Not All Tasks Are Created Equal

Mary Ann Huntley, Ph.D.
Director of Mathematics Outreach & K-12 Education Activities
Department of Mathematics

Cornell University
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Comparing Two Mathematical Tasks

- Martha's Carpeting Task
- The Fencing Task

Think privately and then solve each task. Talk with someone sitting near you about your solution process.

Martha's Carpeting Task

Martha was recarpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

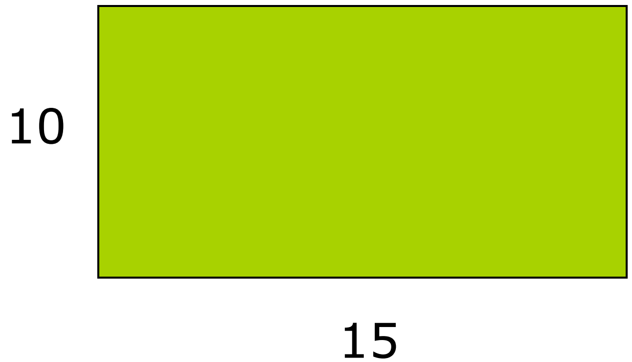
Martha's Carpeting Task
Using the Area Formula

$A = (\ l\ )\ (\ w\ )$

$A = (15 \text{ ft})\ (10 \text{ ft})$

$A = 150$ square feet

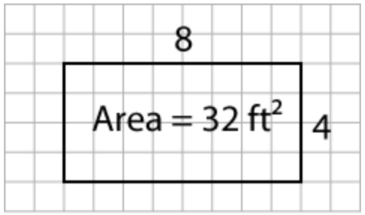
Martha's Carpeting Task
Drawing a Picture

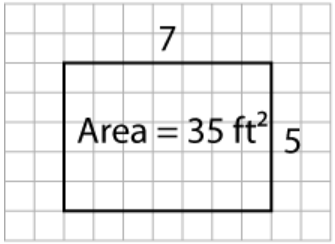


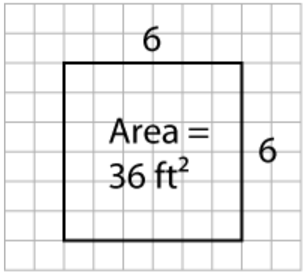
The Fencing Task

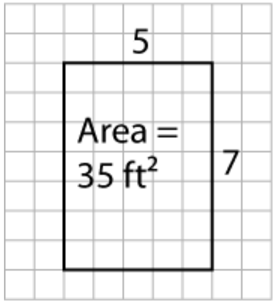
- Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits.
  - If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
  - How long would each of the sides of the pen be if they had only 16 feet of fencing?
  - How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

