Getting Students to Think: The Role of Interactive Graphics

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Outline

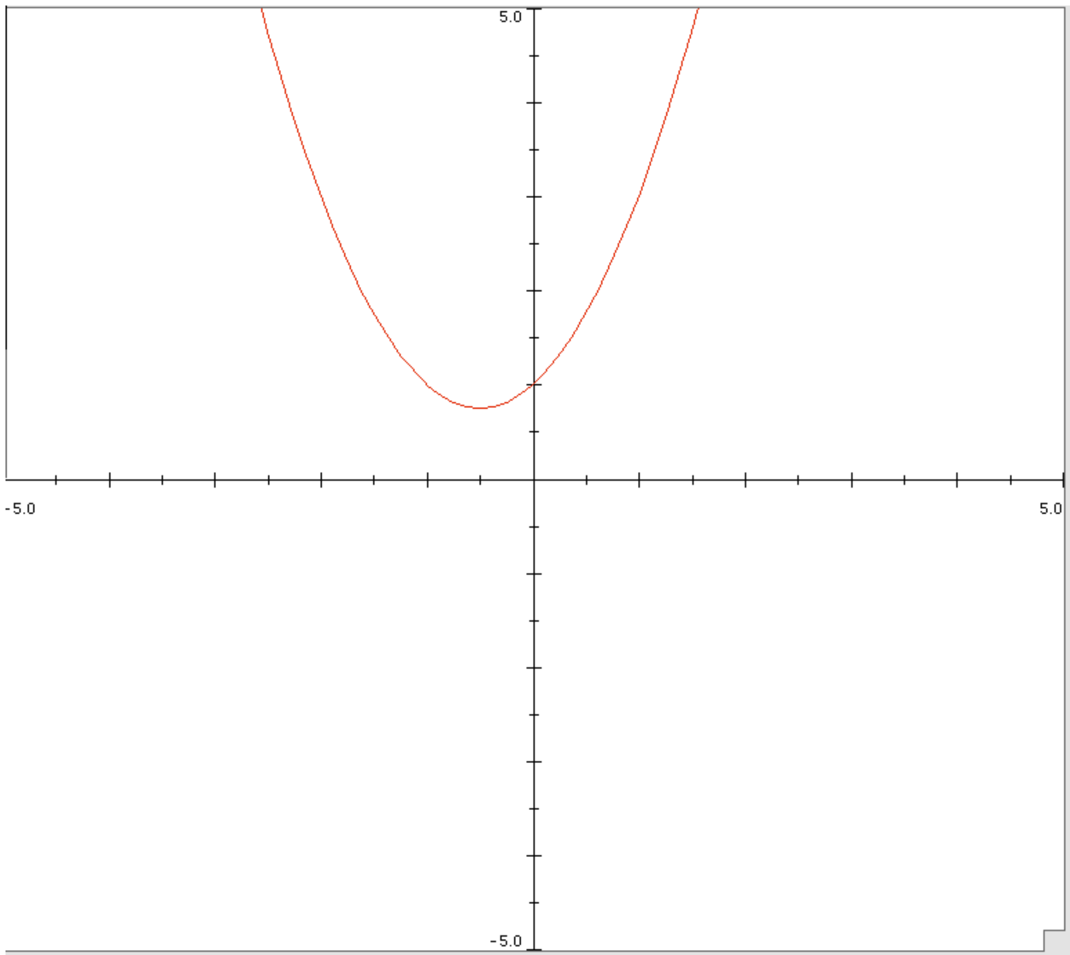
- Graphing
- Following a Direction Field
- Interpreting Graphs
- Discussion

Hubert Hohn
Massachusetts College of Art

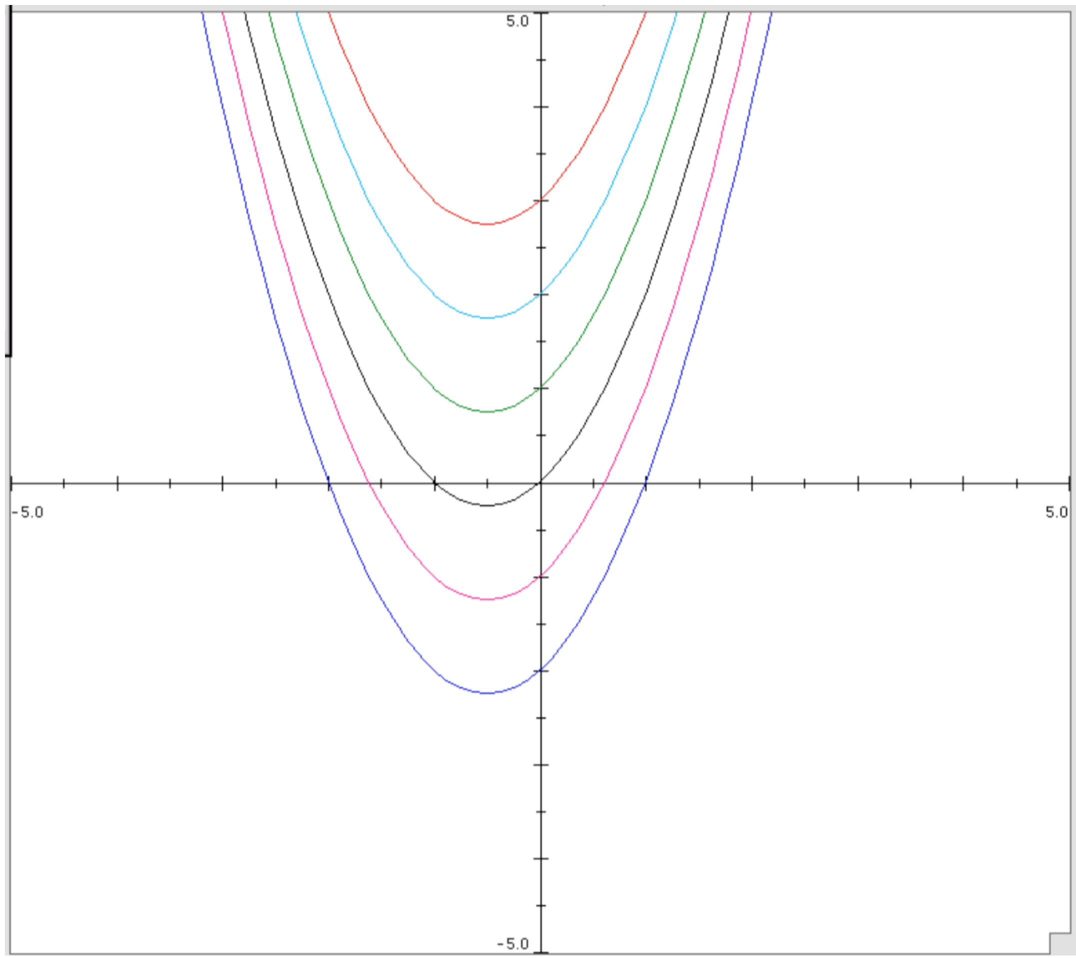
- *Chaos* course on Mass. Learn Pike
- Tools for texts and courses:
  - MIT (Steve Strogatz, Haynes Miller)
  - Boston University (Paul Blanchard, Bob Devaney, Glen Hall)
  - University of Connecticut (Edmund Tomastik)
  - U Mass Boston (Linda Kime, Judy Clark, Beverly Michaels)
  - *Interactive Differential Equations*
  - (Strogatz, McDill, Cantwell, West)

