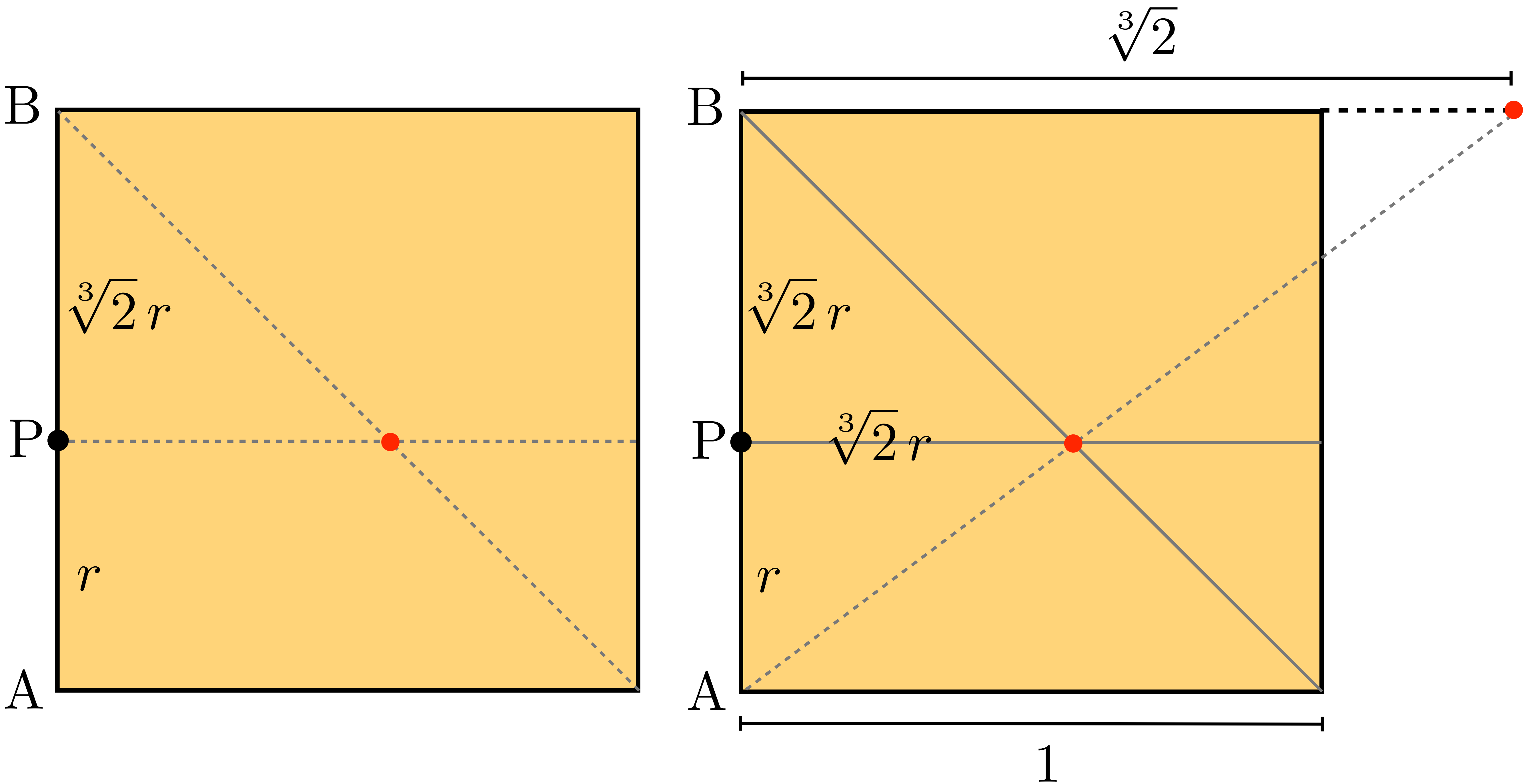


$$BP = \sqrt[3]{2}\ AP$$

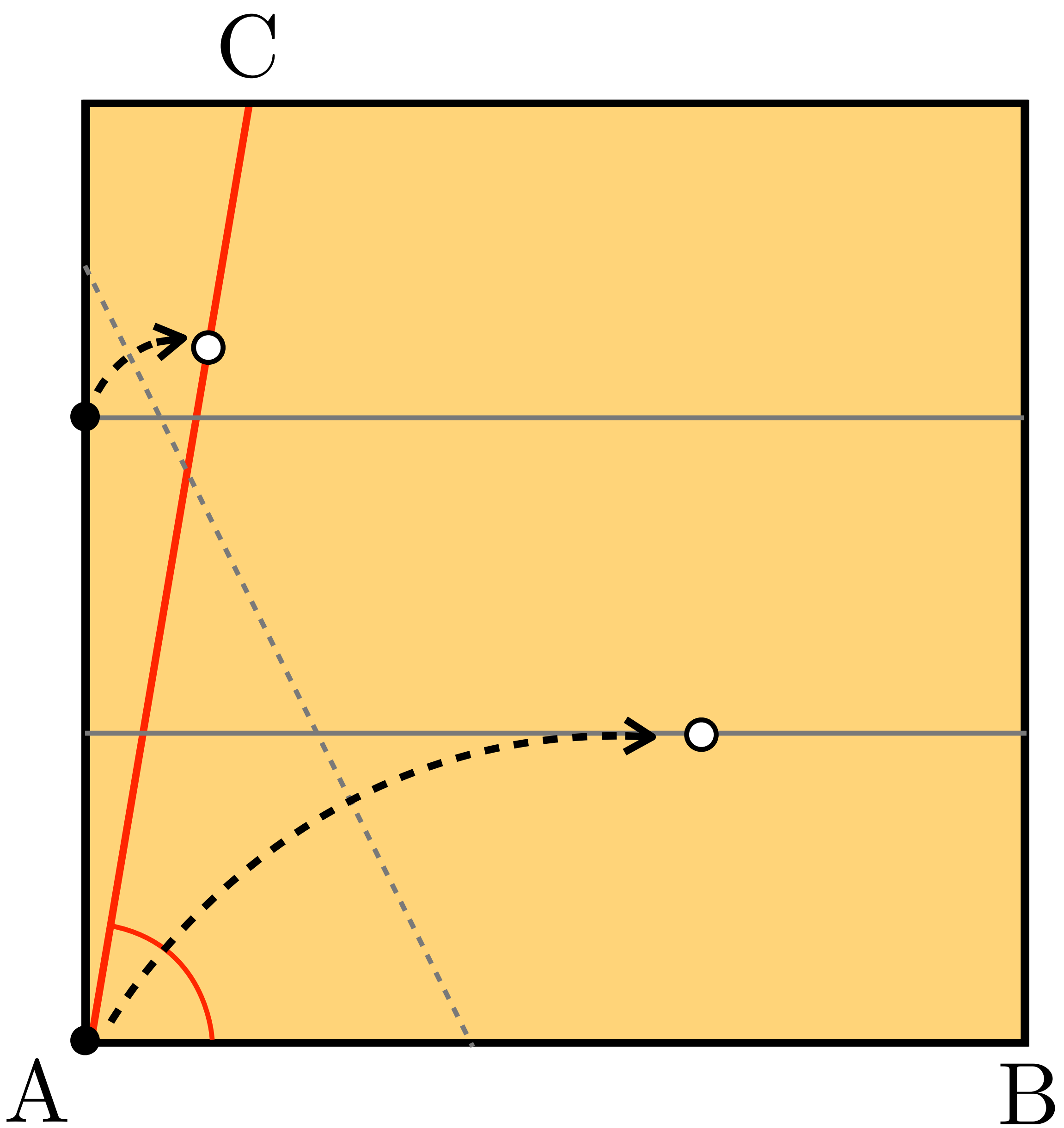

B
$\sqrt[3]{2}\,r$
P
r
A


B
$\sqrt[3]{2}\,r$
P
r
A
B
$\sqrt[3]{2}\,r$
P
r
A

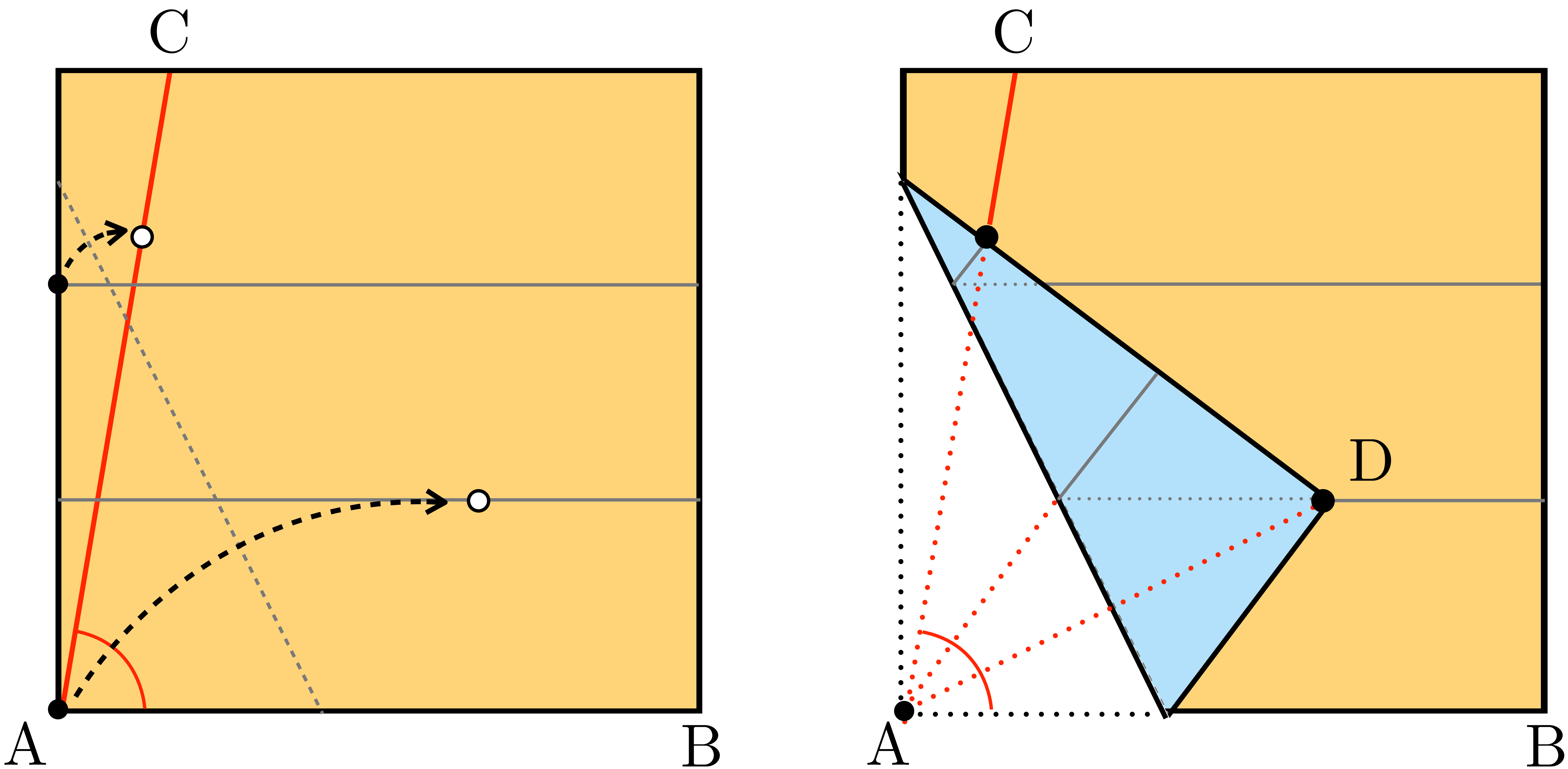

B
$\sqrt[3]{2}\,r$
P
r
A
$\sqrt[3]{2}$
B
$\sqrt[3]{2}\,r$
P
r
A
1


B
$\sqrt[3]{2}\,r$
P
r
A
$\sqrt[3]{2}$
B
$\sqrt[3]{2}\,r$
P
$\sqrt[3]{2}\,r$
r
A
1

Angle trisection:



Angle trisection:



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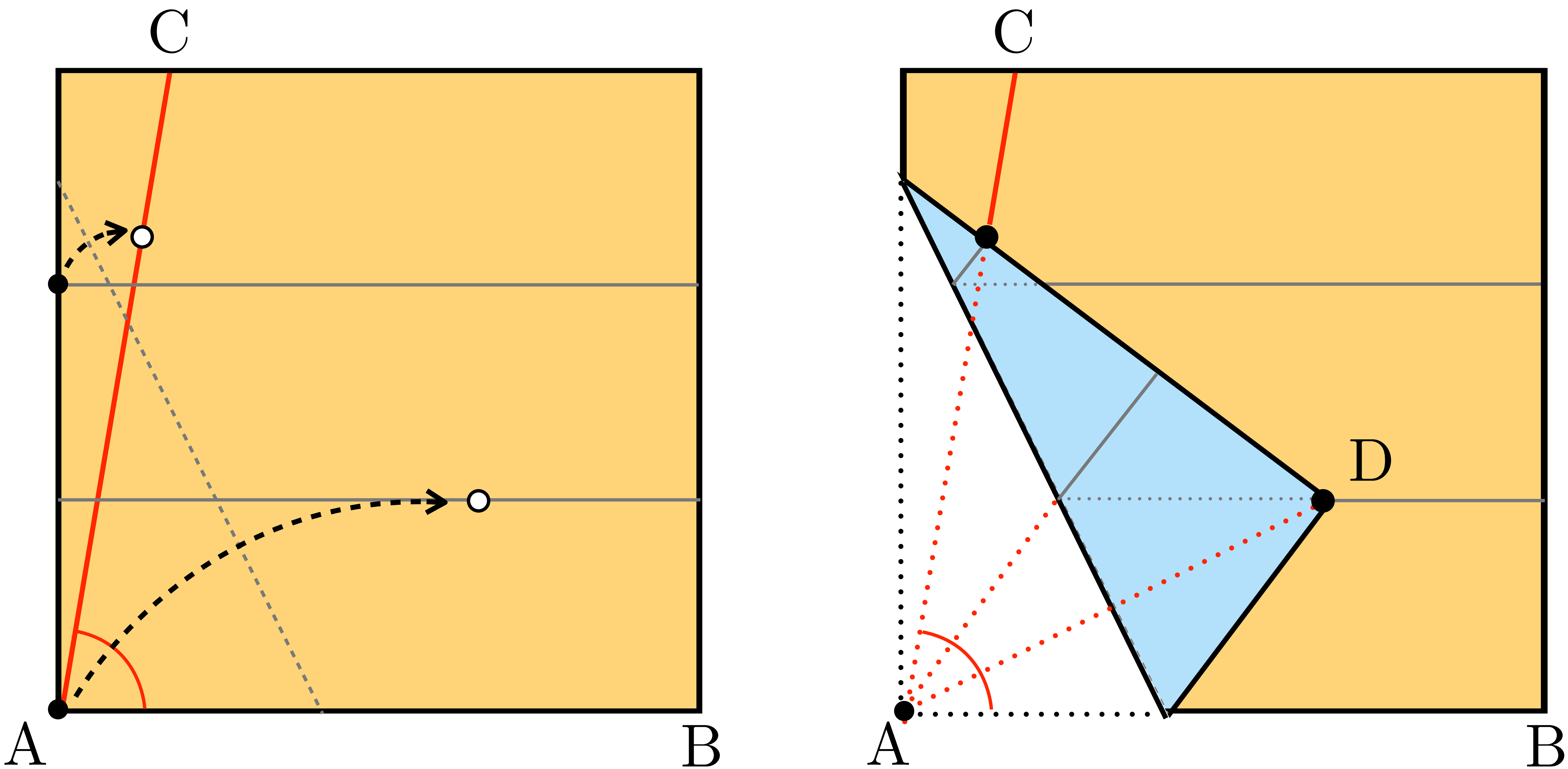
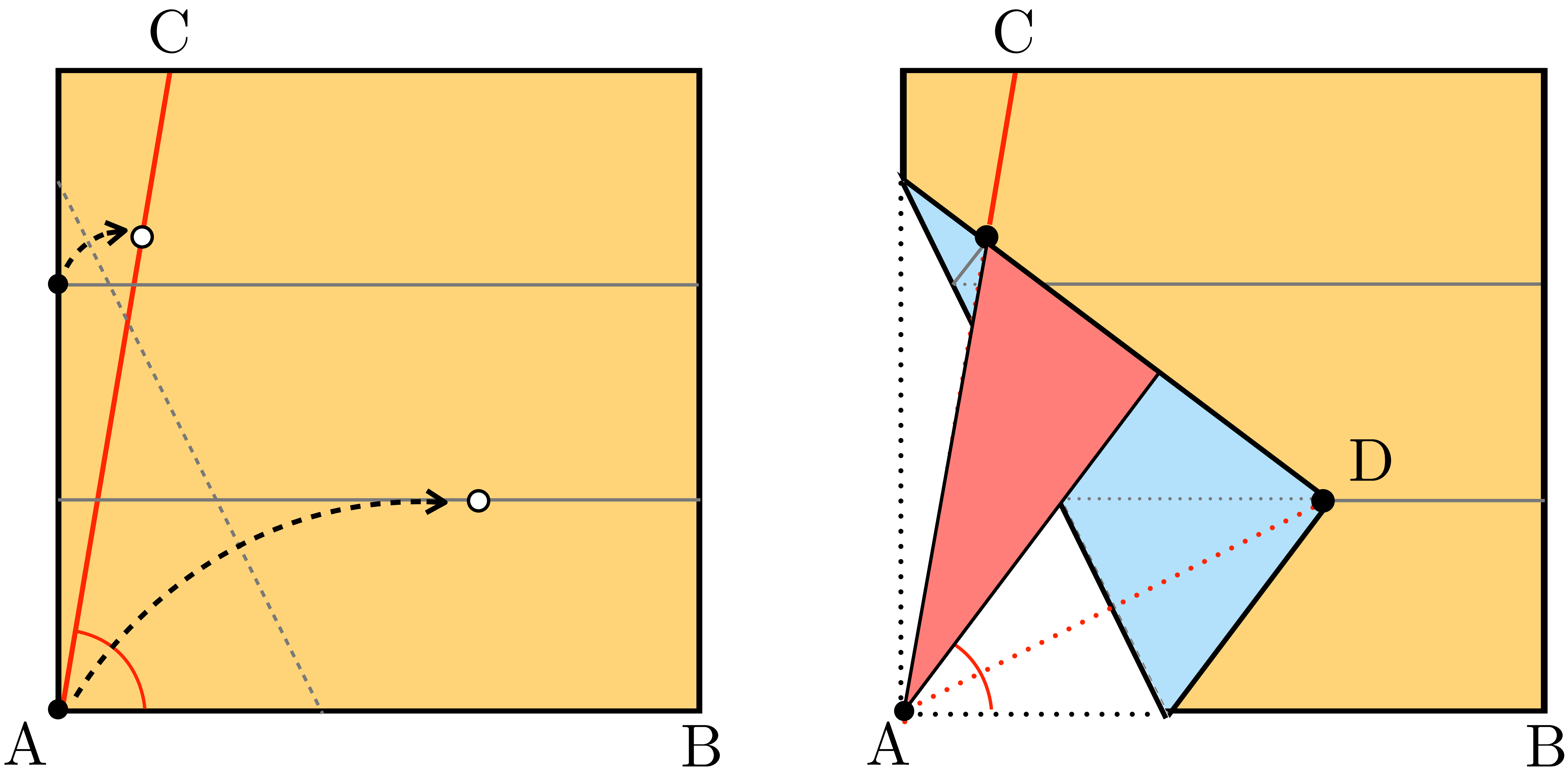


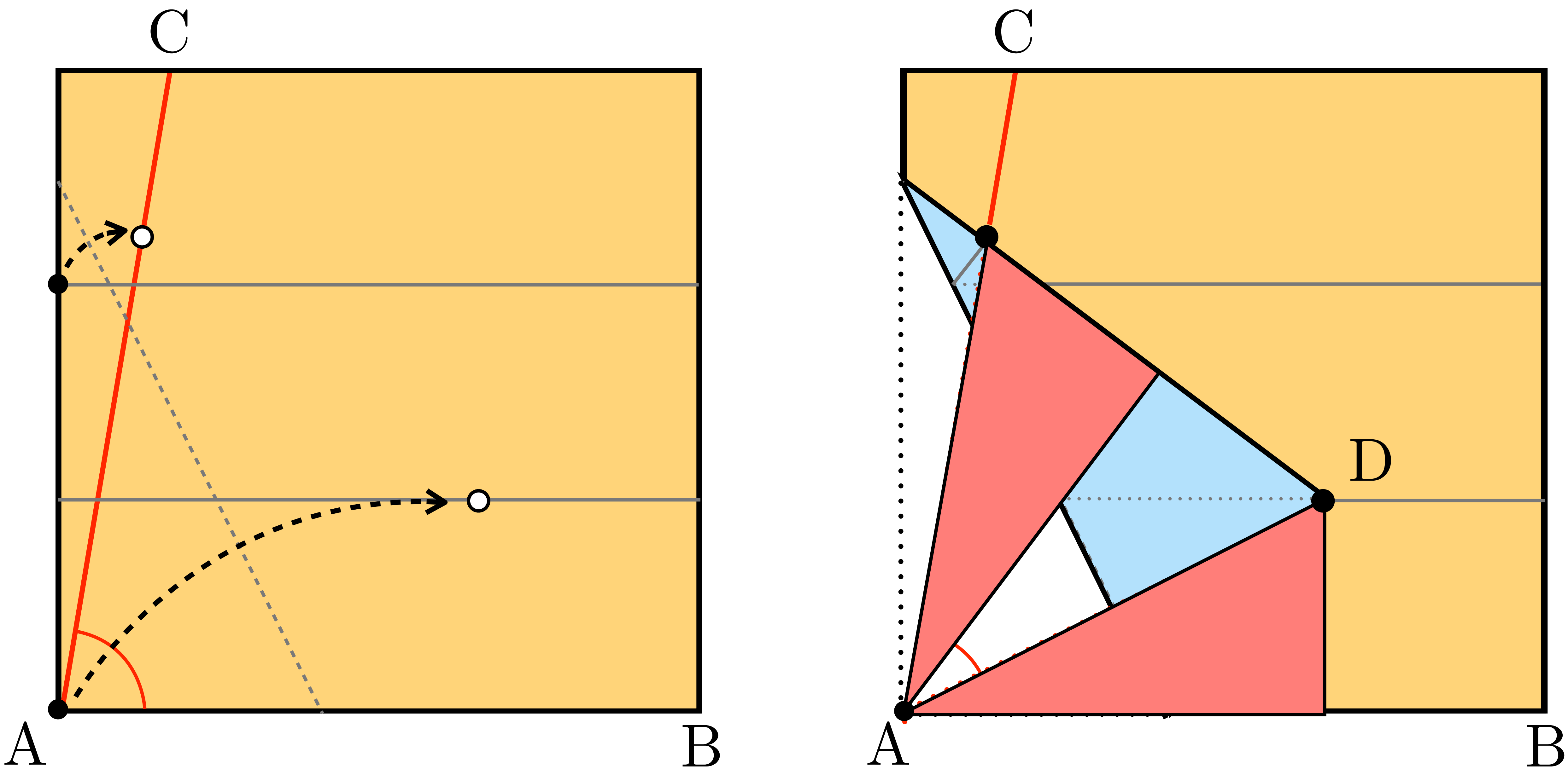
$$\hat{BAD} = \frac{1}{3}\ \hat{BAC}$$

Angle trisection:



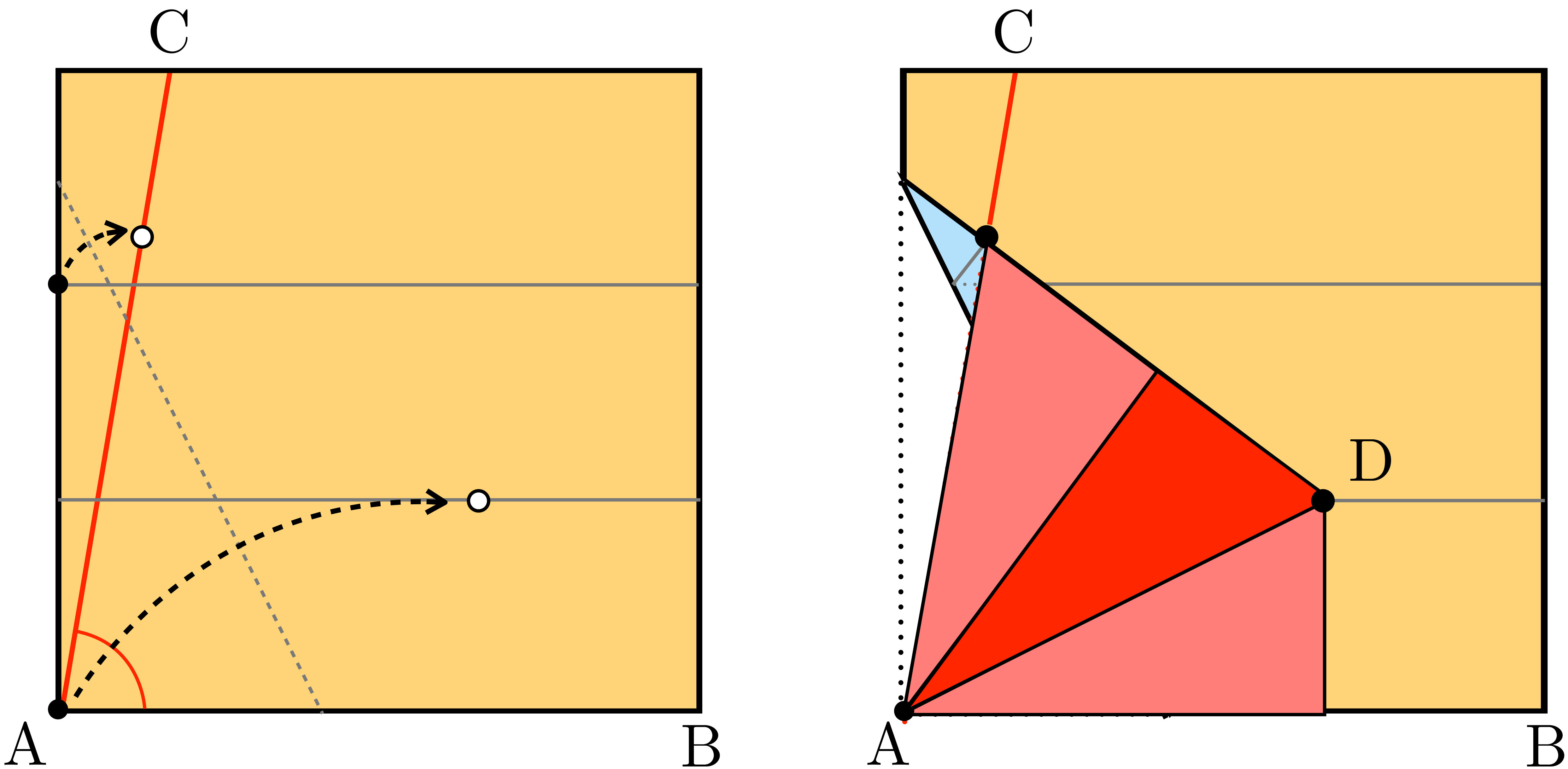
$$\mathrm{B\hat{A}D} = \frac{1}{3}\ \mathrm{B\hat{A}C}$$

Angle trisection:



$$B\hat{A}D = \frac{1}{3}\ B\hat{A}C$$

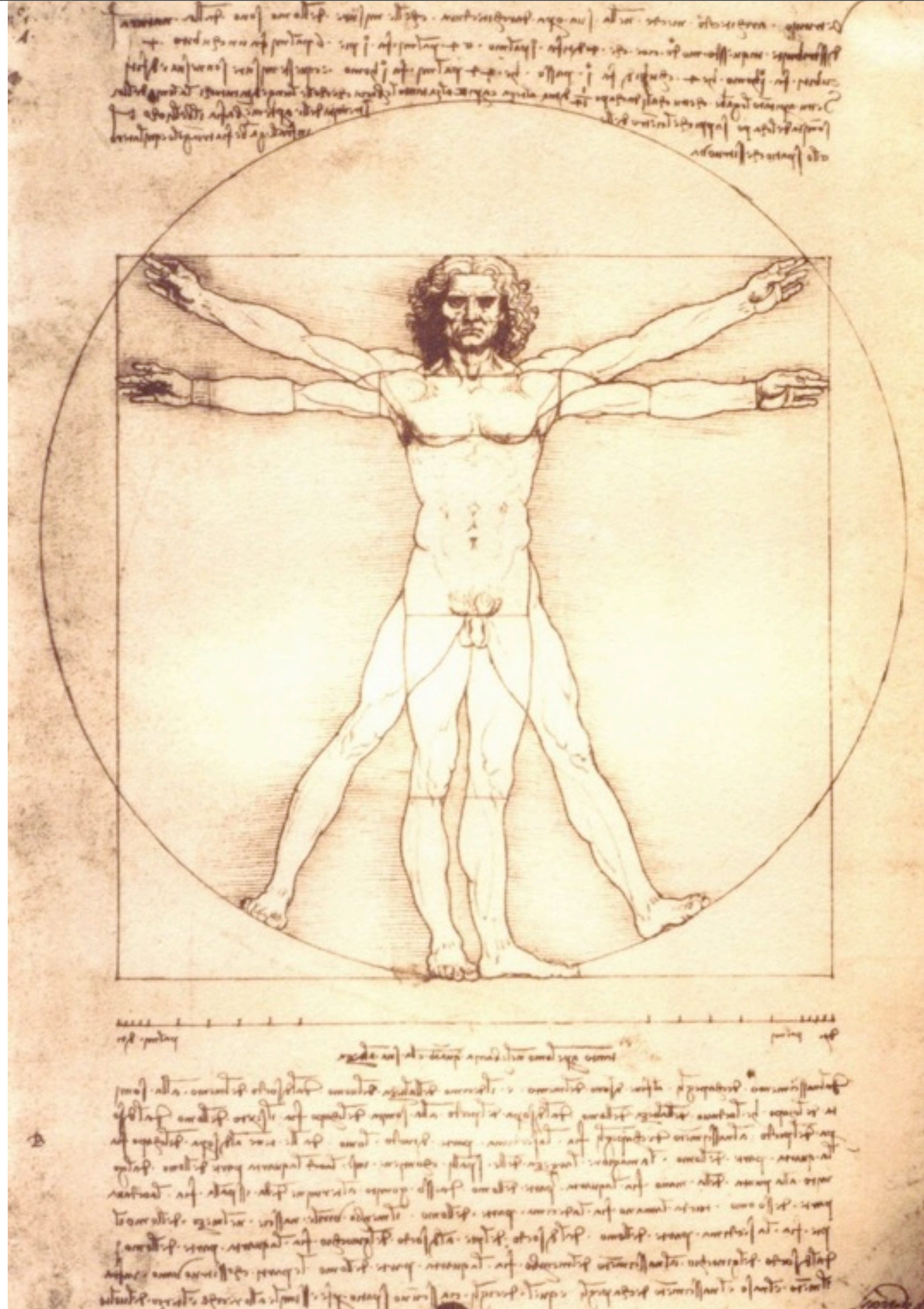
Angle trisection:



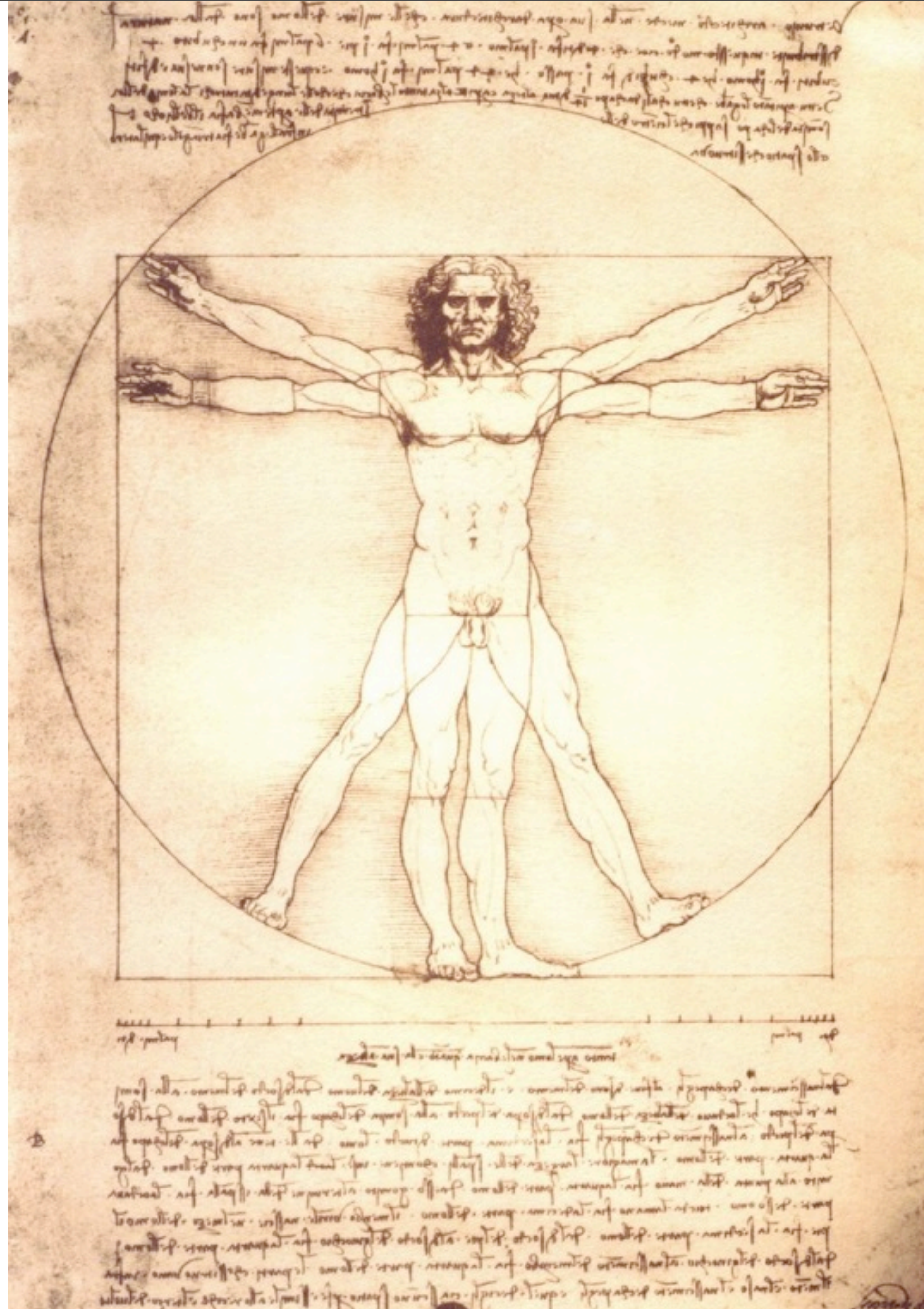
$$\mathrm{B\hat{A}D} = \frac{1}{3}\ \mathrm{B\hat{A}C}$$

And...?