The Fencing Task
Diagrams on Grid Paper





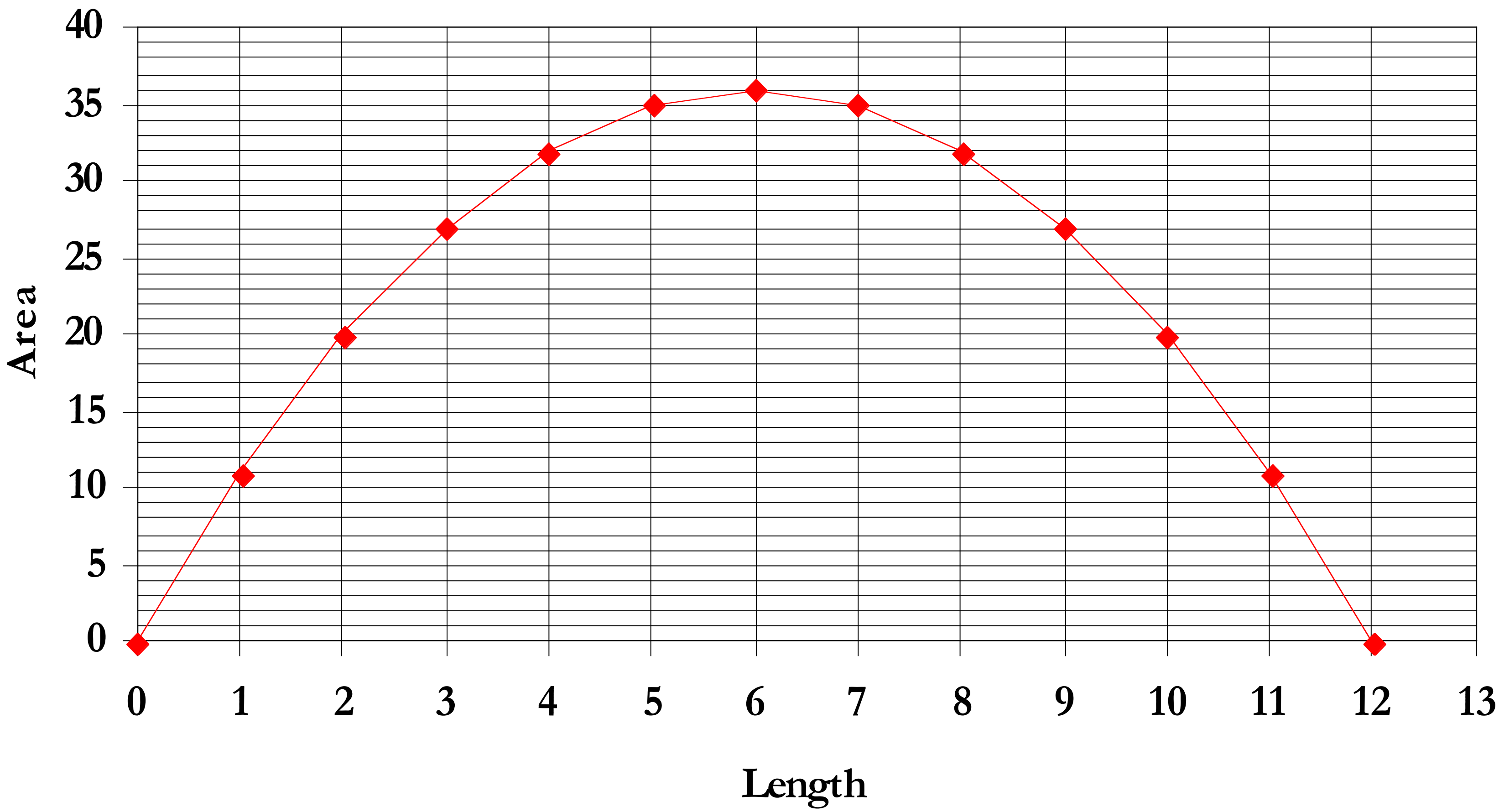




The Fencing Task
Using a Table

Length	Width	Perimeter	Area
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36
7	5	24	35

The Fencing Task
Graph of Length and Area



Comparing Two Mathematical Tasks

Work in Pairs

How are Martha's Carpeting Task and the Fencing Task the same and how are they different?

Similarities and Differences

Similarities

- Both are “area” problems
- Both require prior knowledge of area

Differences

- The amount of thinking and reasoning required
- The number of ways the problem can be solved
- Way in which the area formula is used
- The need to generalize
- The range of ways to enter the problem

Mathematical Tasks:
A Critical Starting Point for Instruction

Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking***.

Stein, Smith, Henningsen, & Silver (2000)

Mathematical Tasks

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray,
Oliver, & Human (1997)

Mathematical Tasks

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars (1995)

Mathematical Tasks

If we want students to develop the capacity to think, reason, and problem solve then we need to start with high-level, cognitively complex tasks.