Parabola ax^2+bx+c

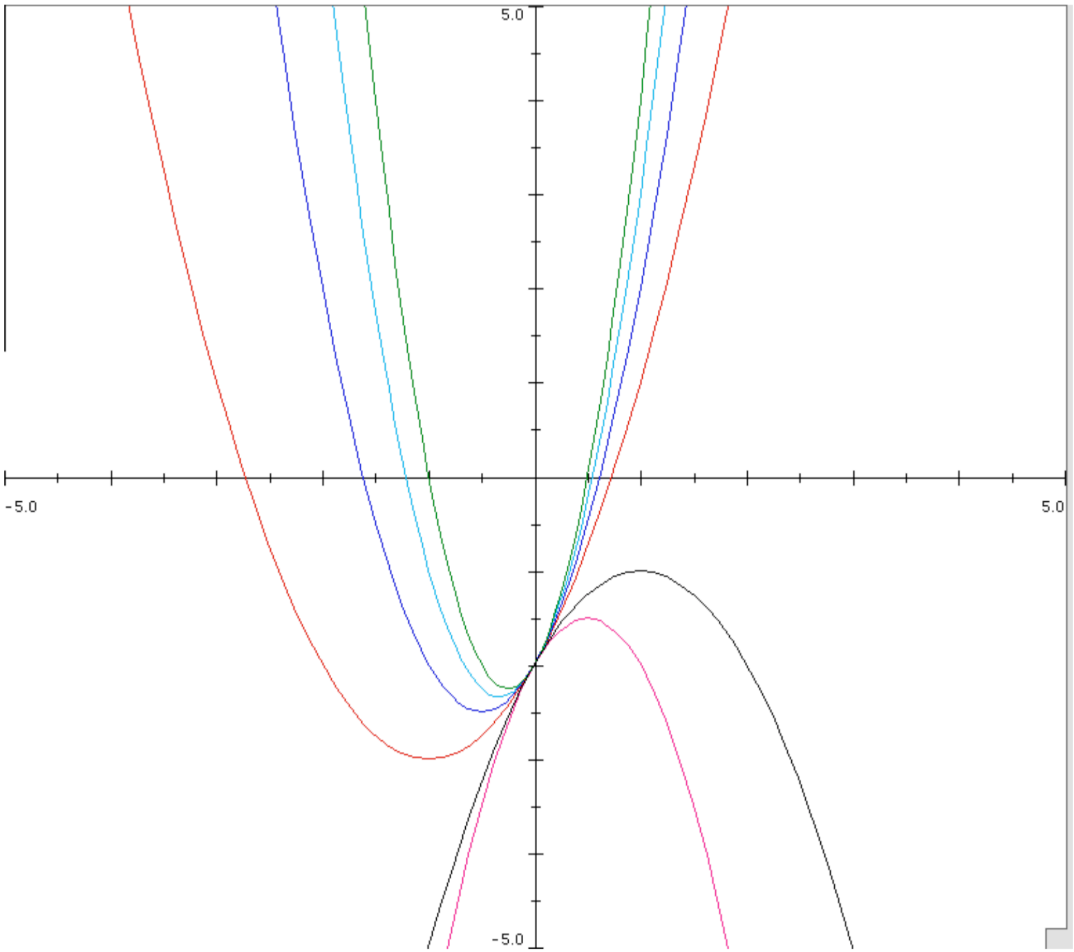
(here shown for a = b = c = 1)