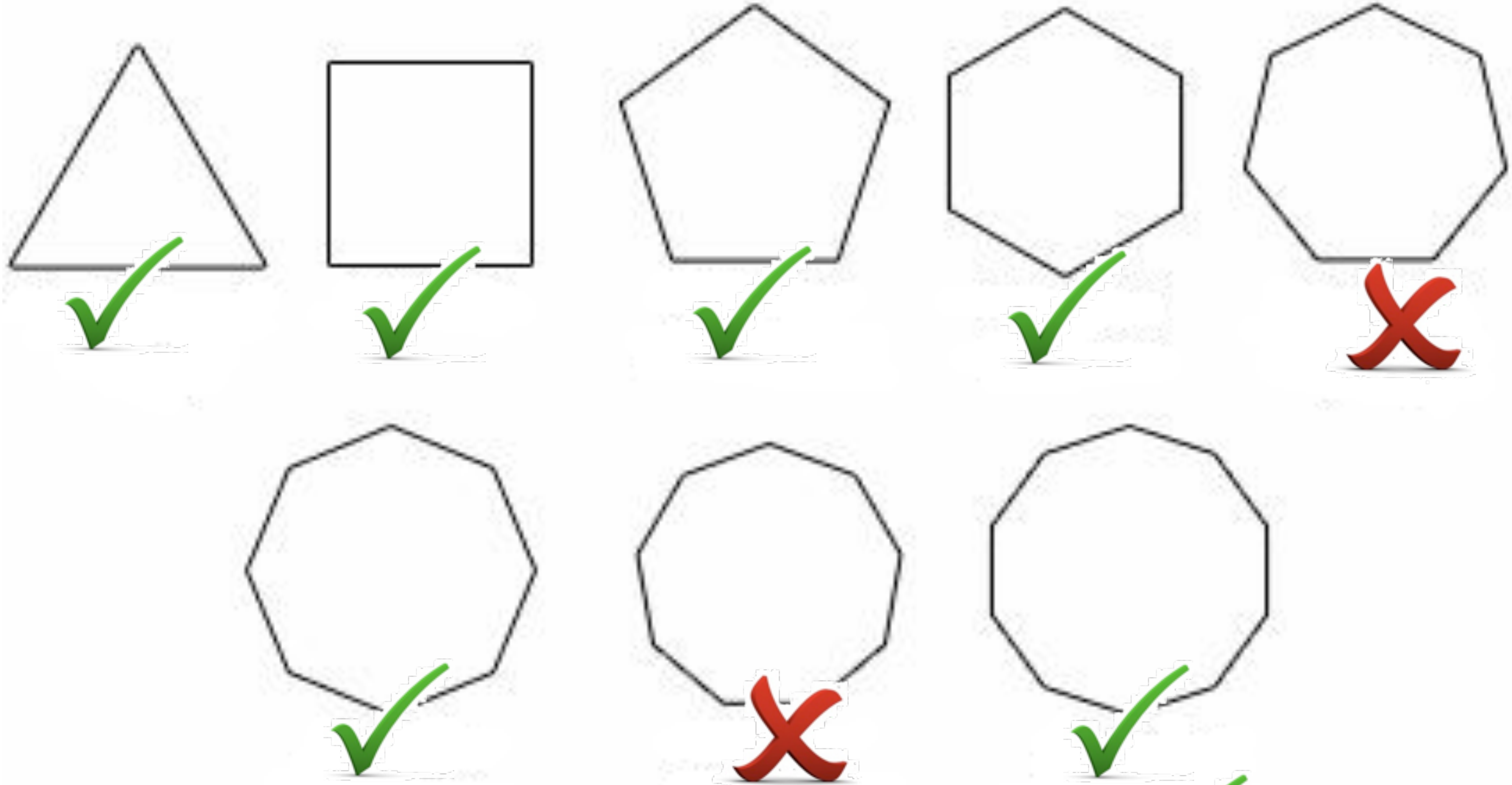
And...?



And...?



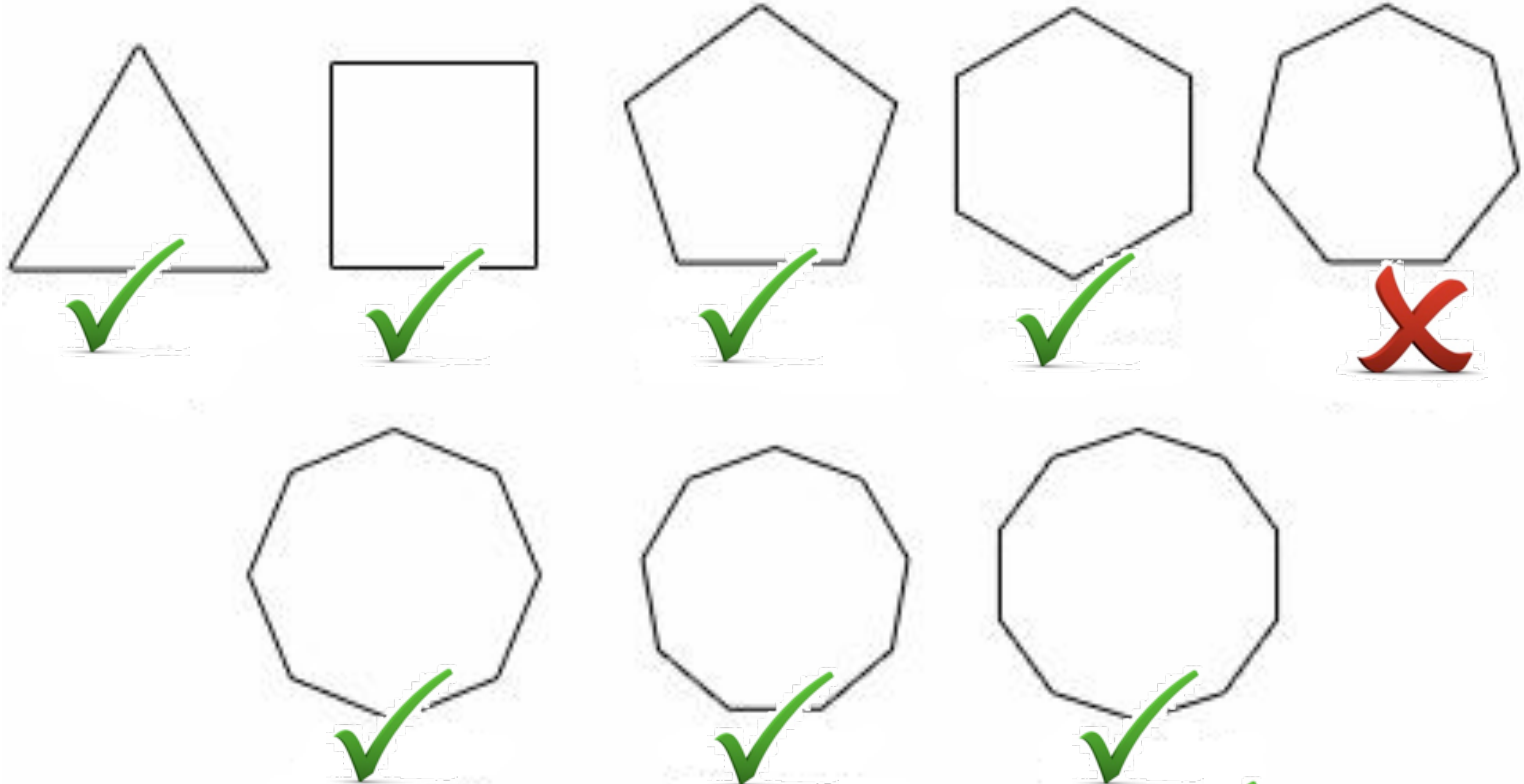
No.



$n = 2^k \times$ product of distinct Fermat primes 

$$F_n = 2^{2^n} + 1$$

$F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537, F_5$ is not prime, ...



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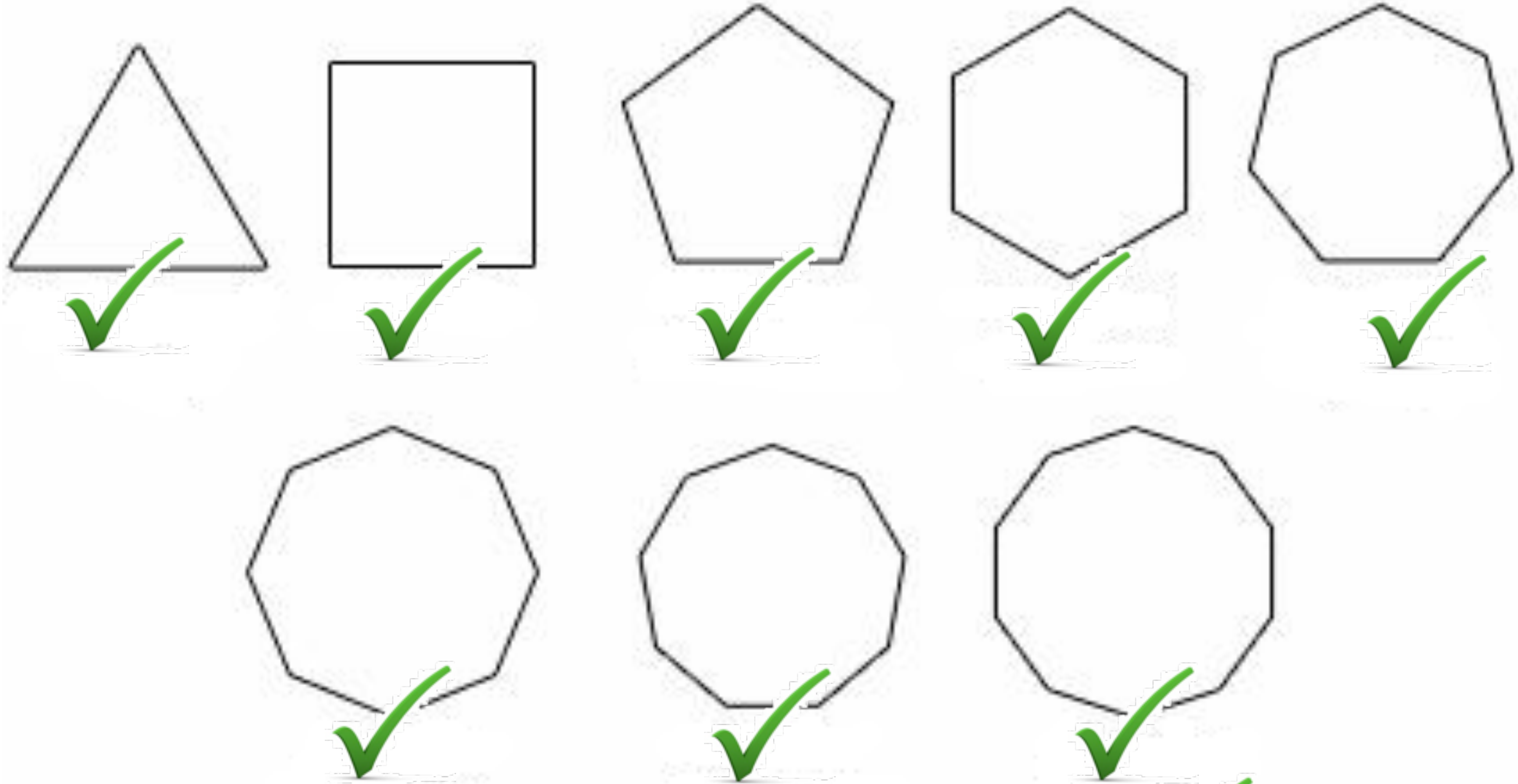

