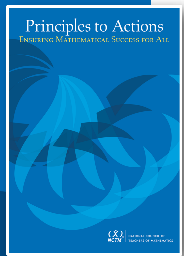

Effective Mathematics Teaching Practices

Diane J. Briars
Immediate Past President
National Council of Teachers of Mathematics
dbriars@nctm.org

Cornell University Department of Mathematics and the NYS Master Teacher Program
March 25, 2017

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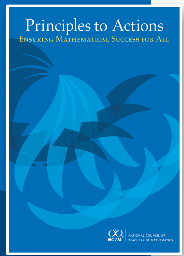

FYI

Electronic copies of slides and handouts are available by request
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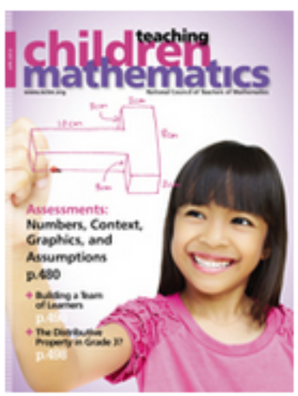

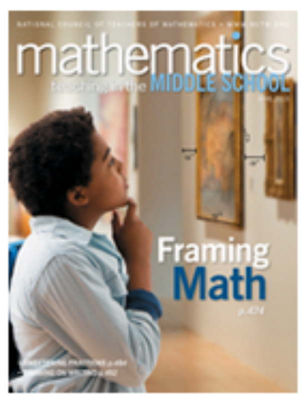

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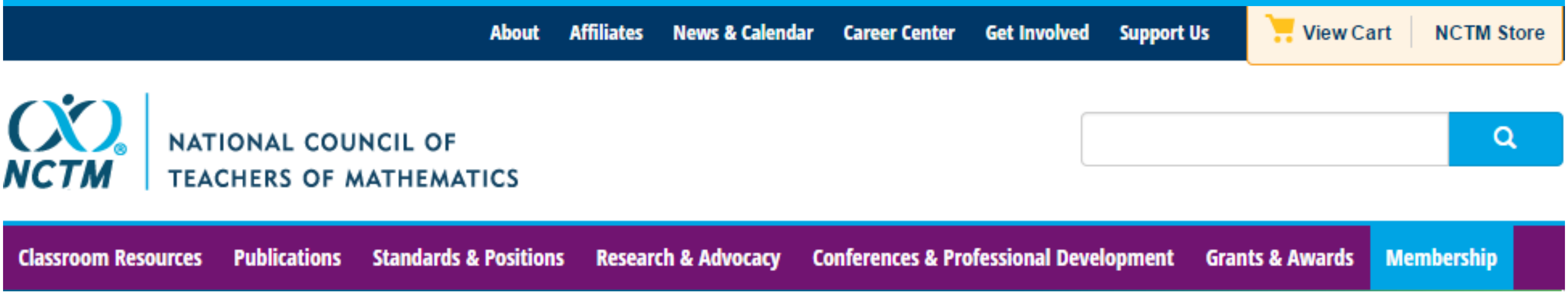

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Facilitating Meaningful Mathematical Discourse (Pre-K–Grade 12)

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Supporting Students' Productive Struggle (Pre-K–Grade 12)

July 20–22, 2017

Baltimore, MD

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2017 Regional Conferences

- **Orlando**

  October 18-20, 2017

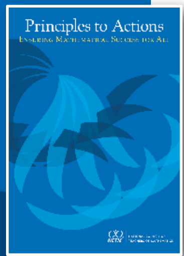
- **Chicago**

  November 29–December 1, 2017

2017 Innov8 Conference

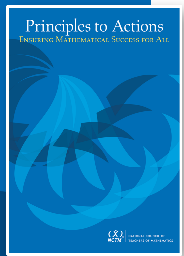
Breaking Barriers:
Actionable approaches to reach each and every learner in mathematics

Las Vegas, November 15-17, 2017

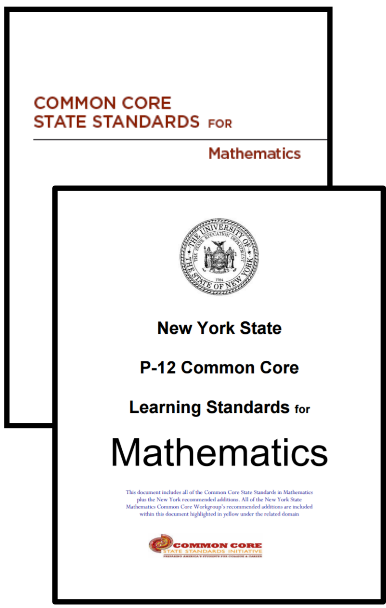


Agenda

- Discuss key shifts in instructional practice required to implement Common Core State Standards for Mathematics and other college and career-ready standards.
- Read and analyze a short case of a teacher (Mr. Donnelly or Ms. Culver) who is attempting to support his/her students' learning.
- Discuss effective teaching practices as described in NCTM's *Principles to Actions: Ensuring Mathematical Success for All*, and relate them to the case.
- Explore selected practices in greater depth.