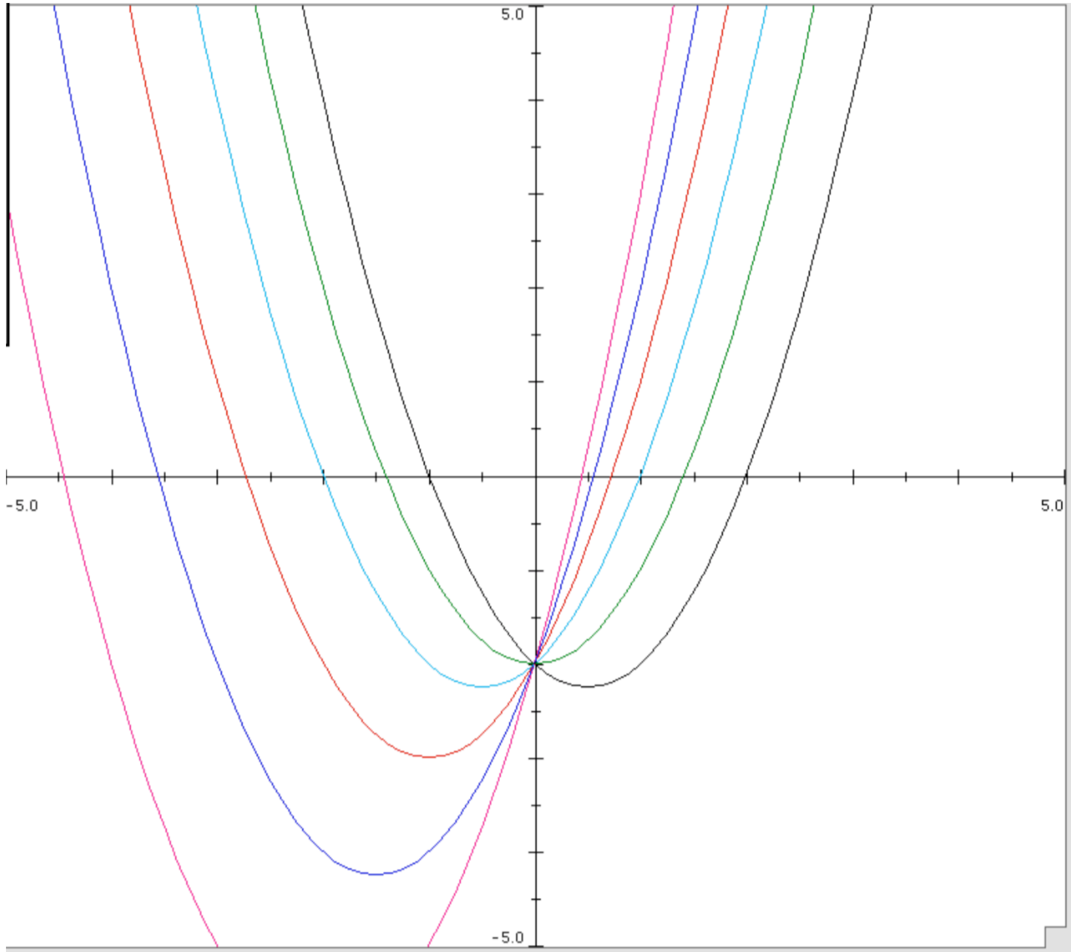
ax^2+bx+c ; changing c



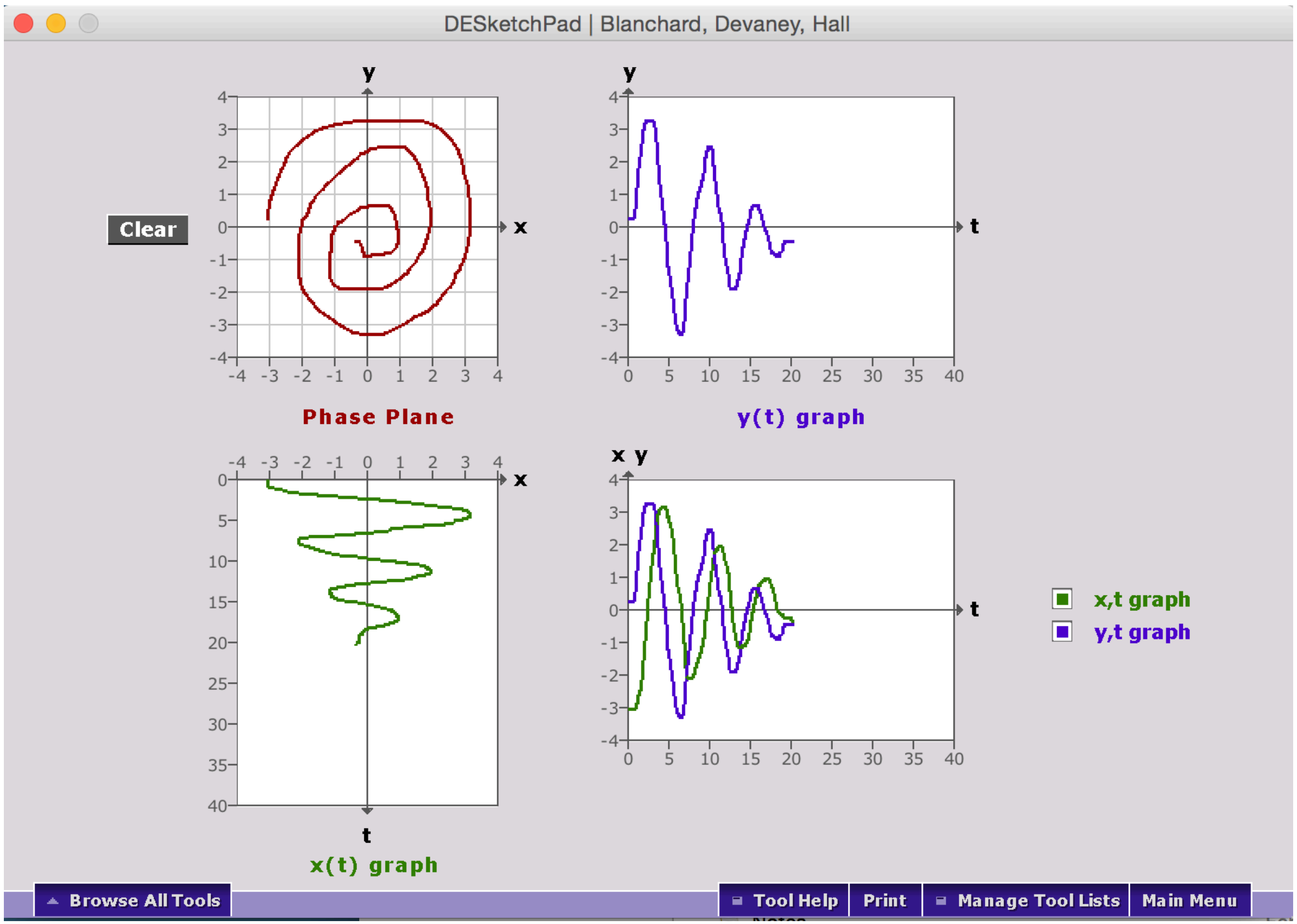
ax^2+bx+c ; changing a



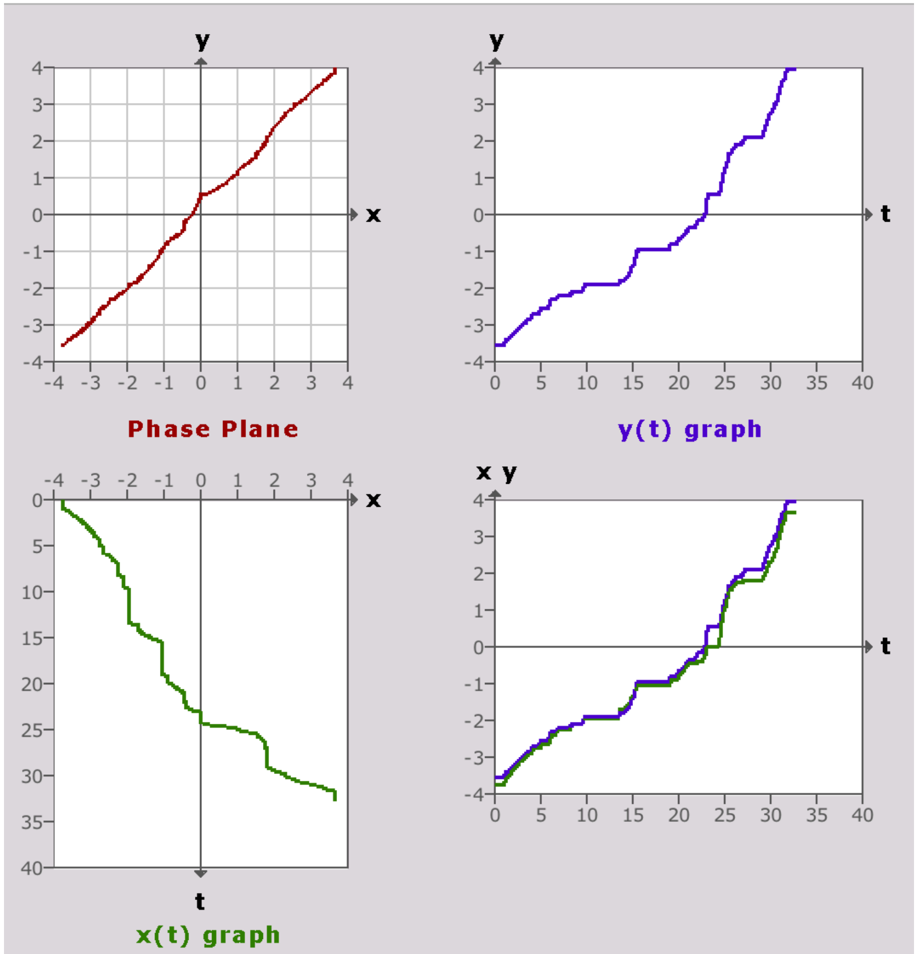
ax^2+bx+c ; changing b



Relating xy , xt , yt graphs



Different results for “straight” line