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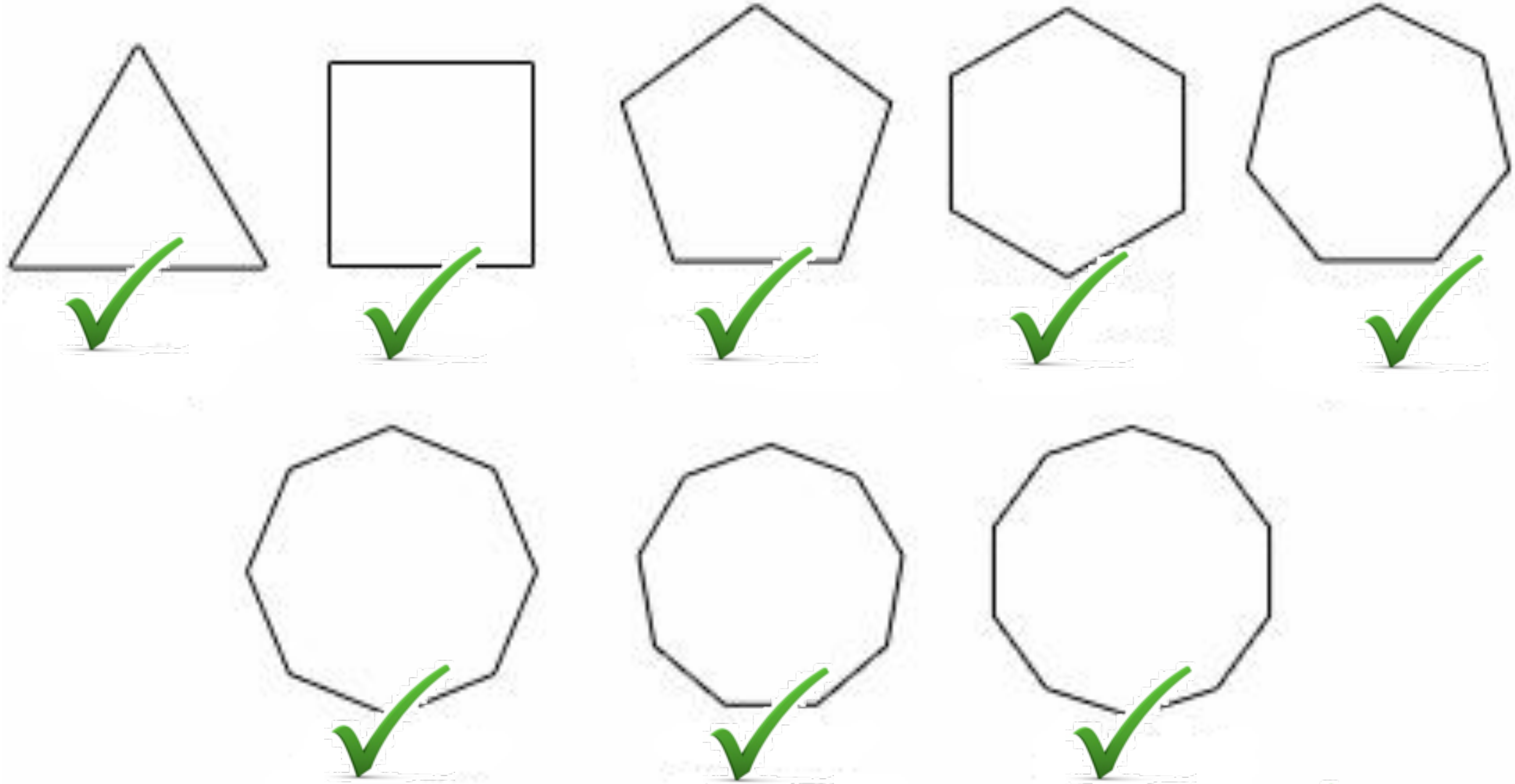
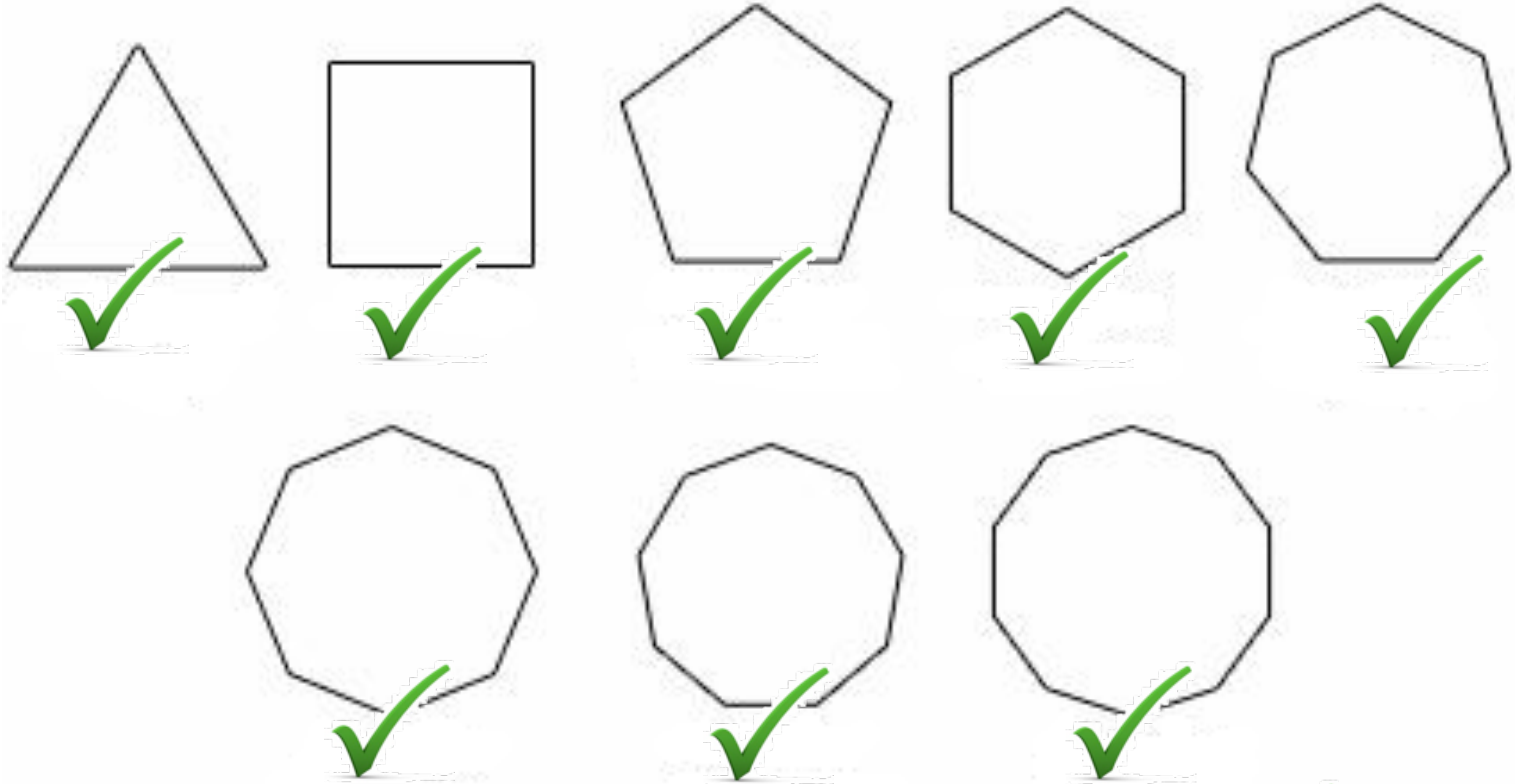
$$F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537, F_5 \text{ is not prime}, \ldots$$



$n = 2^k 3^l \times$ product of distinct Pierpont primes

$$F_n = 2^{2^n} + 1$$

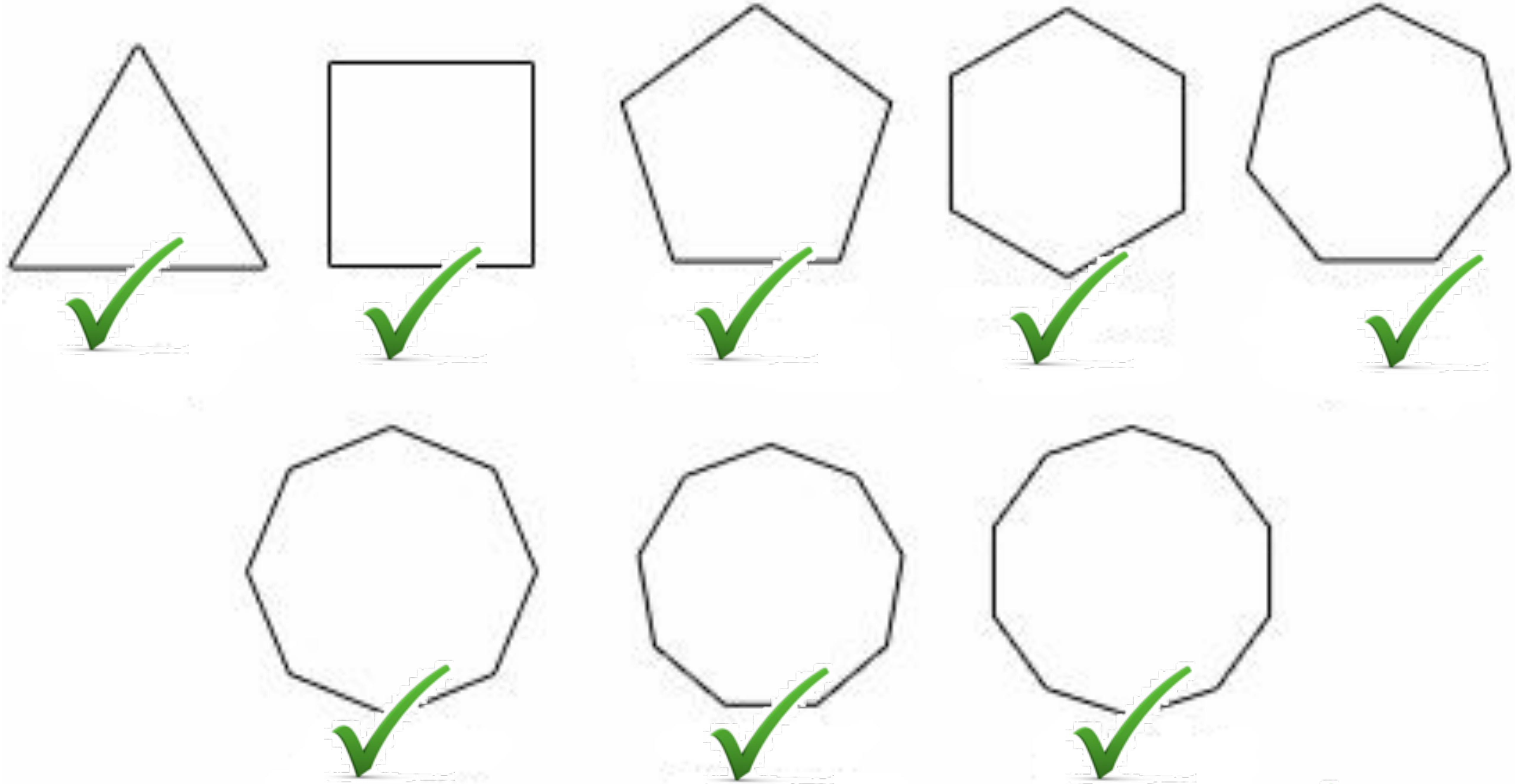
$F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537, F_5$ is not prime, $\ldots$



$n = 2^k 3^l \times$ product of distinct Pierpont primes

$2^u 3^v + 1$

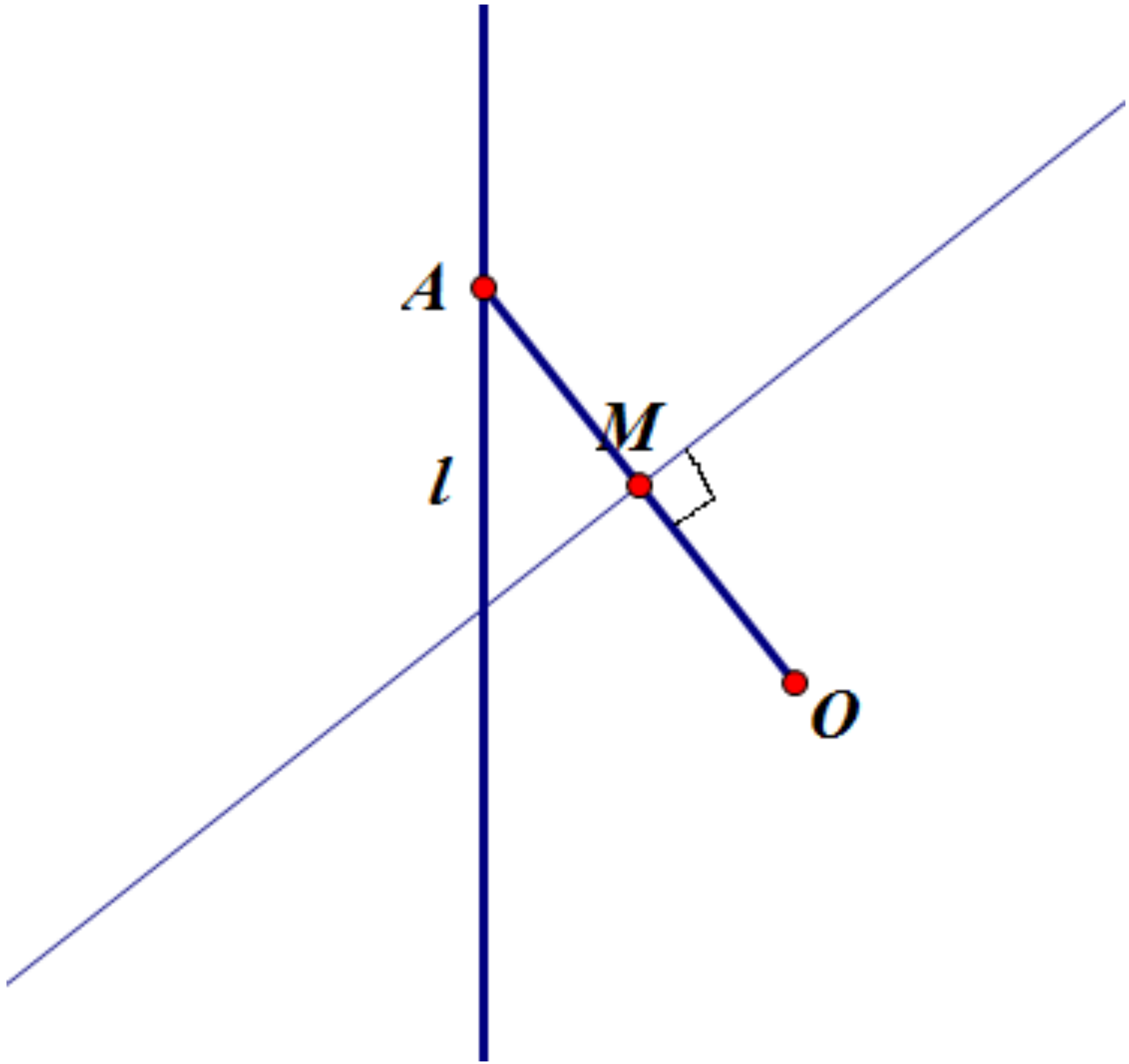
$F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537, F_5$ is not prime, $\ldots$