Stein & Lane (1996)

Levels of Cognitive Demand & The Mathematical Tasks Framework

Linking to Research: The QUASAR Project

QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) – a national project that was aimed at improving mathematics instruction for students attending middle schools in economically disadvantaged communities in ways that emphasized thinking, reasoning, problem solving, and the communication of mathematical ideas

QUASAR Math Task Analysis Guide

- Low-Level Tasks
  - memorization
  - procedures without connections to meaning
    (e.g., Martha's Carpeting Task)
- High-Level Tasks
  - procedures with connections to meaning
  - doing mathematics (e.g., The Fencing Task)

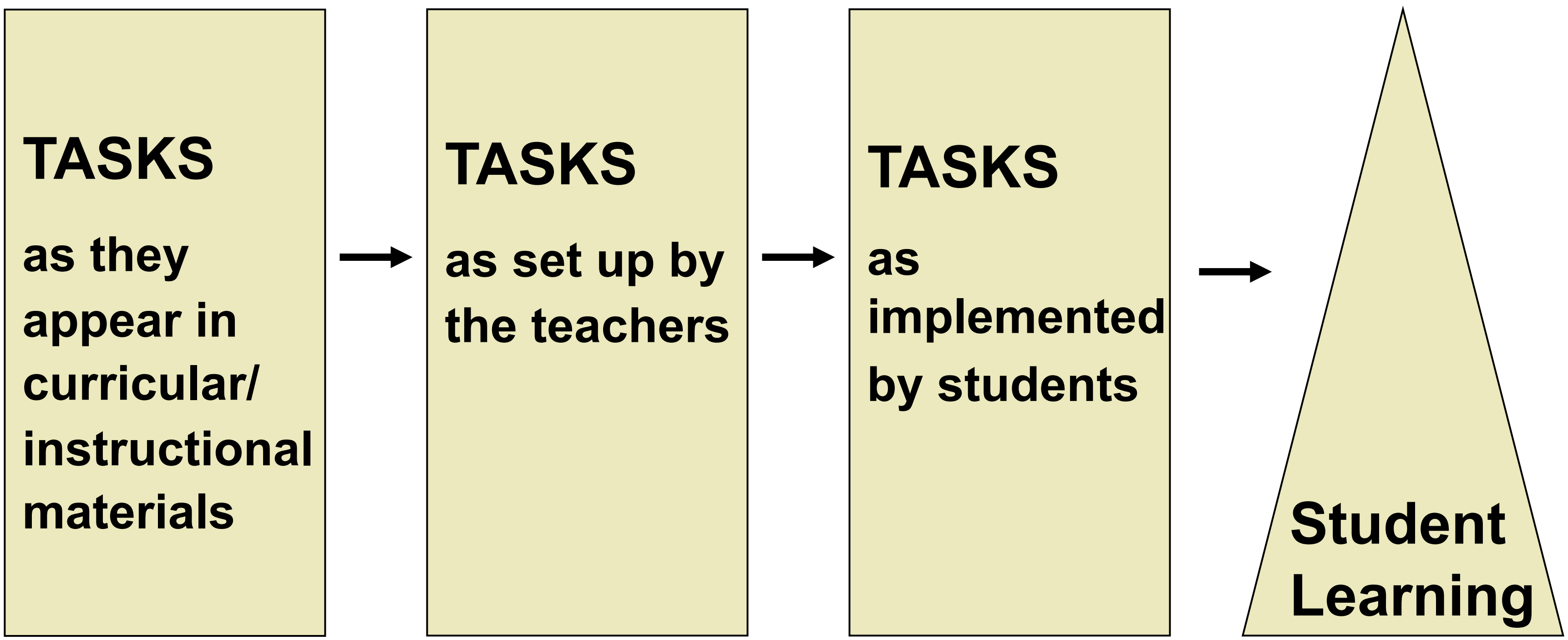
Math Task Sort Activity

THINK-PAIR-SHARE

Classify Tasks A-H using the

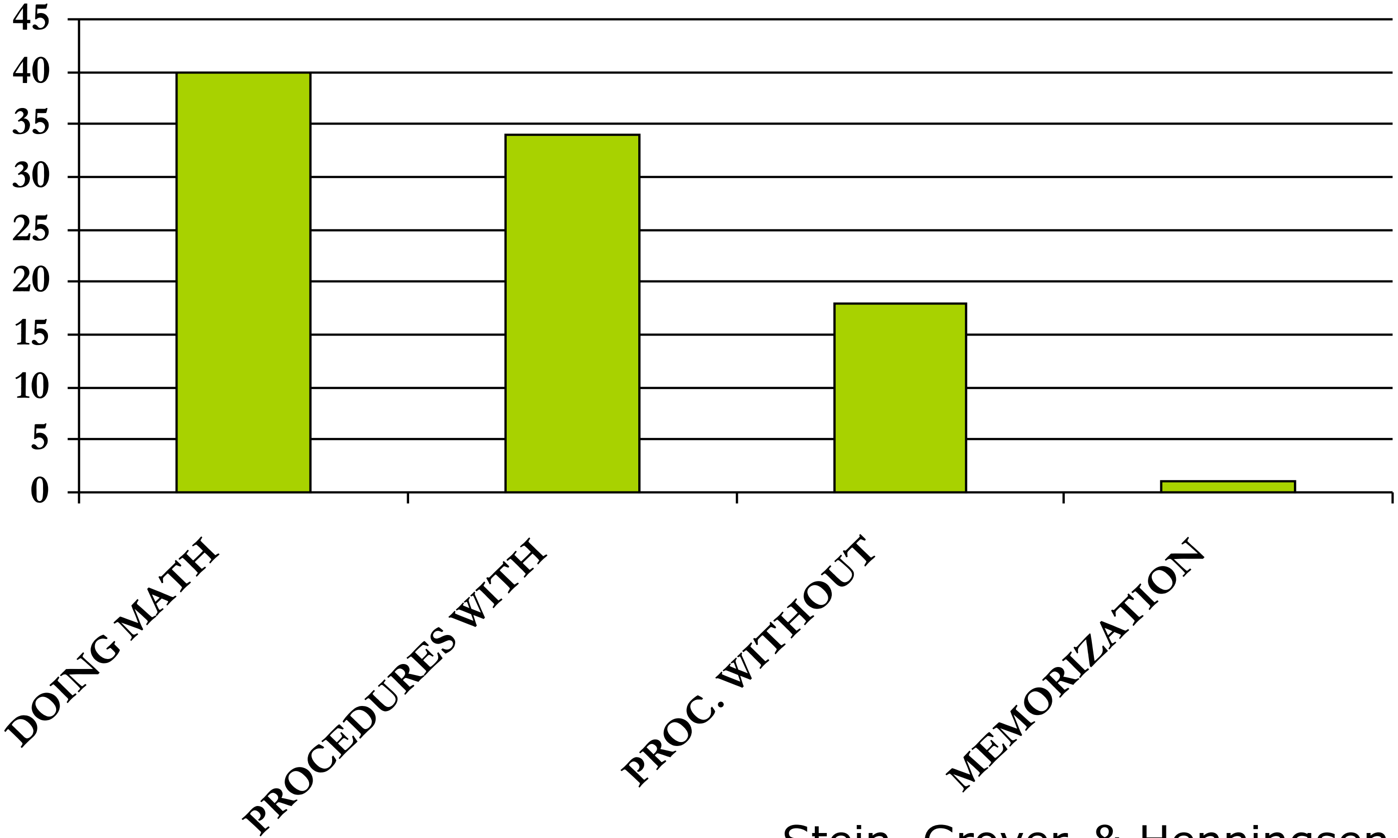
QUASAR Math Task Analysis Guide

The Mathematical Tasks Framework



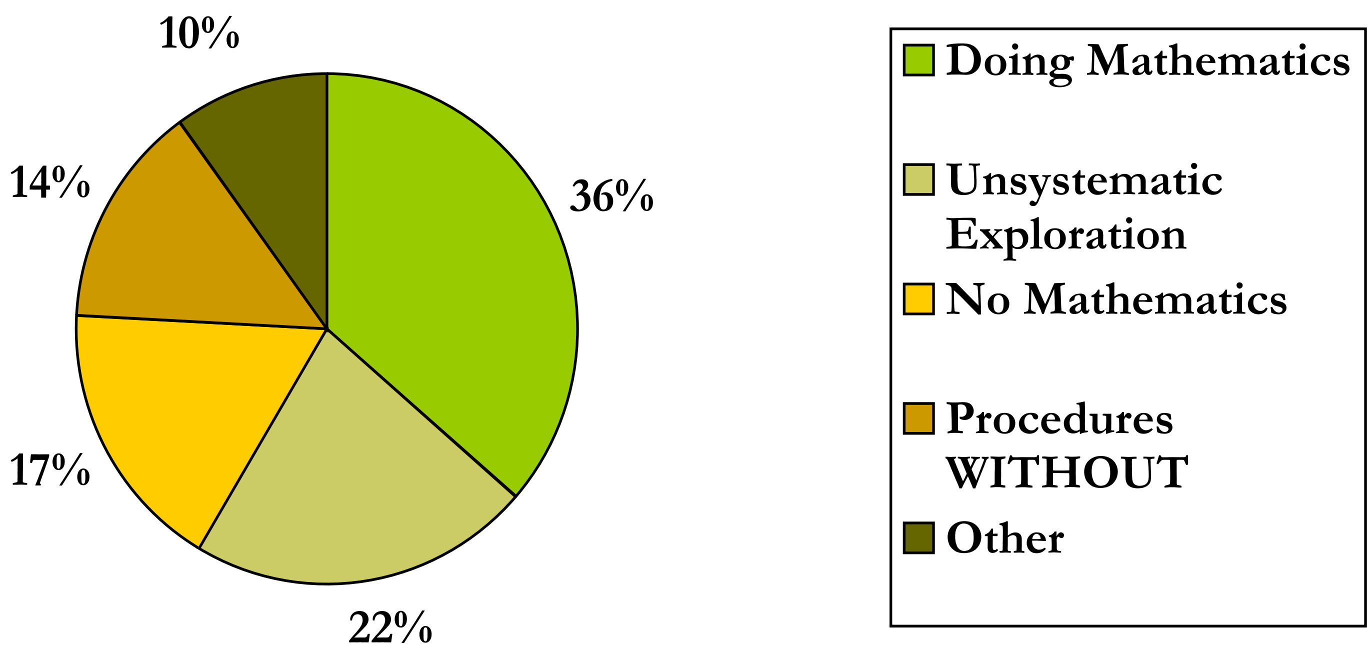
Stein, Smith, Henningsen, & Silver (2000, p. 4)