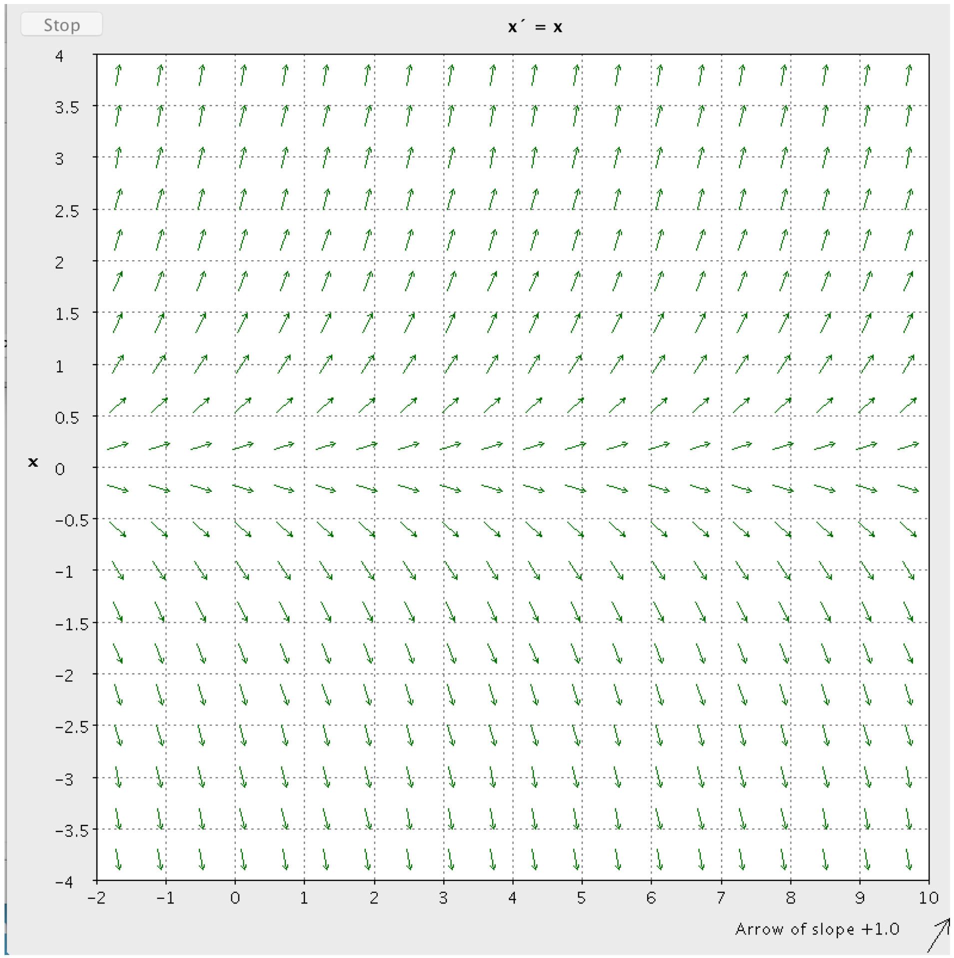


Direction Fields

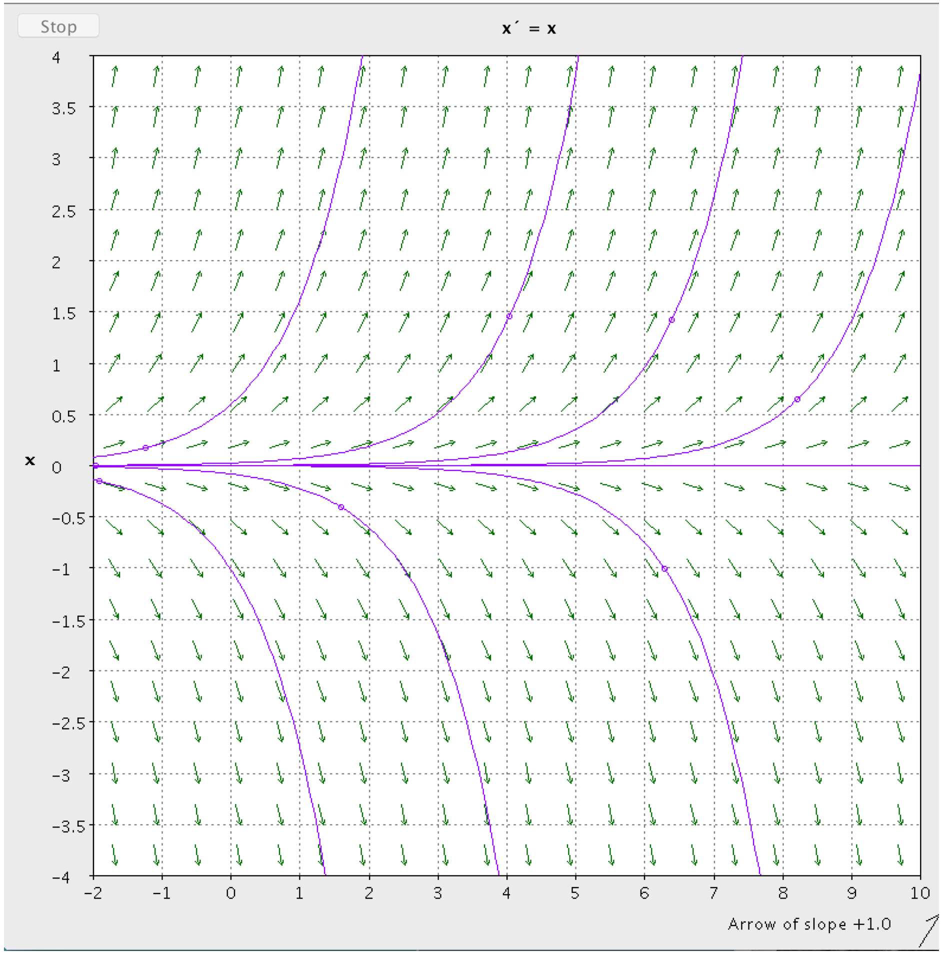
For a population problem where per capita growth rate is constant,

$$\frac{P'}{P} = a \quad \text{or} \quad \frac{\Delta P}{\Delta t} = P' = aP \,.$$