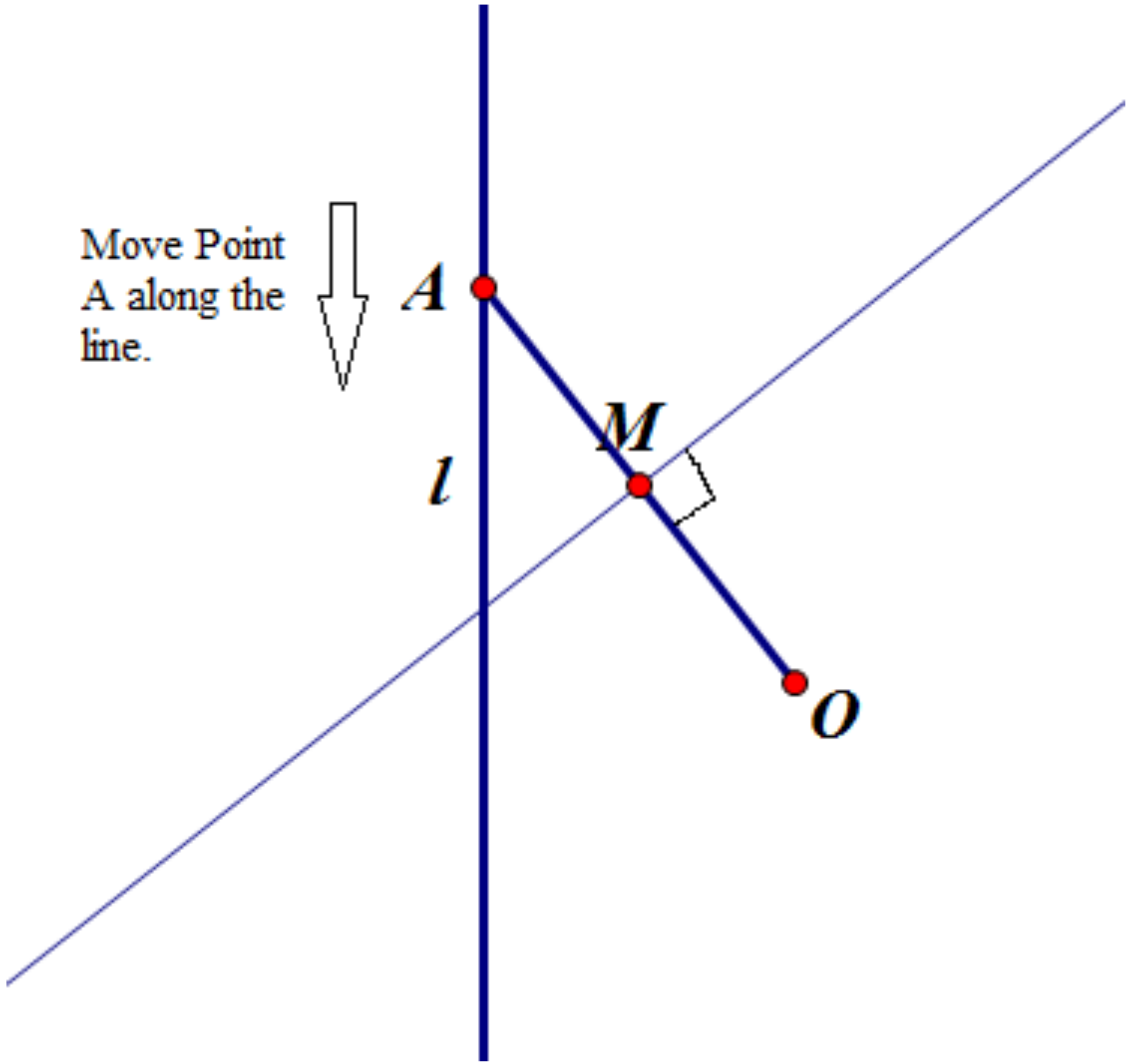


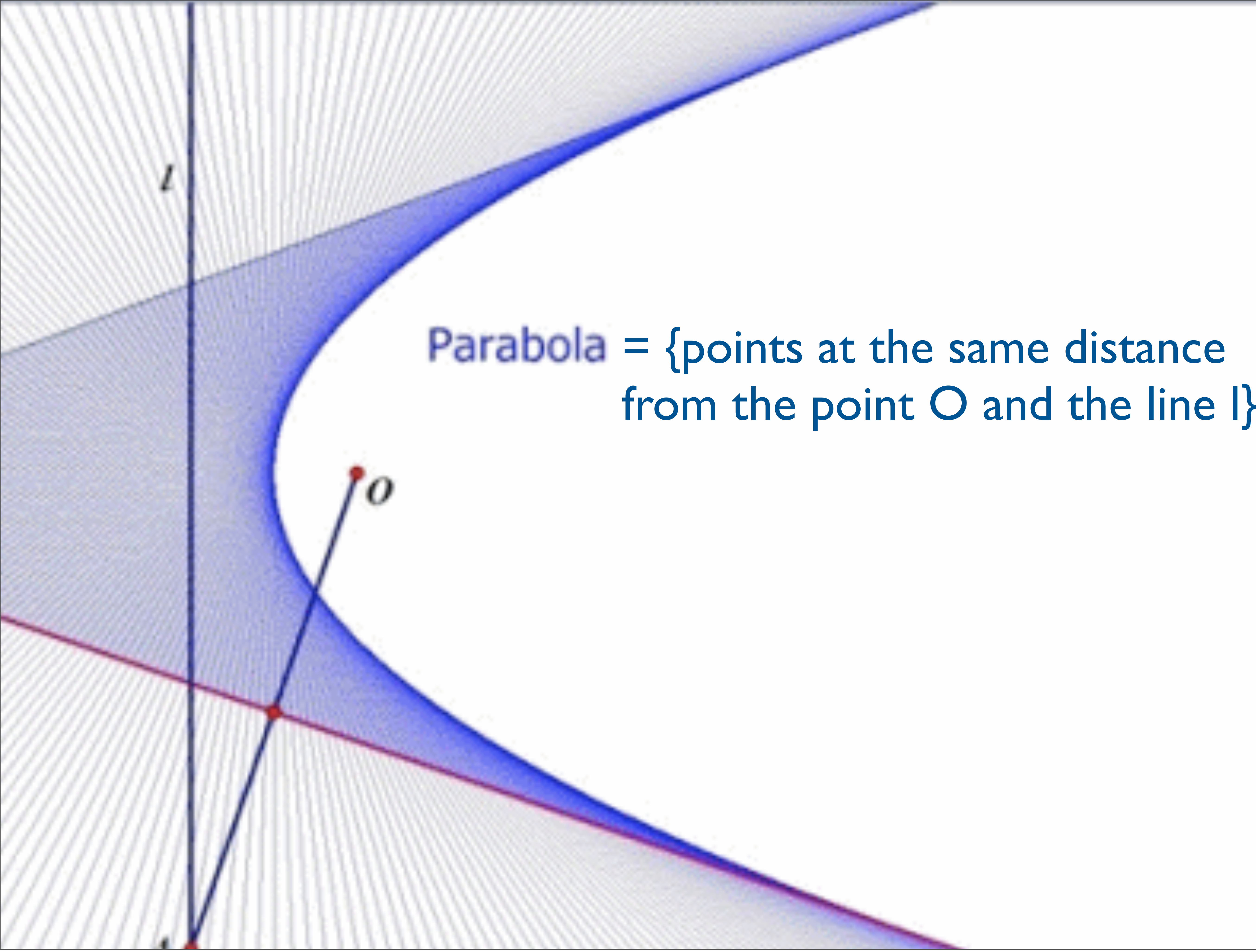
$n = 2^k 3^l \times$ product of distinct Pierpont primes