High Quality Standards Are Necessary, but Insufficient, for Effective Teaching and Learning


COMMON CORE STATE STANDARDS FOR
Mathematics

New York State
P-12 Common Core
Learning Standards for
Mathematics

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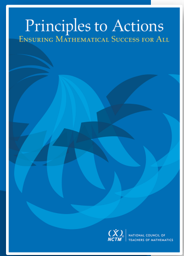
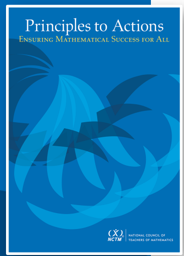


Key Features of CCSS-M

- **Focus:** Focus strongly where the standards focus.
- **Coherence**: Think across grades, and link to major topics
- **Rigor:** In major topics, pursue conceptual understanding, procedural skill and fluency, and application
- **Standards for Mathematical Practice**



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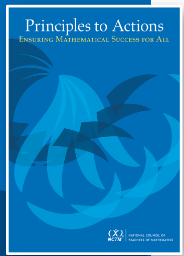


Curriculum Standards, Not Assessment Standards

Understand and apply properties of operations and the relationship between addition and subtraction. (1.OA)

3. Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

4. **Understand** subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Curriculum Standards, Not Assessment Standards

Define, evaluate, and compare functions. (8.F)

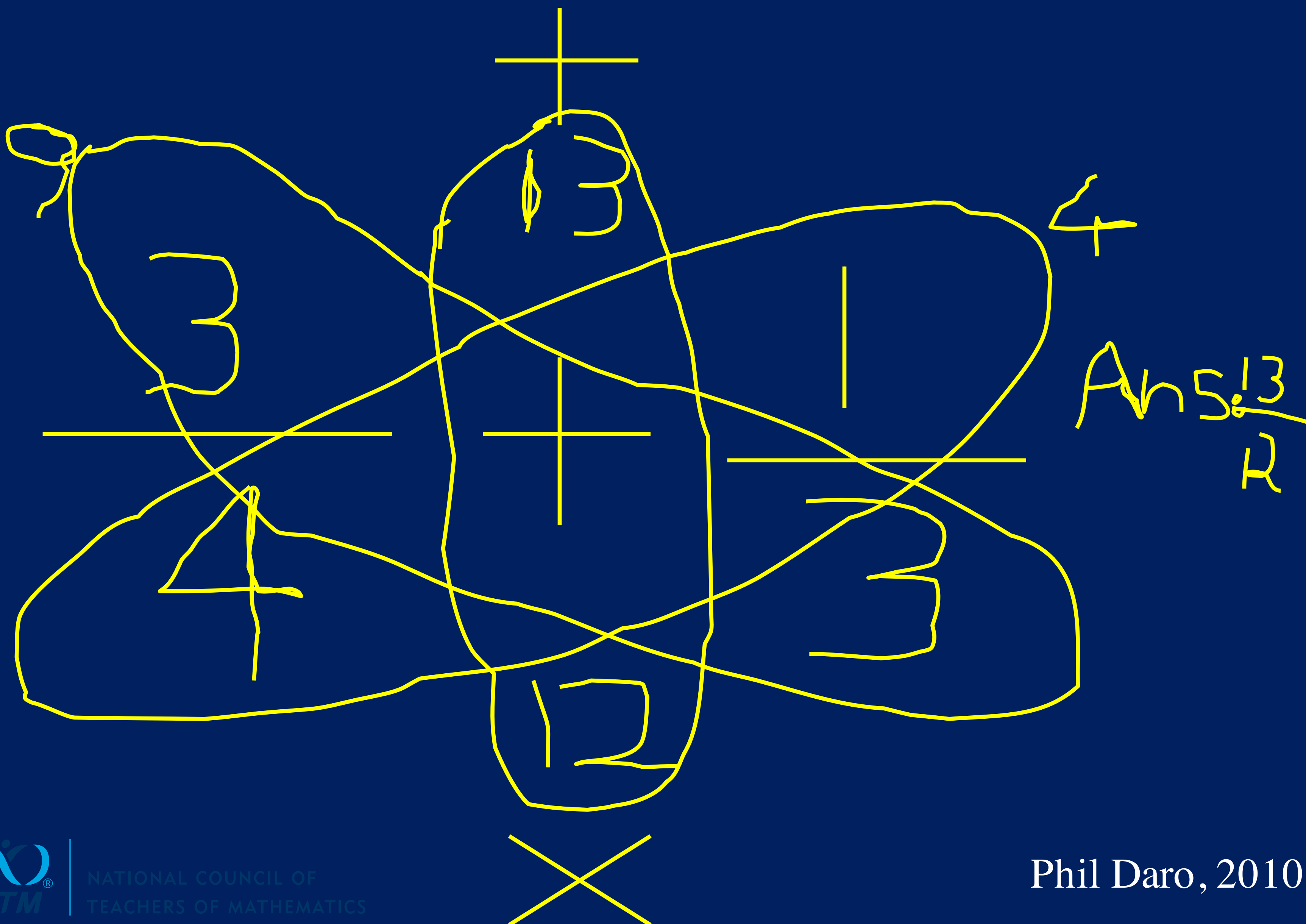
1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Curriculum Standards, Not Assessment Standards

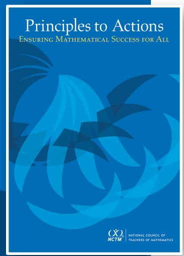
Define, evaluate, and compare functions. (8.F)

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

$$\frac{3}{4} + \frac{1}{3}$$

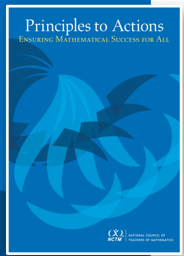


Phil Daro, 2010



Other “Butterflies”?