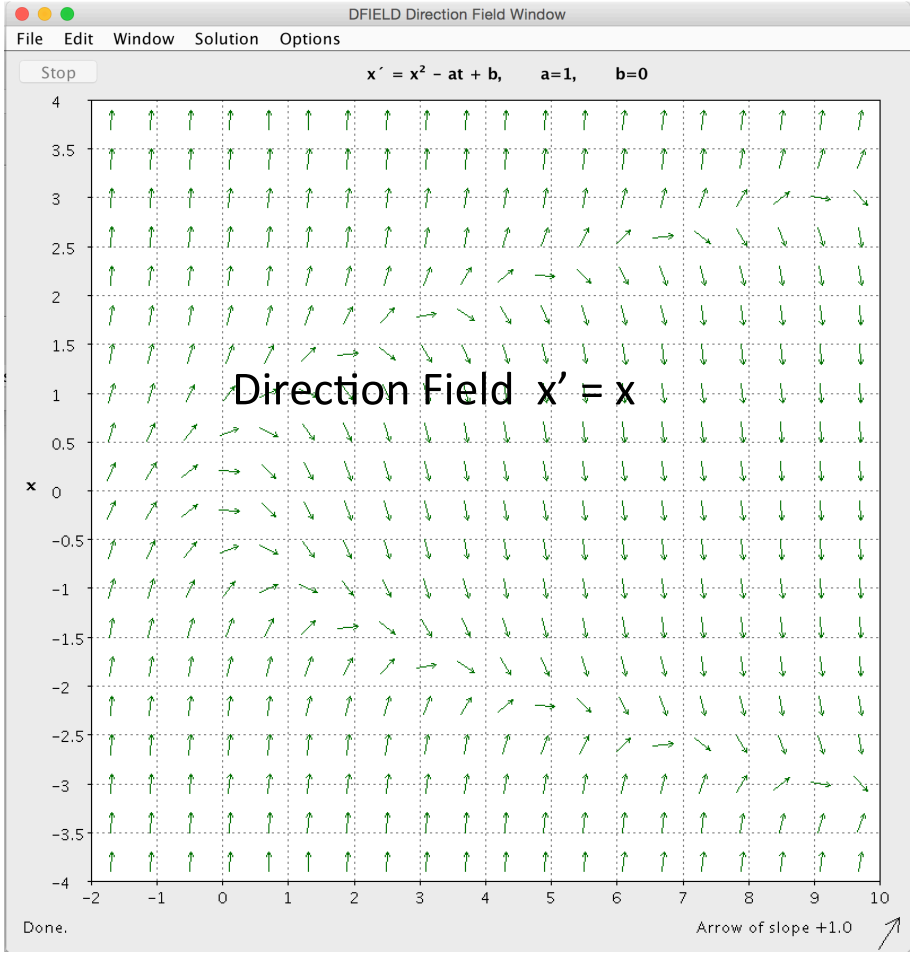
Direction Field x’ = x



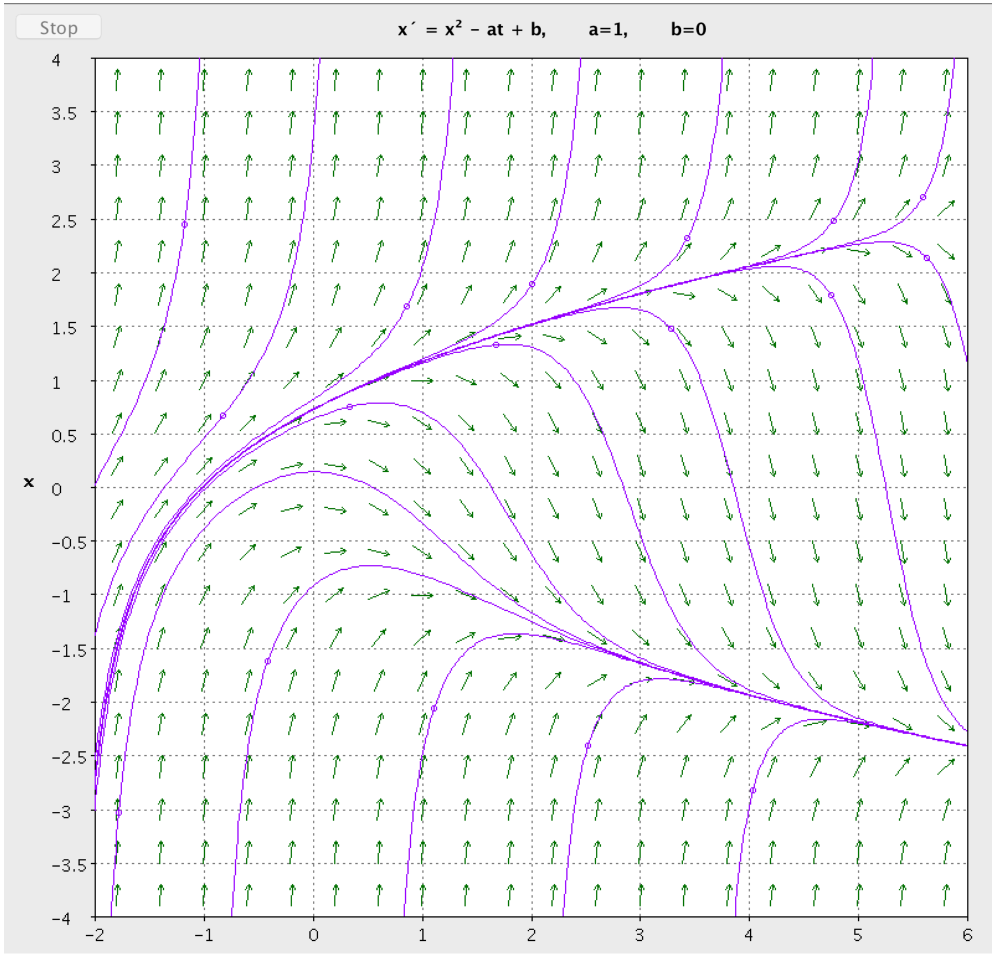
x’ = x   solutions



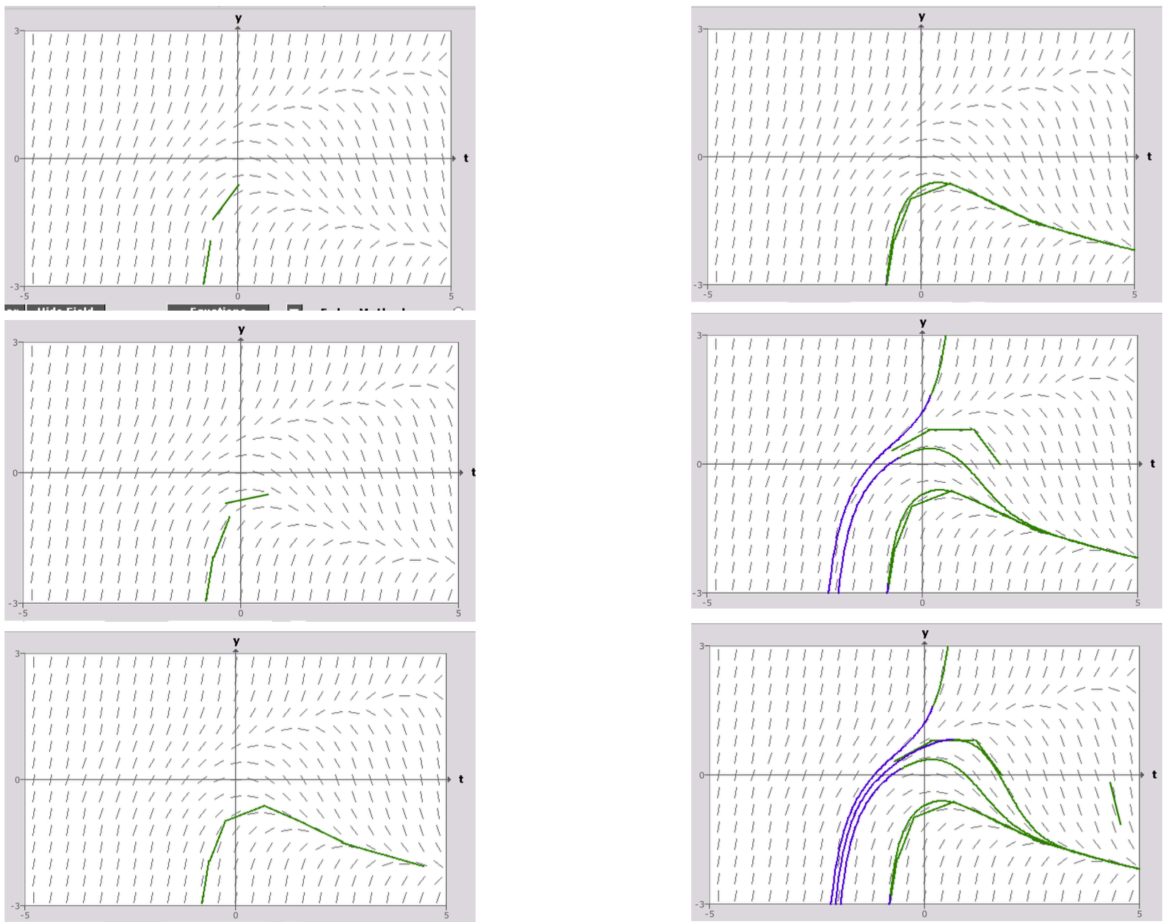
Direction Field x' = x^2 - t



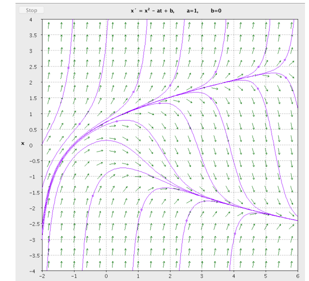
x’ = x^2 – t   solutions



$x' = x^2 - t$ drawing solutions



$x' = x^2 - t$

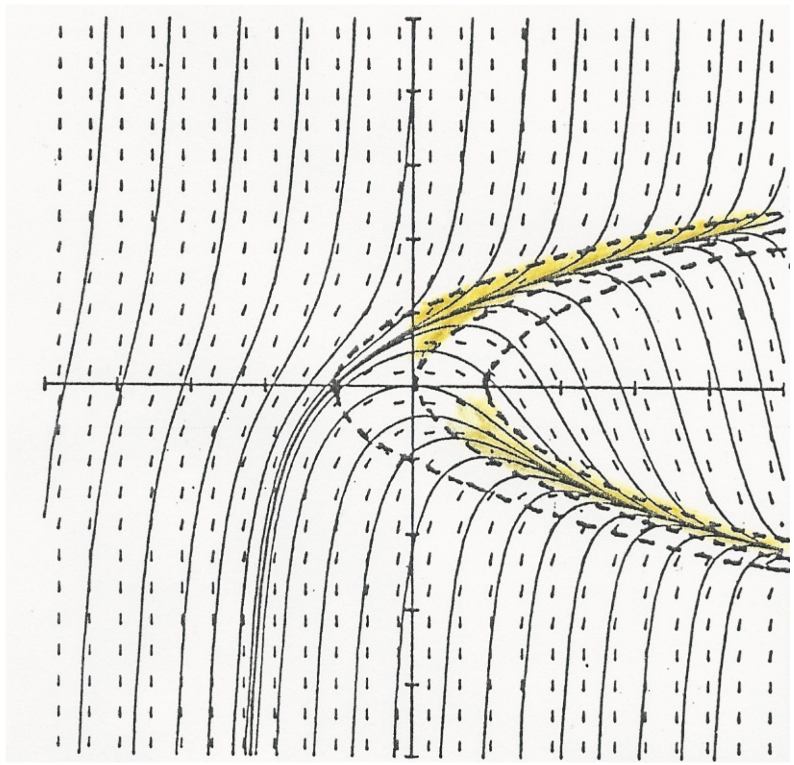


This equation is one of our favorites because, despite its simple appearance, it has *no* solutions that can be written in terms of ordinary functions. But this does *not* mean there are no solutions!

The direction field is given everywhere, so with the graphics capability of taking tiny steps through the slope field, we can *see* the solutions and analyze their behaviors.