Cognitive Demands at Set Up



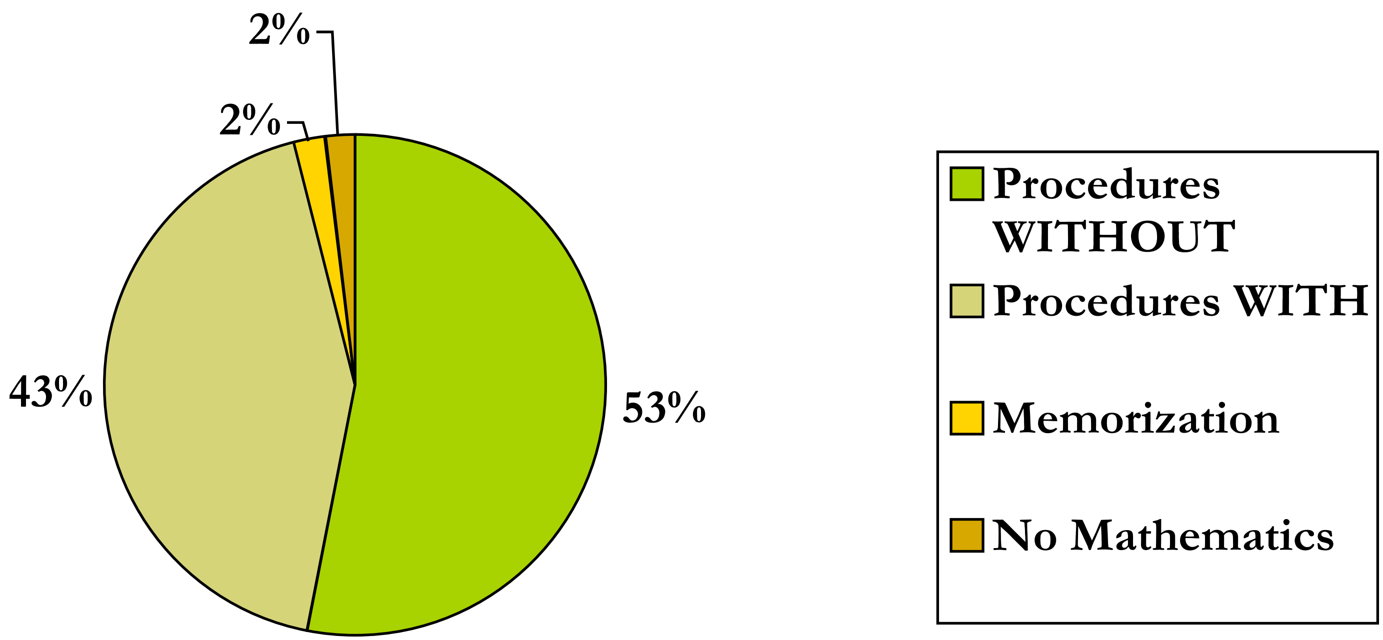
Stein, Grover, & Henningsen (1996)

The Fate of Tasks Set Up as Doing Mathematics



Stein, Grover, & Henningsen (1996)

The Fate of Tasks Set Up as Procedures WITH Connections to Meaning



Stein, Grover, & Henningsen (1996)

Factors Associated with the Maintenance and Decline of High-Level Cognitive Demands

- Routinizing problematic aspects of the task
- Shifting the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
- Providing insufficient time to wrestle with the demanding aspects of the task or so much time that students drift into off-task behavior
- Engaging in high-level cognitive activities is prevented due to classroom management problems
- Selecting a task that is inappropriate for a given group of students
- Failing to hold students accountable for high-level products or processes

Stein, Grover & Henningsen (1996)

Factors Associated with the Maintenance and Decline of High-Level Cognitive Demands

- Scaffolding of student thinking and reasoning
- Providing a means by which students can monitor their own progress
- Modeling of high-level performance by teacher or capable students
- Pressing for justifications, explanations, and/or meaning through questioning, comments, and/or feedback
- Selecting tasks that build on students' prior knowledge
- Drawing frequent conceptual connections
- Providing sufficient time to explore

Stein, Grover & Henningsen (1996)

Factors Associated with the Maintenance and Decline of High-Level Cognitive Demands

Decline

- Routinizing problematic aspects of the task
- Shifting the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
- Providing insufficient time to wrestle with the demanding aspects of the task or so much time that students drift into off-task behavior
- Engaging in high-level cognitive activities is prevented due to classroom management problems
- Selecting a task that is inappropriate for a given group of students
- Failing to hold students accountable for high-level products or processes

Maintenance

- Scaffolding of student thinking and reasoning
- Providing a means by which students can monitor their own progress
- Modeling of high-level performance by teacher or capable students
- Pressing for justifications, explanations, and/or meaning through questioning, comments, and/or feedback
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Stein, Grover & Henningsen (1996)