- FOIL
- Cross multiplication
- Division of fractions: Keep-change-flip, KFC, etc.
- $a - b = a + {}^{-}b$: Keep-change-change
- Division algorithm: Does McDonalds Sell Cheese Burgers?
- Key words



	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

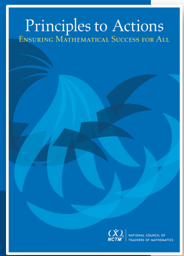
Key Instructional Shift

From emphasis on:

How to get answers

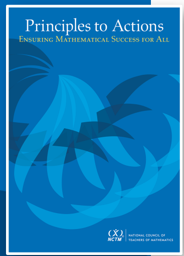
To emphasis on:

Understanding mathematics



Why Focus on Understanding?

- Understanding facilitates initial learning and retention.
- Understanding supports appropriate application and transfer.
- Understanding is an explicit expectation of CCSS-M.



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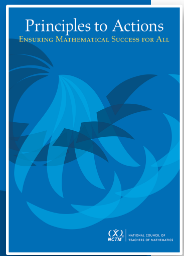

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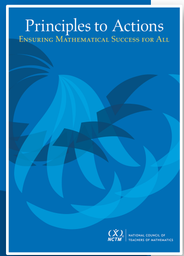

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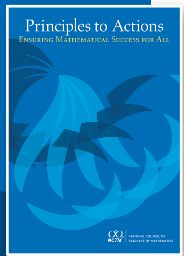
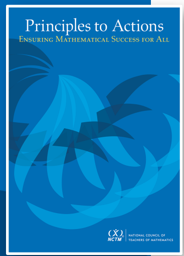
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Implementing CCSS-M Requires

Instructional practices that promote students' development of conceptual understanding and proficiency in the Standards for Mathematical Practice.



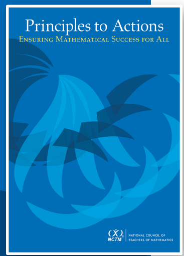
We Must Focus on Instruction

- Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum."

  (Ball & Forzani, 2011, p. 17)

- "Teaching has 6 to 10 times as much impact on achievement as all other factors combined ... Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points."

  Schmoker (2006, p.9)



Principles to Actions: Ensuring Mathematical Success for All

- Describes the **supportive conditions, structures, and policies** required to give all students the power of mathematics
- Focuses on **teaching and learning**
- Emphasizes engaging students in **mathematical thinking**
- Describes how to ensure that mathematics achievement is maximized **for every student**
- Is not specific to any standards; **it’s universal**

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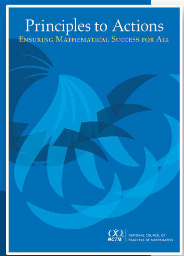

Guiding Principles for School Mathematics

1. Teaching and Learning

Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.

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Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism

→ Essential Elements of Effective Math Programs

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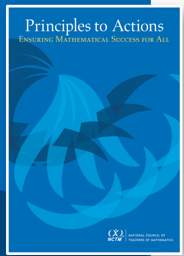


Looking into the Classroom

Select a problem:

Candy Jar Problem (Grade 7)

A candy jar contains 5 Jolly Ranchers (JRs) and 13 Jawbreakers (JBs). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know.

Pay It Forward Problem (Algebra 1)

In the movie "Pay it Forward", a student, Trevor, comes up with an idea that he thought could change the world. He decides to do a good deed for three people and then each of the three people would do a good deed for three more people and so on. He believed that before long there would be good things happening to billions of people. At stage 1 of the process, Trevor completes three good deeds. How does the number of good deeds grow from stage to stage? How many good deeds would be completed at stage 5? Describe a function that would model the Pay It Forward process at *any* stage.

Looking into the Classroom

Select a problem:

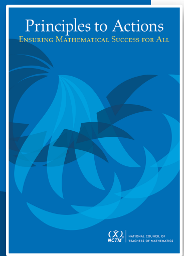
- Candy Jar—Grade 7
- Pay It Forward—Algebra 1

Please work the problem as if you were a student.

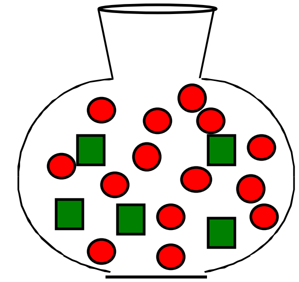
When done, share your work with people at your table.

Discuss:

- What mathematics learning could this task support?



Candy Jar Task



Common Core State Standards

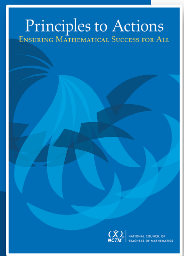
7.RP:2. Recognize and represent proportional relationships between quantities.

a. **Decide whether two quantities are in a proportional relationship**, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. **Identify the constant of proportionality (unit rate) in tables**, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. **Represent proportional relationships by equations**. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.



Pay It Forward Task

Common Core State Standards:

Standard HSF-BF.A.1 Write a function that describes a relationship between two quantities.

> **BF.A.1.A** Determine an explicit expression, a recursive process, or steps for calculation from a context.