Such qualitative analysis has revolutionized the field of differential equations. Differential equations are easy to set up, but the majority are nonlinear equations that traditional solution methods fail to solve analytically. Now the important thing is the ability to interpret the graphs and explain the behaviors of the solutions – e.g., what do the curves mean? when are they stable?

Qualitative solution behaviors



$x^2 - t = 1$

$x^2 - t = 0$

$x^2 - t = -1$

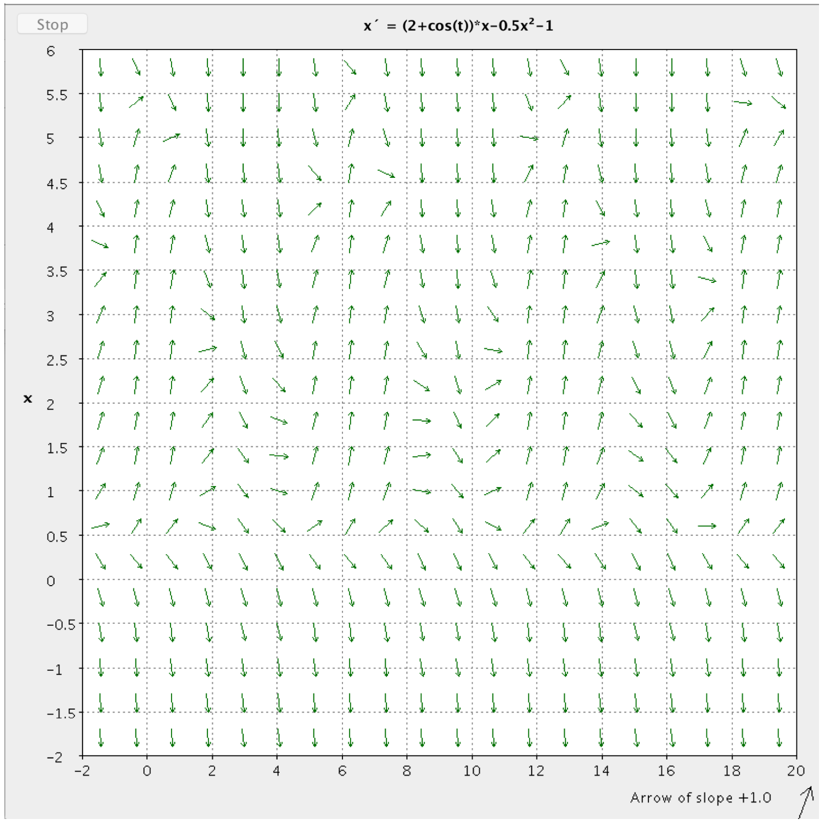
A refined population model

For x' = rate of change in population per unit time

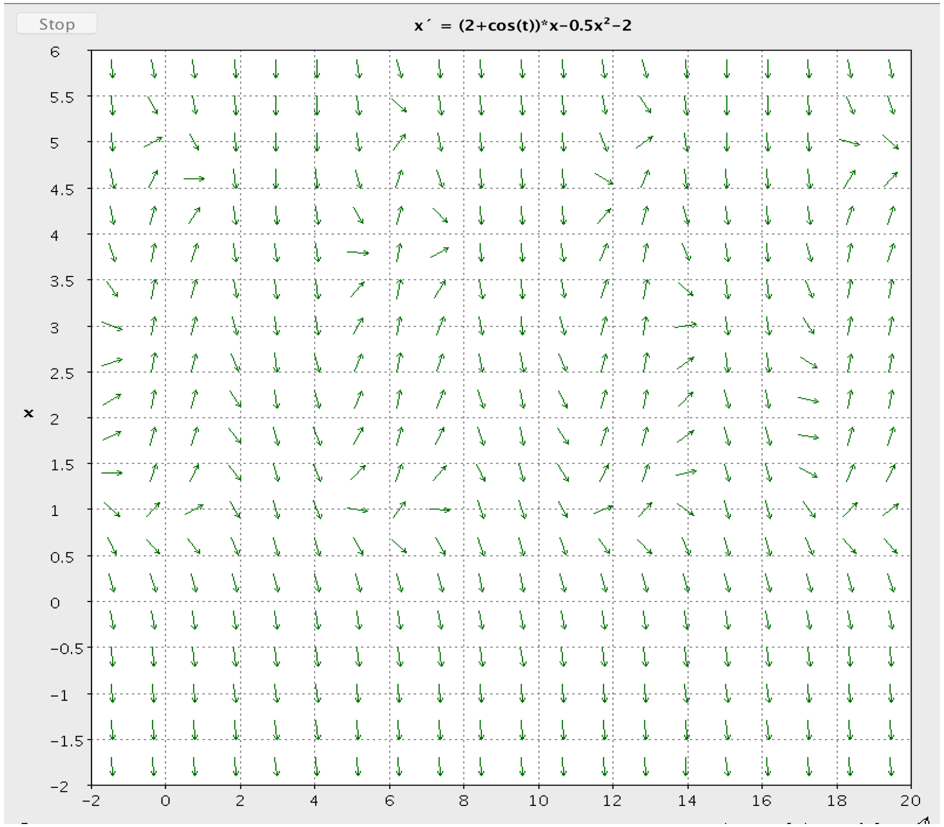
$$x' = (2+\cos t)\,x - 0.5\,x^2 - C$$

natural, positive rate of growth, with seasonal fluctuation	population decrease due to crowding	constant rate of decrease due to hunting (# licenses?)