$$2^u 3^v + 1$$

$$2, 3, 5, 7, 13, 17, 19, 37, 73, 97, 109, 163, 193, 257, \ldots$$

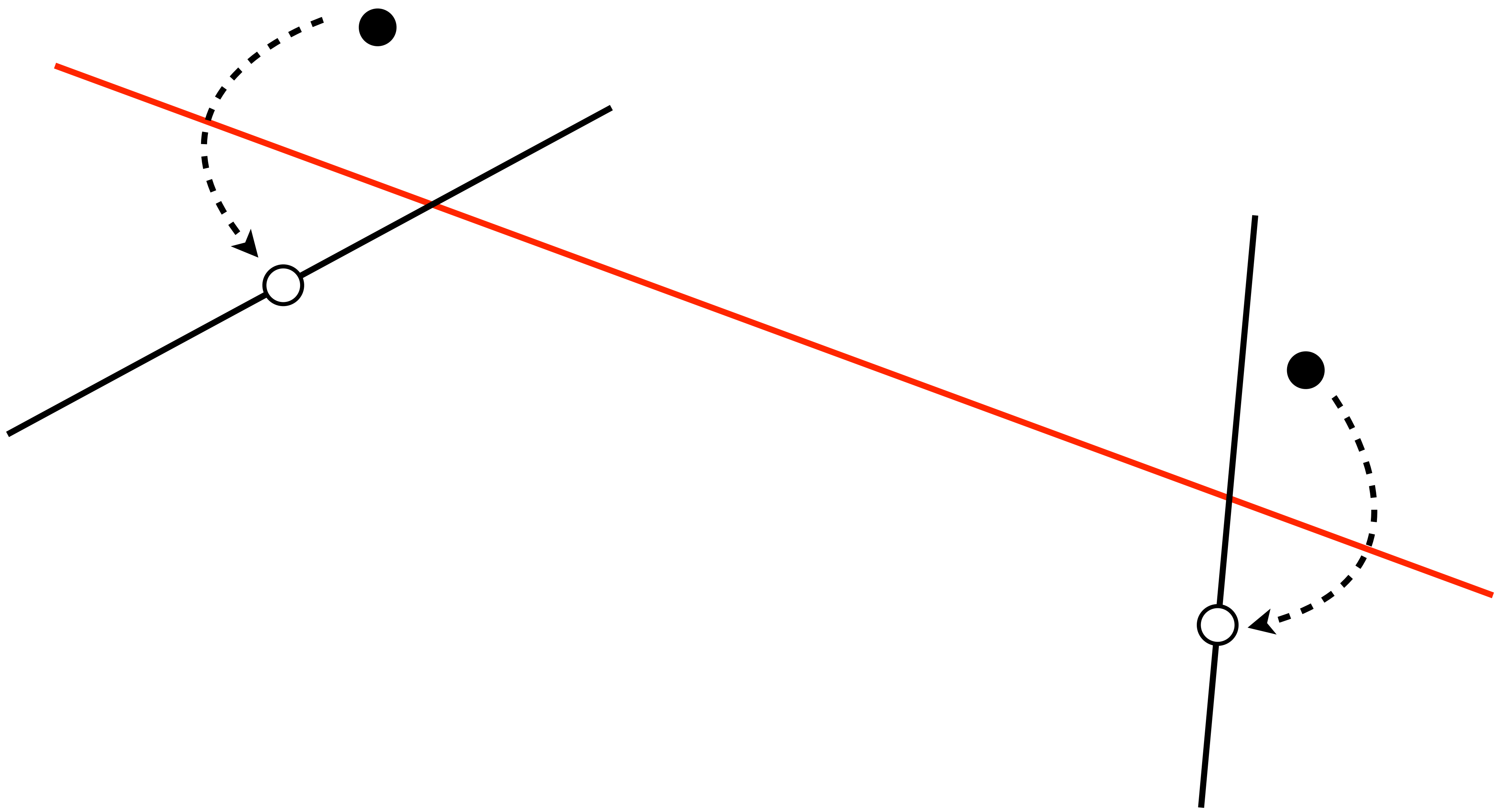

A
M
l
O


Move Point
A along the
line.
A
M
l
O



Parabola = {points at the same distance from the point O and the line l}

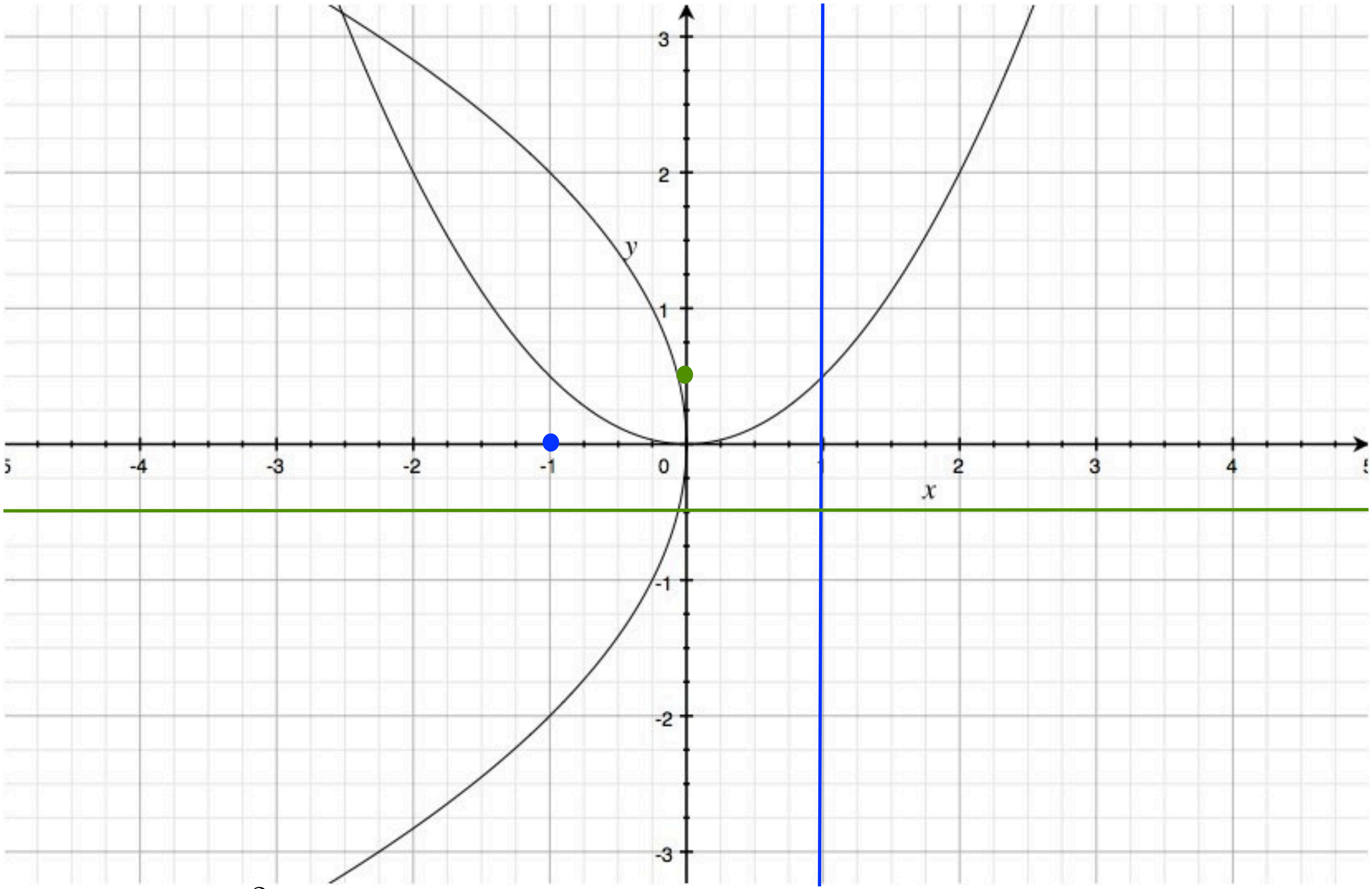
Axiom 6:



directrix: $y = -1/2$

focus: $(0, 1/2)$

$x^2 = 2y$



$y^2 = -4x$

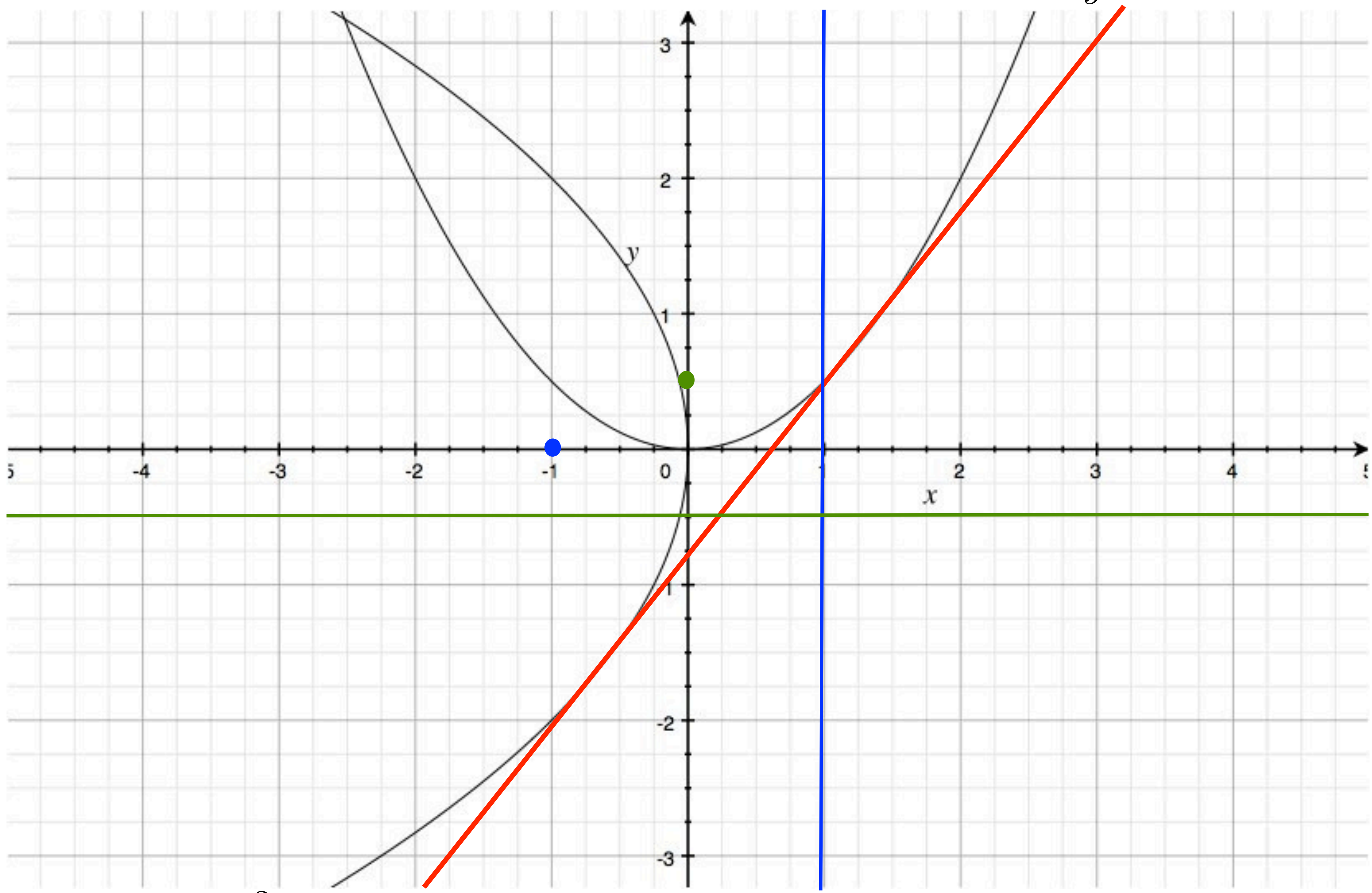
focus: $(-1, 0)$

directrix: $x = 1$

directrix: $y = -1/2$

focus: $(0, 1/2)$

$x^2 = 2y$



$y^2 = -4x$

focus: $(-1, 0)$

directrix: $x = 1$

$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx}$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4$$

$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx} \quad \rightsquigarrow x = m$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4 \rightsquigarrow y = \frac{-2}{m}$$

$$x^2 = 2y \quad \leadsto 2x = 2\frac{dy}{dx} \quad \leadsto x = m \quad \leadsto (x, y) = (m, \frac{m^2}{2})$$

$$y^2 = -4x \leadsto 2y\frac{dy}{dx} = -4 \leadsto y = \frac{-2}{m} \leadsto (x, y) = (\frac{-1}{m^2}, \frac{-2}{m})$$

$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx} \quad \rightsquigarrow x = m \quad \rightsquigarrow (x, y) = (m, \frac{m^2}{2})$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4 \rightsquigarrow y = \frac{-2}{m} \rightsquigarrow (x, y) = (\frac{-1}{m^2}, \frac{-2}{m})$$

$$y = mx + b$$

$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx} \quad \rightsquigarrow x = m \quad \rightsquigarrow (x, y) = (m, \frac{m^2}{2})$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4 \rightsquigarrow y = \frac{-2}{m} \rightsquigarrow (x, y) = (\frac{-1}{m^2}, \frac{-2}{m})$$

$$y = mx + b \qquad \begin{aligned} &\rightsquigarrow \frac{m^2}{2} = mm + b \\ &\rightsquigarrow \frac{-2}{m} = m\frac{-1}{m^2} + b \end{aligned}$$

$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx} \quad \rightsquigarrow x = m \quad \rightsquigarrow (x, y) = (m, \frac{m^2}{2})$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4 \rightsquigarrow y = \frac{-2}{m} \rightsquigarrow (x, y) = (\frac{-1}{m^2}, \frac{-2}{m})$$

$$y = mx + b \qquad \begin{matrix} \rightsquigarrow \dfrac{m^2}{2} = mm + b \\ \rightsquigarrow \dfrac{-2}{m} = m\dfrac{-1}{m^2} + b \end{matrix} \qquad \rightsquigarrow b = -\frac{m^2}{2} = -\frac{1}{m}$$