Standard HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

> **LE.A.1.A** Prove that linear functions grow by equal differences over equal intervals, and that **exponential functions grow by equal factors over equal intervals**.

Standard HSF-LE.A.2. Construct linear and **exponential functions**, including arithmetic and geometric sequences, **given** a graph, **a description of a relationship**, or two input-output pairs (include reading these from a table).

Standard HSF.LE.B.5. Interpret the parameters in a linear or **exponential function in terms of a context**.

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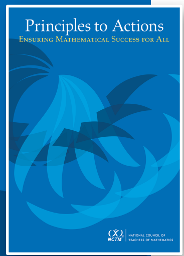

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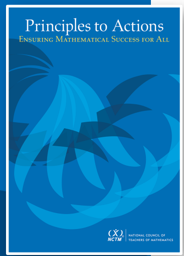
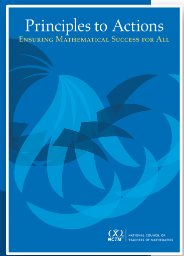


Looking into the Classroom

- Read the case for your problem:
  - Mr. Donnelly
  - Ms. Culver
- Make note of what the teacher did before or during instruction to support his/her students’ learning and understanding.
- Talk with a neighbor about the actions and interactions that you identified as supporting student learning.



Principle on Teaching and Learning

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

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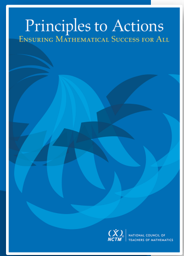

Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

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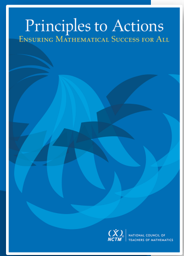

Establish Mathematics Goals To Focus Learning

Learning Goals should:

- Clearly state what it is students are to learn and understand about mathematics as the result of instruction;
- Be situated within learning progressions; and
- Frame the decisions that teachers make during a lesson.

Formulating clear, explicit learning goals sets the stage for everything else.

(Hiebert, Morris, Berk, & Janssen, 2007, p.57)

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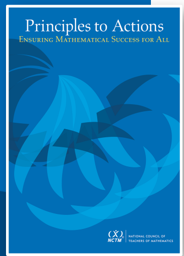

Goals to Focus Learning

Mr. Donnelly's goal for students' learning:

Students will recognize that quantities that are in a proportional (multiplicative) relationship grow at a constant rate and that there are three key strategies that could be used to solve problems of this type – scaling up, scale factor, and unit rate.

Ms. Culver's goal for students' learning:

Students will understand that exponential functions grow by equal factors over equal intervals and that in the general equation $y = b^x$, the exponent (x) tells you how many times to use the base (b) as a factor.

How does each goal align with this next teaching practice?

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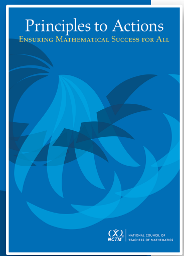

Implement Tasks that Promote Reasoning and Problem Solving

Mathematical tasks should:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students' current understanding; and
- Have multiple entry points.

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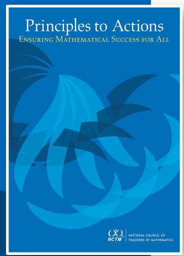
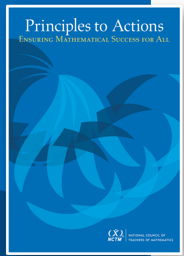


Why Tasks Matter

- Tasks form the basis for students' opportunities to learn what mathematics is and how one does it;
- Tasks influence learners by directing their attention to particular aspects of content and by specifying ways to process information;
- The level and kind of thinking required by mathematical instructional tasks influences what students learn; and
- Differences in the level and kind of thinking of tasks used by different teachers, schools, and districts, is a major source of inequity in students' opportunities to learn mathematics.



Why Tasks Matter

- Tasks form the basis for students' opportunities to learn what mathematics is and how one does it;
- Tasks influence learners by directing their attention to particular aspects of content and by specifying ways to process information;
- The level and kind of thinking required by mathematical instructional tasks influences what students learn; and
- **Differences in the level and kind of thinking of tasks used by different teachers, schools, and districts, is a major source of inequity in students' opportunities to learn mathematics.**

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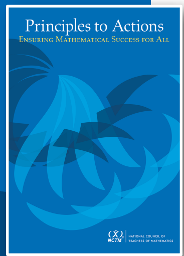

Compare Candy Jar and Pay It Forward to:

Finding the Missing Value

Find the value of the unknown in each of the proportions shown below.

$$\frac{5}{2} = \frac{y}{10} \qquad \frac{a}{24} = \frac{7}{8}$$

$$\frac{n}{8} = \frac{3}{12} \qquad \frac{30}{6} = \frac{b}{7}$$

$$\frac{5}{20} = \frac{3}{d} \qquad \frac{3}{x} = \frac{4}{28}$$

Petoskey Population

The population of Petoskey, Michigan, was 6076 in 1990 and was growing at the rate of 3.7% per years. The city planners want to know what the population will be in the year 2025. Write and evaluate an expression to estimate this population.

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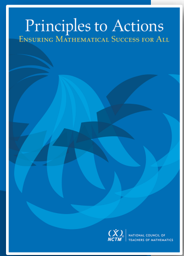

Candy Jar and Pay It Forward

What features of these tasks contributed to them meeting these criteria:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students' current understanding; and
- Have multiple entry points.