$x' = (2+\cos t)\, x - 0.5\, x^2 - 1$

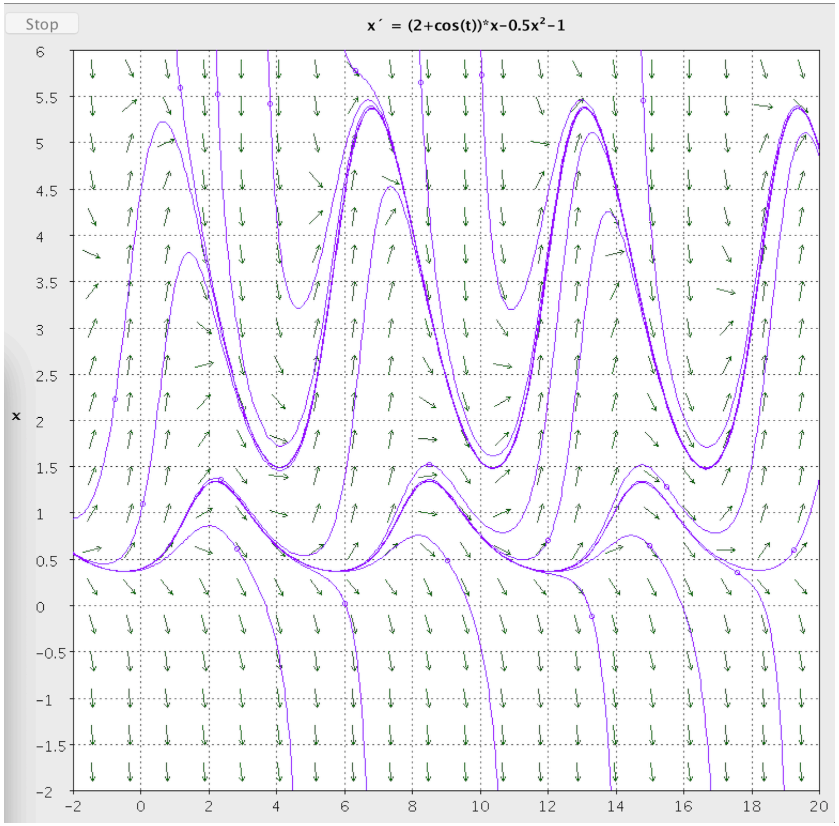


$x' = (2+\cos t)\, x - 0.5\, x^2 - 2$



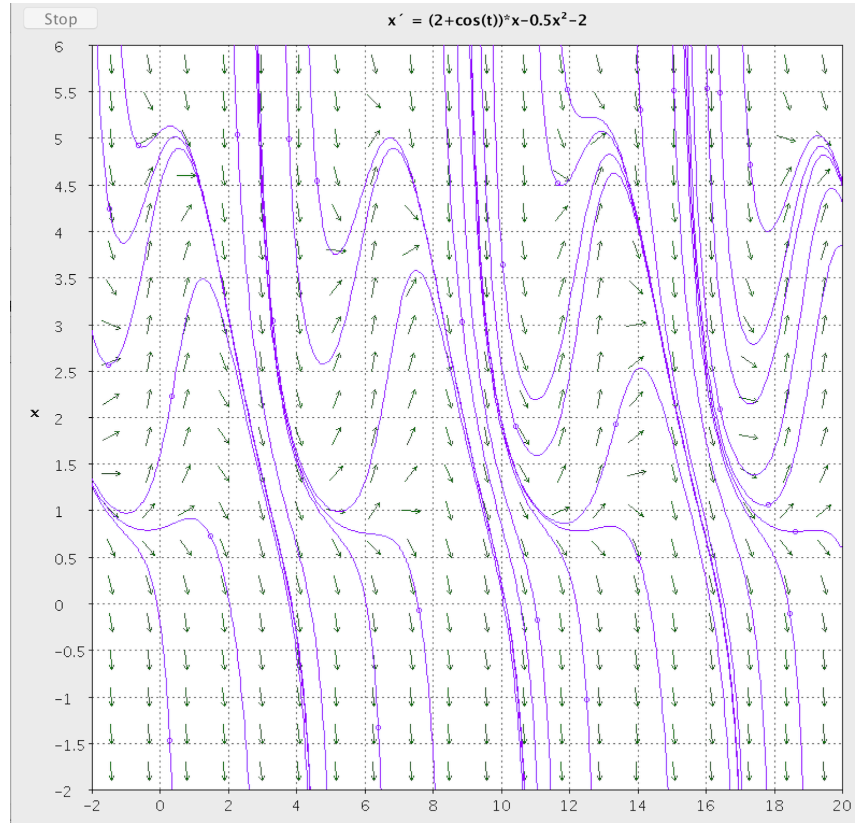
Solutions

x’ = (2+cos t) x - 0.5 x^2 - 1



Many sustainable solutions

x’ = (2+cos t) x - 0.5 x^2 - 2



NO sustainable solutions

Interpreting Graphs

Pendulum Example (Hu Hohn) for Linear

Let θ = angle from vertical. Then we have

$$\theta = \text{position} = x$$

$$\dot{\theta} = \text{velocity} = y$$

$$\ddot{\theta} = \text{acceleration.}$$

From physics, the simplest model gives

$$\ddot{\theta} = -\theta.$$

More Pendulum Models

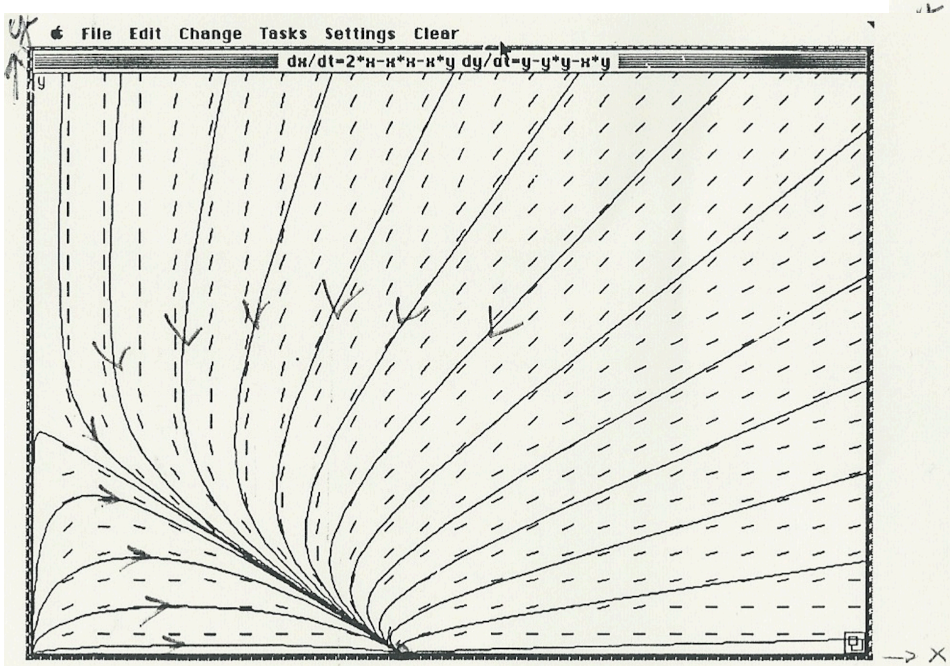
- Nonlinear $\ddot{\theta} = -\sin\theta$
- Damped $\ddot{\theta} = -\sin\theta - b\dot{\theta}$
- Forced Damped