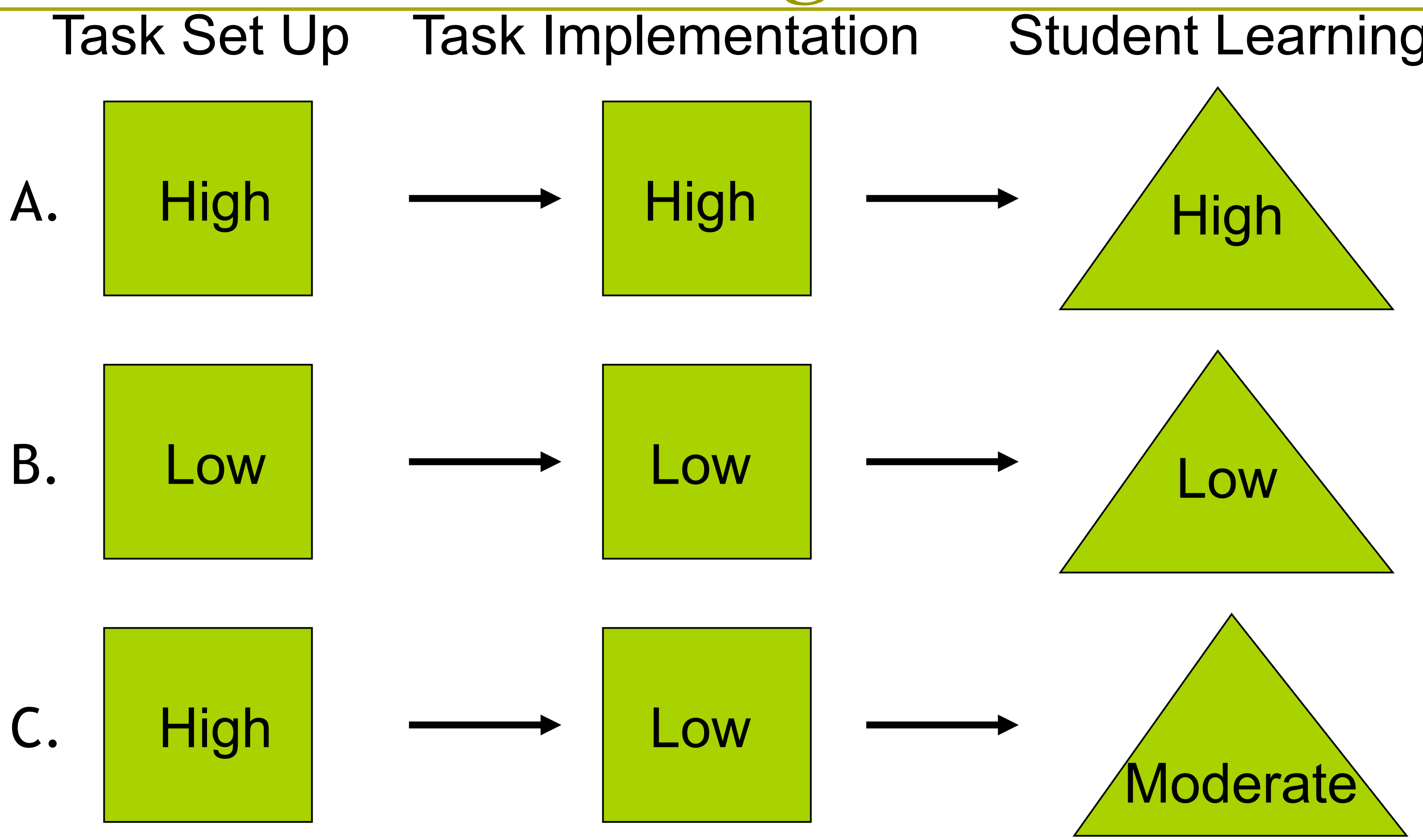
Does Maintaining Cognitive Demand Matter?

YES

QUASAR research shows . . .

- That maintaining the cognitive complexity of instructional tasks through the task enactment phase is associated with **higher student achievement.**
- That students who performed the best on project-based measures of reasoning and problem solving were in classrooms in which tasks were more likely to be set up **and** enacted at high levels of cognitive demand (Stein & Lane, 1996).