Core Instructional Issue

Do *all* students have the opportunity to engage in mathematical tasks that promote students' attainment of the mathematical practices on a regular basis?

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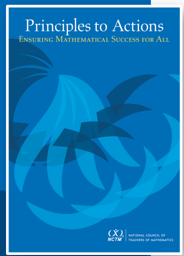
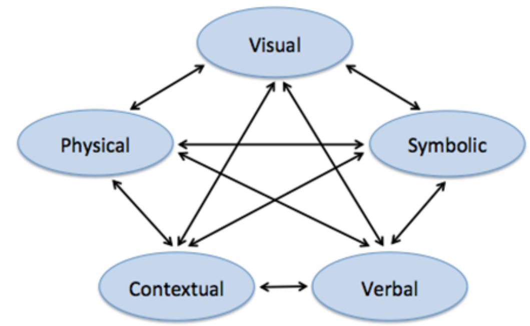


Use and Connect Mathematical Representations



Different Representations should:

- Be introduced, discussed, and connected;
- Focus students' attention on the structure or essential features of mathematical ideas; and
- Support students' ability to justify and explain their reasoning.

Strengthening the ability to move between and among these representations improves the growth of children's concepts.

Lesh, Post, Behr, 1987

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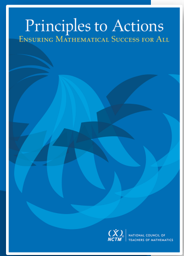

How might students benefit from connecting representations?

Group 1 (incorrect, additive)

100 JRs is 95 more than the 5 we started with. So we will need 95 more JBs than the 13 we started with.

5 JRs + 95 JRs = 100 JRs
13 JBs + 95 JBs = 108 JBs

Groups 3 and 5 (scale factor)

You had to multiply the five JRs by 20 to get 100, so you'd also have to multiply the 13 JBs by 20 to get 260.

$$\begin{array}{l} 5 \text{ JRs} \xrightarrow{(\times 20)} 100 \text{ JRs} \\ 13 \text{ JBs} \xrightarrow[(\times 20)]{} 260 \text{ JBs} \end{array}$$

Groups 4 and 7 (scaling up)

JR	JB	JR	JB
5	13	55	143
10	26	60	156
15	39	65	169
20	52	70	182
25	65	75	195
30	78	80	208
35	91	85	221
40	104	90	234
45	117	95	247
50	130	100	260

Group 2 (unit rate)

Since the ratio is 5 JRs for 13 JBs, we divided 13 by 5 and got 2.6. So that would mean that for every 1 JR there are 2.6 JBs. So then you just multiply 2.6 by 100.

$$\begin{array}{l} 1 \text{ JR} \xrightarrow{(\times 100)} 100 \text{ JRs} \\ 2.6 \text{ JBs} \xrightarrow[(\times 100)]{} 260 \text{ JBs} \end{array}$$

Group 6 (scaling up)

JRs	5	10	20	40	80	100
JBs	13	26	52	104	208	260

We started by doubling both the number of JRs and JBs. But then when we got to 80 JRs, we didn't want to double it anymore because we wanted to end up at 100 JRs, and doubling 80 would give us too many. So we noticed that if we added 20 JRs:52 JBs and 80 JRs:208 JBs, we would get 100 JRs:260 JBs.

Group 8 (scaling up)

We drew 100 JRs in groups of 5. Then we put 13 JBs with each group of 5 JRs. We then counted the number of JBs and found we had used 260 of them.

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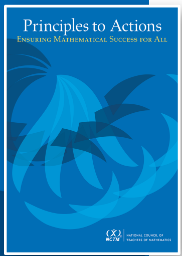


How might students benefit from connecting representations?

Group 1
(equation - incorrect)

$y = 3x$

At every stage there are three times as many good deeds as there were in the previous stage.

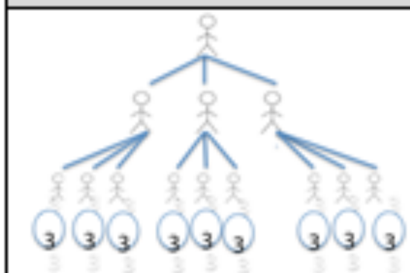
Group 2
(table like Group 6 & 7 and **equation**)

$y = 3^x$

Group 3
(diagram like Group 4 and **table**)

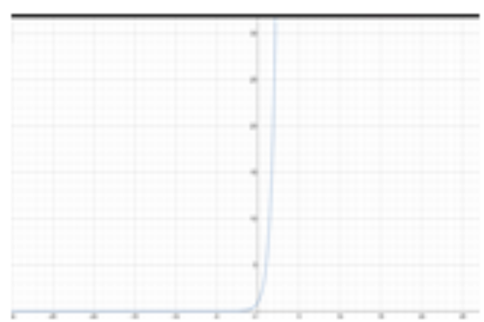
x (stages)		y (deeds)
1	3	3
2	3 x 3	9
3	3 x 3 x 3	27
4	3 x 3 x 3 x 3	81
5	3 x 3 x 3 x 3 x 3	243

Group 4
(diagram)



So the next stage will be 3 times the number there in the current stage so 27 x 3. It is too many to draw. You keep multiplying by 3.