$$\ddot{\theta} = -\sin\theta - b\dot{\theta} + A\cos\omega t\sin\theta$$

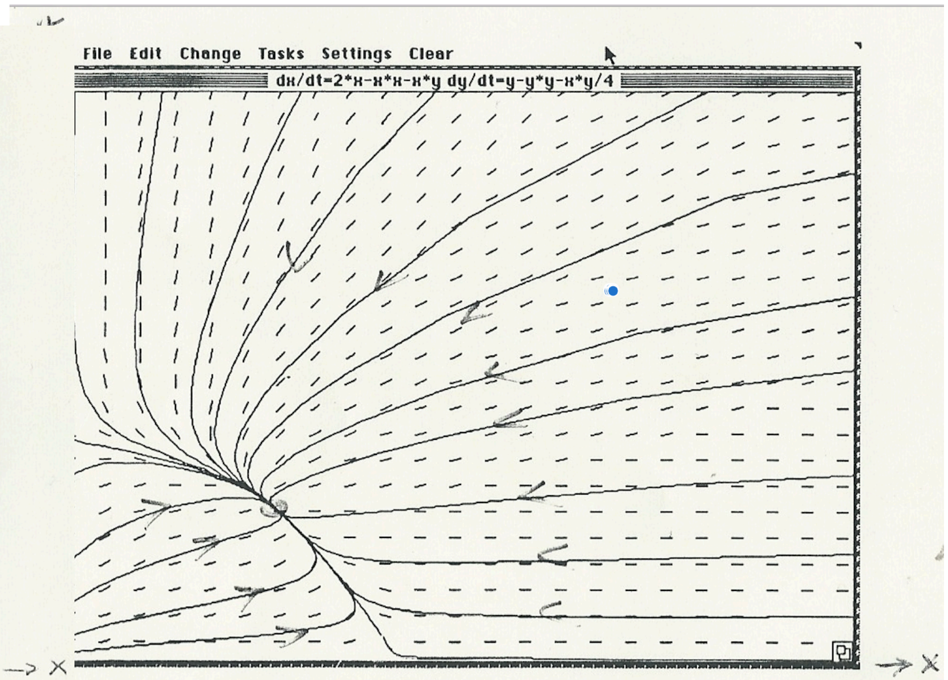
Predator/Prey

Two species, only last coefficient has changed.
Are you an ecologist or a pest control specialist? Which do *you* want?

$x' = x(2 - x - y)$, $y' = y(1-y-x)$

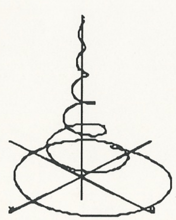


$x' = x(2 - x - y)$, $y' = y(1-y-x/4)$

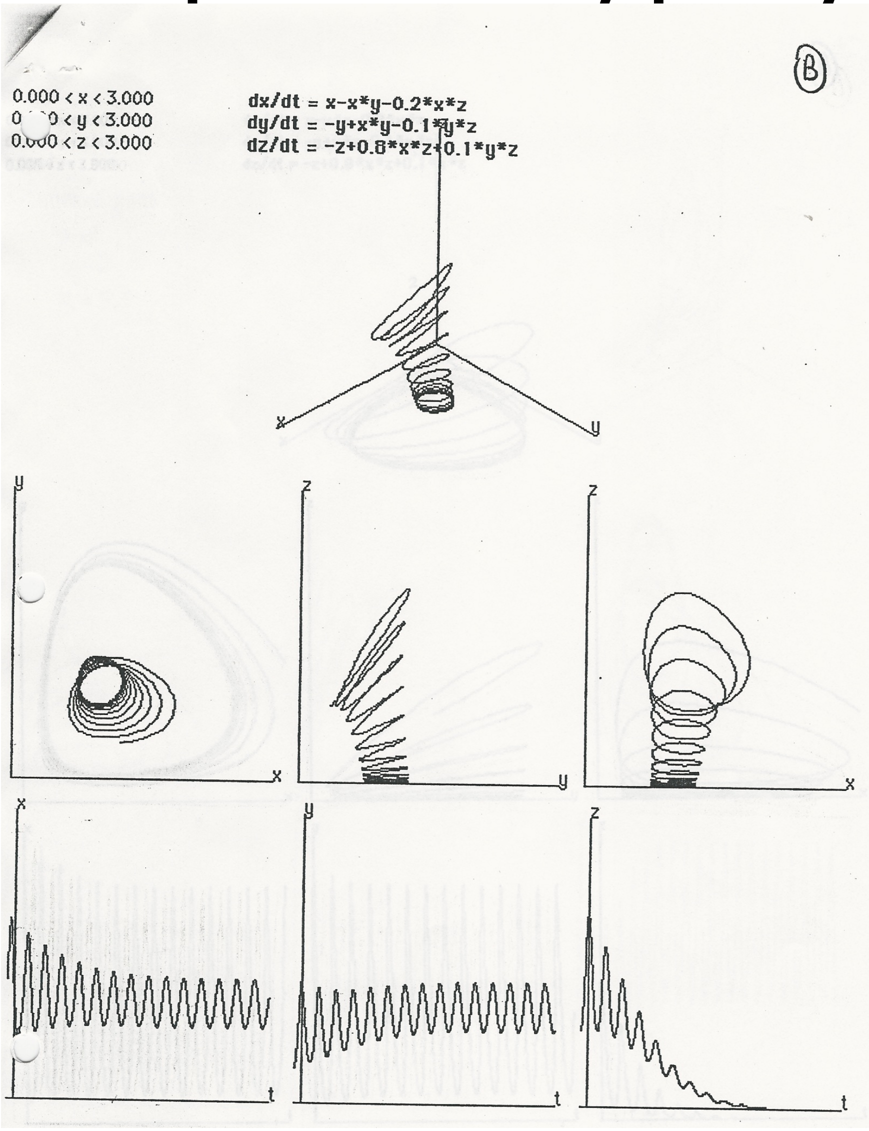


why *interactive* graphics?

- "to get it in your bones"
  (Rod Smart, University of Wisconsin)



3 species predator/prey model



Resources

- *Interactive Differential Equations* (free)

  (Hohn, Strogatz, McDill, Cantwell, West)

  Phase Plane Drawing (precursor of DE Sketchpad), Pendulums

  http://www.aw-bc.com/ide

- *DE Tools* (Blanchard, Devaney, Hall) (not free)

  DE Sketchpad, HPGSolver, Pendulums

  http://www.cengagebrain.com/shop/isbn/9780495562009

Thanks for listening and thinking!