Patterns of Set up, Implementation, and Student Learning



Stein & Lane (1996)

TIMSS Video Study

- Higher-achieving countries implemented a greater percentage of high level tasks in ways that maintained the demands of the task (Stigler & Hiebert, 2004).

TIMSS Video Study

- Approximately 17% of the problem statements in the U.S. suggested a focus on mathematical connections or relationships. This percentage is within the range of many higher-achieving countries (i.e., Hong Kong, Czech Republic, Australia).
- Virtually **none** of the making-connections problems in the U.S. were discussed in a way that made the mathematical connections or relationships visible for students. Mostly, they turned into opportunities to apply procedures. Or, they became problems in which even less mathematical content was visible (i.e., only the answer was given).

TIMSS Video Mathematics Research Group (2003)

Boaler & Staples (2008)

- The success of students in the high-achieving school was due in part to the high cognitive demand of the curriculum and the teachers' ability to maintain the level of demand during enactment through questioning.

Conclusion

- Not all tasks are created equal — they provide different opportunities for students to learn mathematics.
- High level tasks are the most difficult to carry out in a consistent manner.
- Engagement in cognitively challenging mathematical tasks leads to the greatest learning gains for students.
- Professional development is needed to help teachers build the capacity to enact high level tasks in ways that maintain the rigor of the task.

Additional Articles and Books about the Mathematical Tasks Framework

Research Articles

Boston, M.D., & Smith, M.S., (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40*, 119-156.

Stein, M.K., Grover, B.W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*, 455-488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*, 50 - 80.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*, 524-549.

Additional Articles and Books about the Mathematical Tasks Framework

Practitioner Articles

Stein, M. K., & Smith, M.S. (1998). Mathematical tasks as a framework for reflection. *Mathematics Teaching in the Middle School*, *3*, 268-275.

Smith, M.S., & Stein, M.K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, *3*, 344-350.

Henningsen, M., & Stein, M.K. (2002). Supporting students' high-level thinking, reasoning, and communication in mathematics. In J. Sowder & B. Schappelle (Eds.), *Lessons learned from research* (pp. 27 – 36*).* Reston VA: National Council of Teachers of Mathematics.

Smith, M.S., Stein, M.K., Arbaugh, F., Brown, C.A., & Mossgrove, J. (2004). Characterizing the cognitive demands of mathematical tasks: A sorting task. In G.W. Bright and R.N. Rubenstein (Eds.), *Professional development guidebook for perspectives on the teaching of mathematics* (pp. 45-72). Reston, VA: NCTM.

Additional Books about the Mathematical Tasks Framework

Books

Stein, M.K., Smith, M.S., Henningsen, M., & Silver, E.A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Smith, M.S., Silver, E.A., Stein, M.K., Boston, M., Henningsen, M., & Hillen, A. (2005). *Cases of mathematics instruction to enhance teaching (Volume I: Rational Numbers and Proportionality).* New York: Teachers College Press.

Smith, M.S., Silver, E.A., Stein, M.K., Henningsen, M., Boston, M., & Hughes,E. (2005). *Cases of mathematics instruction to enhance teaching (Volume 2: Algebra as the Study of Patterns and Functions).* New York: Teachers College Press.

Smith, M.S., Silver, E.A., Stein, M.K., Boston, M., & Henningsen, M. (2005). *Cases of mathematics instruction to enhance teaching (Volume 3: Geometry and Measurement).* New York: Teachers College Press.

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Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record*, *110*, 608-645.

Hiebert, J., Carpenter, T.P., Fennema, D., Fuson, K.C., Wearne, D., Murray, H., Olivier, A., Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Lappan, G., & Briars, D.J. (1995). How should mathematics be taught? In I. Carl (Ed.), *75 years of progress: Prospects for school mathematics* (pp. 131-156). Reston, VA: National Council of Teachers of Mathematics.

Stigler, J.W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership*, *61*(5), 12-16.

TIMSS Video Mathematics Research Group. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study.* Washington, DC: NCES.