$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx} \quad \rightsquigarrow x = m \quad \rightsquigarrow (x, y) = (m, \frac{m^2}{2})$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4 \rightsquigarrow y = \frac{-2}{m} \rightsquigarrow (x, y) = (\frac{-1}{m^2}, \frac{-2}{m})$$

$$y = mx + b \qquad \rightsquigarrow \frac{m^2}{2} = mm + b$$

$$\rightsquigarrow \frac{-2}{m} = m\frac{-1}{m^2} + b$$

$$\rightsquigarrow b = -\frac{m^2}{2} = -\frac{1}{m}$$

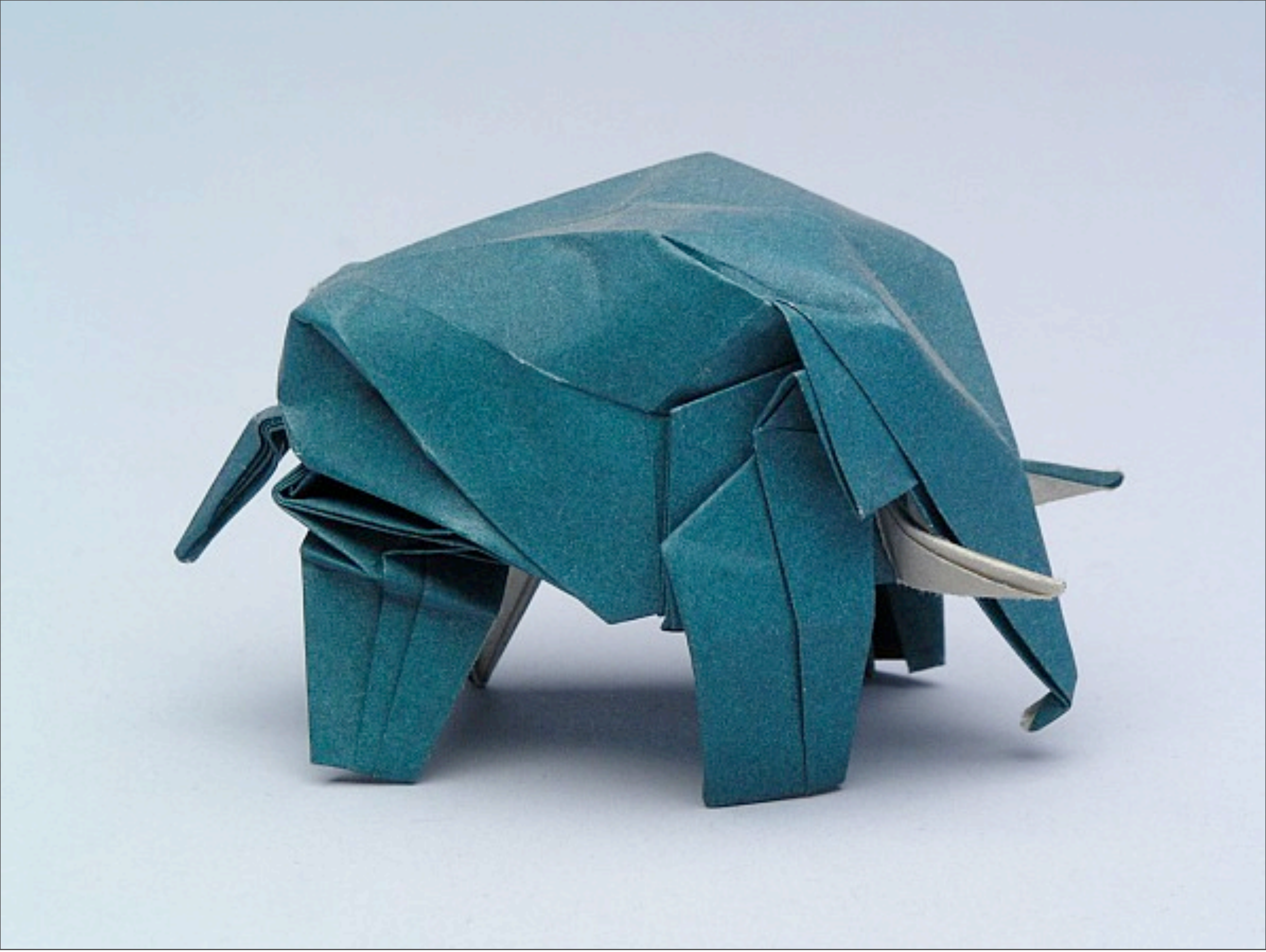
$$\rightsquigarrow m^3 = 2$$

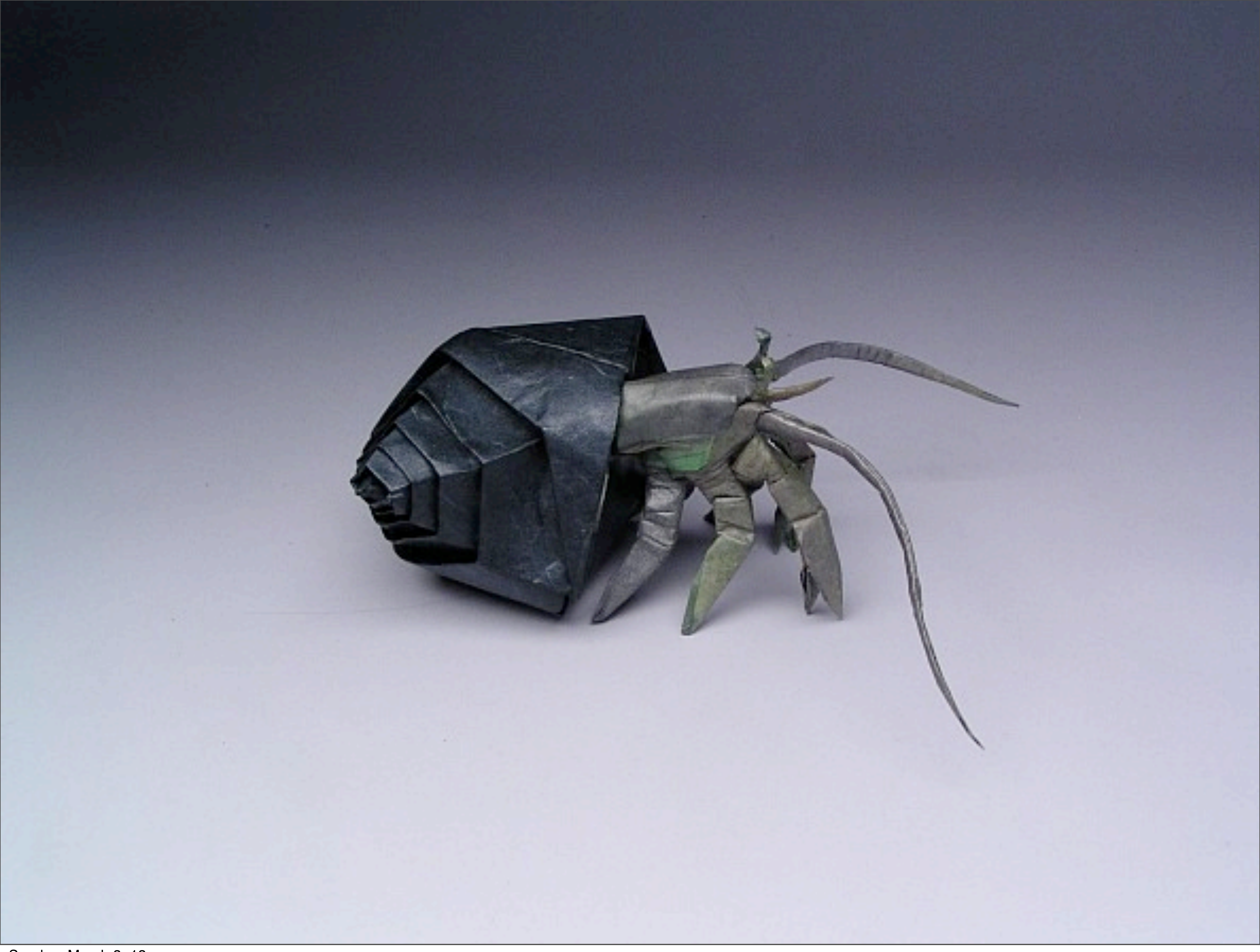
$$x^2 = 2y \quad \rightsquigarrow 2x = 2\frac{dy}{dx} \quad \rightsquigarrow x = m \quad \rightsquigarrow (x, y) = (m, \frac{m^2}{2})$$

$$y^2 = -4x \rightsquigarrow 2y\frac{dy}{dx} = -4 \rightsquigarrow y = \frac{-2}{m} \rightsquigarrow (x, y) = (\frac{-1}{m^2}, \frac{-2}{m})$$

$$y = mx + b \qquad \begin{array}{l} \rightsquigarrow \dfrac{m^2}{2} = mm + b \\ \rightsquigarrow \dfrac{-2}{m} = m\dfrac{-1}{m^2} + b \end{array} \qquad \rightsquigarrow b = -\frac{m^2}{2} = -\frac{1}{m}$$

$$\rightsquigarrow m^3 = 2 \quad \rightsquigarrow m = \sqrt[3]{2}$$



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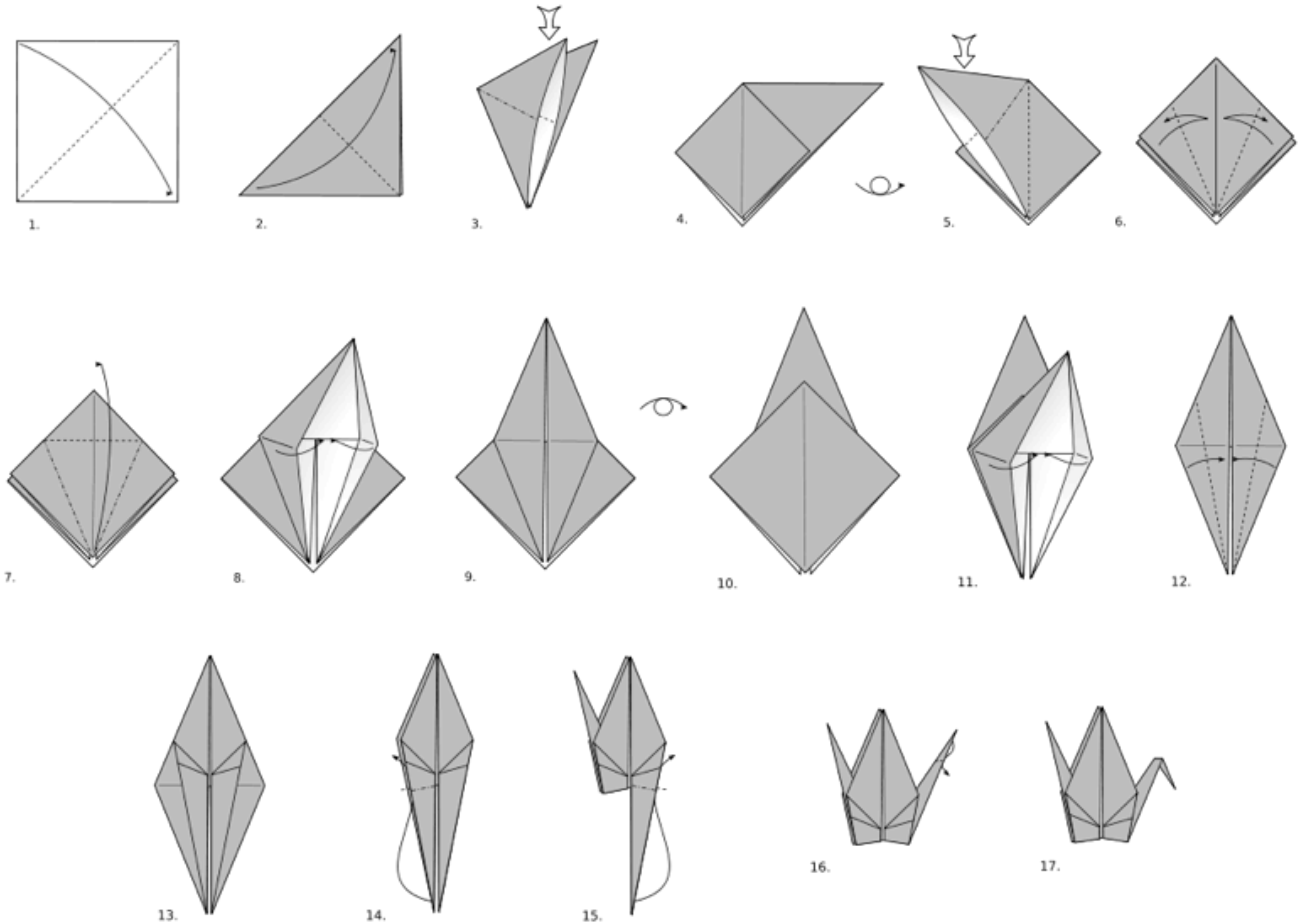
Orizuru

Traditional Japanese Model
Diagram by Andrew Hudson

Public Diagram Project



References:

- Euclid's elements: http://aleph0.clarku.edu/~djoyce/java/elements/elements.html (with explanations of the proofs and a geometry applet) and http://www.math.ubc.ca/~cass/Euclid/byrne.html ("in which coloured diagrams and symbols are used instead of letters for the greater ease of learners")

- Wikipedia page about the origami axioms:
http://en.wikipedia.org/wiki/Huzita%E2%80%93Hatori_axioms

- Wikipedia page about origami math results:
http://en.wikipedia.org/wiki/Mathematics_of_paper_folding

- Robert Lang's webpage has loads of materials (including many of the photos of fancy origami): http://www.langorigami.com/ . In particular, this paper has a lot a information: http://www.langorigami.com/science/math/hja/origami_constructions.pdf

- A TED talk about origami math things, but not quite the same content as this talk:
http://www.ted.com/talks/robert_lang_folds_way_new_origami.html

- Folding a regular heptagon!
http://www.math.sjsu.edu/~alperin/TotallyRealHeptagon.pdf