Group 5
(table like Group 6 & 7 and **graph**)



Groups 6 and 7
(table)

X (stages)	Y (deeds)
1	3
2	9
3	27
4	81
5	243

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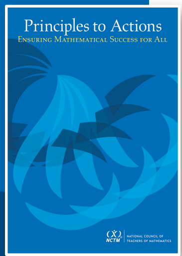

Facilitate Meaningful Mathematical Discourse

Mathematical Discourse should:

- Build on and honor students' thinking;
- Provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and
- Advance the mathematical learning of the whole class.

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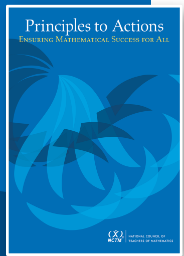

Facilitate Meaningful Mathematical Discourse

Discussions that focus on cognitively challenging mathematical tasks, namely those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano & Inagaki, 1991; Michaels, O'Connor, & Resnick, 2008).

Smith, Hughes, Engle & Stein, 2009, p. 549

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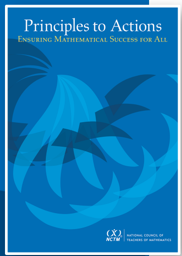
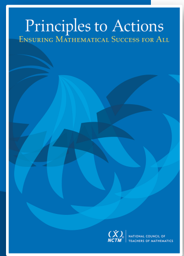
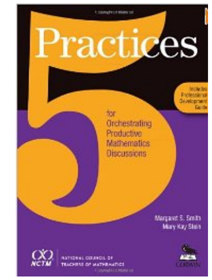


Meaningful Discourse

What did Mr. Donnelly and Ms. Culver do (before or during the discussion) that may have positioned him/her to engage students in a productive discussion?



Five Practices for Orchestrating Productive Discussions



- ***Anticipating*** likely student responses
- ***Monitoring*** students' actual responses
- ***Selecting*** particular students to present their mathematical work during the whole class discussion
- ***Sequencing*** the student responses
- ***Connecting*** different students' responses—to each other and to key mathematical ideas.

Smith & Stein, 2011

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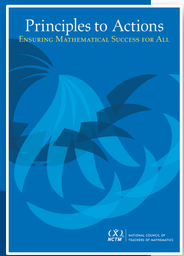

Planning with the Student in Mind

- Anticipate solutions, thoughts, and responses that students might develop as they struggle with the problem
- Generate questions that could be asked to promote student thinking during the lesson, and consider the kinds of guidance that could be given to students who showed one or another types of misconception in their thinking
- Determine how to end the lesson so as to advance students’ understanding

Stigler & Hiebert, 1997

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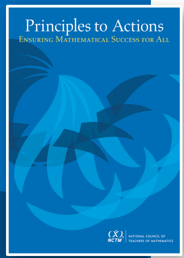

Pose Purposeful Questions

Effective Questions should:

- Reveal students' current understandings;
- Encourage students to explain, elaborate, or clarify their thinking; and
- Make the mathematics more visible and accessible for student examination and discussion.

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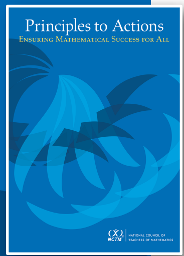
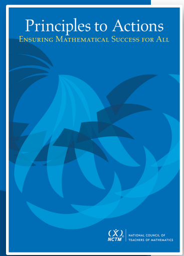


Pose Purposeful Questions

Teachers' questions are crucial *in helping students make connections and learn important mathematics and science concepts. Teachers need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding.*

Weiss & Pasley, 2004



Pose Purposeful Questions

- How did you get that?
- How do you know that?
- Can you explain your idea?
- Why?
- Can you convince us?
- Did anyone get something else?
- Can someone tell me or share with me another way?
- Do you think that means the same things?
- Is there another opinion about this?
- Why did you say that, Justin?

Boaler, J., & Brodie, K. (2004)

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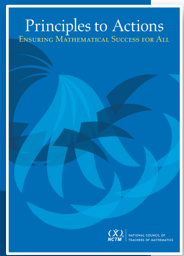

Build Procedural Fluency from Conceptual Understanding

Procedural Fluency should:

- Build on a foundation of conceptual understanding;
- Result in generalized methods for solving problems; and
- Enable students to flexibly choose among methods to solve contextual and mathematical problems.

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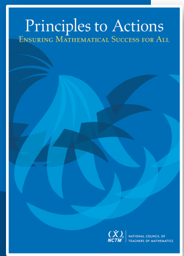

Build Procedural Fluency from Conceptual Understanding

Students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. ***Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.***

Martin, 2009, p. 165

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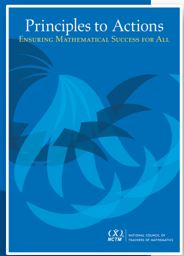

What Research Tells Us

- When procedures are connected with the underlying concepts, students have better retention of the procedures and are more able to apply them in new situations
- Informal methods → general methods → formal algorithms is more effective than rote instruction.
- Engaging students in solving challenging problems is essential to build conceptual understanding.

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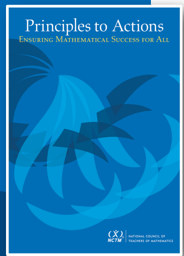
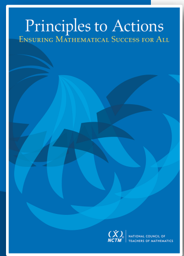


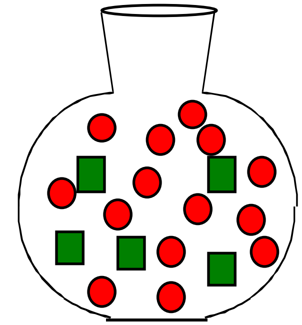
Procedural Fluency

What might we expect the students in Mr. Donnelly's class to be able to do when presented with a missing value problem, after they have had the opportunity to develop a set of strategies through solving a variety of contextual problems like the Candy Jar Task?

$$\frac{5}{13} = \frac{127}{x}$$



Procedural Fluency



$$\frac{5}{13} = \frac{127}{x}$$

$13 \div 5 = 2.6.$
$2.6 \cdot 127 = 330.2$

$\frac{5}{13} = \frac{127}{330.2}$
because
$127 \div = 25.4$
$330.2 \div 13 = 25.4$

$\times 25.4$

JR	5	10	15	20	127
SB	13	26	39	52	330.2

$\times 25.4$

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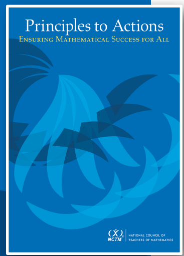

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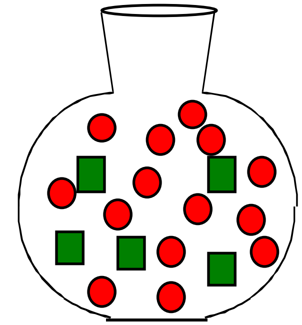


Unit Rate

$$\frac{5}{13} = \frac{127}{x}$$

$13 \div 5 = 2.6.$
$2.6 \cdot 127 = 330.2$

Scale Factor

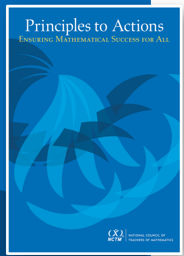
$\frac{5}{13} = \frac{127}{330.2}$
because
$127 \div = 25.4$
$330.2 \div 13 = 25.4$

Scaling Up

×25.4

JR	5	10	15	20	127
JB	13	26	39	52	330.2

×25.4



Building Procedural Fluency from Conceptual Understanding

Finding the Missing Value

Find the value of the unknown in each of the proportions shown below.

$$\frac{5}{2} = \frac{y}{10} \qquad \frac{a}{24} = \frac{7}{8}$$

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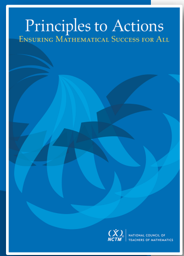
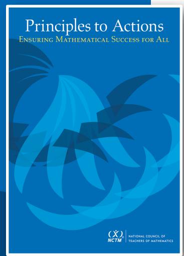


Procedural Fluency

What might we expect the students in Ms. Culver's class to be able to do when presented with a problem like the Petoskey Population, after they have had the opportunity to develop an understanding of what exponential functions do and how they are written?

The population of Petoskey, Michigan, was 6,076 in 1990 and was growing at the rate of 3.7% per year. The city planners want to know what the population will be in the year 2025. Write and evaluate an expression to estimate this population.



Procedural Fluency

- Students would recognize it as an as exponential function in the form y = a(b^x) even if it was in a list of different types of functions
- Students would understand that for each year an additional factor of 1.037 would be multiplied.
- Students would be able to set up the correct function because of their understanding of what exponential functions represent and how they work.
- Students would not interpret b^x as b times x

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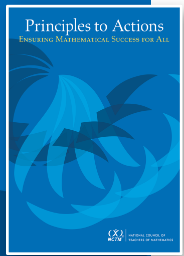

Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

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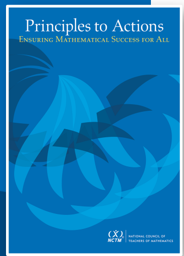

Support Productive Struggle in Learning Mathematics

The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed

Hiebert et al., 1996

By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.

Hiebert & Grouws, 2007, pp. 387-388

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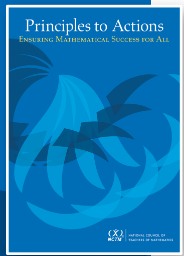

Elicit and Use Evidence of Student Thinking

Evidence should:

- Provide a window into students' thinking;
- Help the teacher determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

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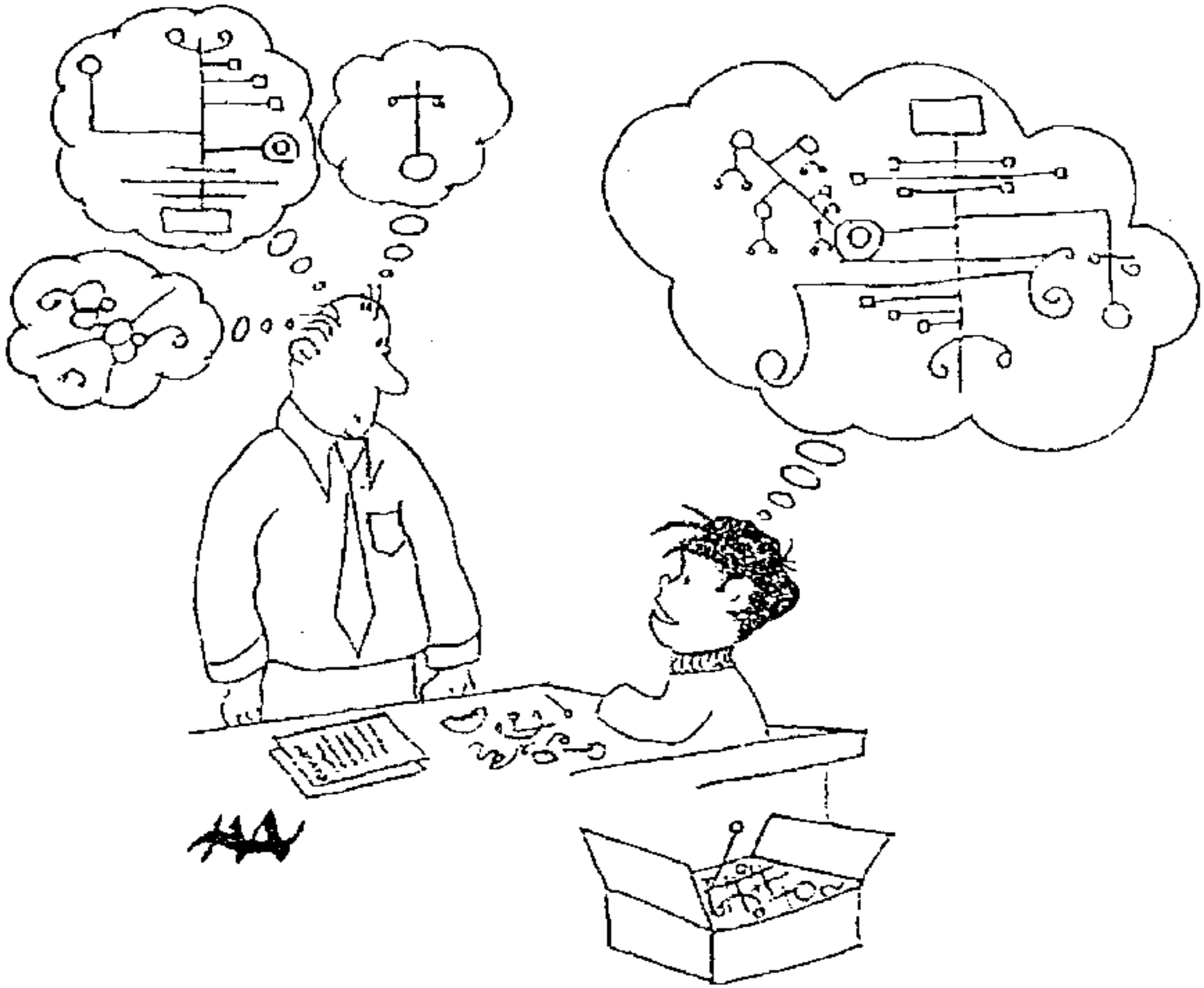
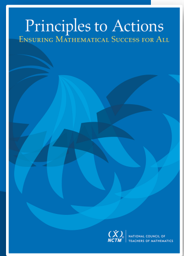
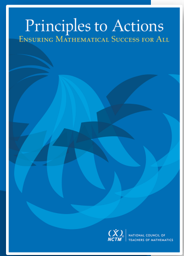


Harold Asturias, 1996



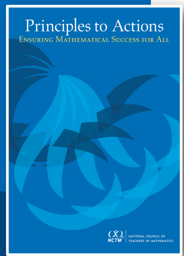
Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

Wiliam, 2007, pp. 1054; 1091



Elicit and Use Evidence of Student Thinking

- Identify specific places during the lesson (cite line numbers) in which Mr. Donnelly or Ms. Culver elicited evidence of student learning.
- Discuss how he or she used or might use that evidence to adjust his/her instruction to support and extend student learning.



Effective
Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

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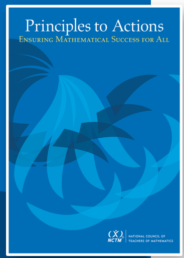
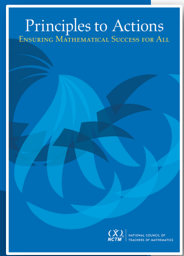


DIGGING DEEPER INTO THE EFFECTIVE TEACHING PRACTICES



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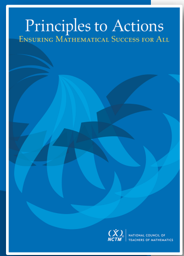

Effective
Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

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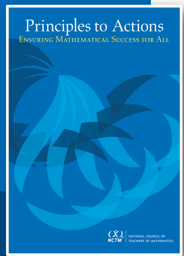
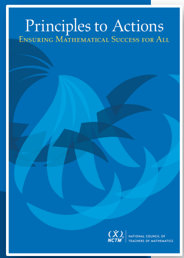


Implement Tasks that Promote Reasoning and Problem Solving

Mathematical tasks should:

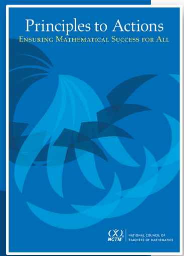
- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students’ current understanding; and
- Have multiple entry points.



The QUASAR Project

- Assisted schools in economically disadvantaged communities to develop instructional programs that emphasize thinking, reasoning and problem solving in mathematics.
- Worked with lowest achieving middle schools in six urban sites.
- Studied the impact of high quality curricula and professional development upon student achievement.

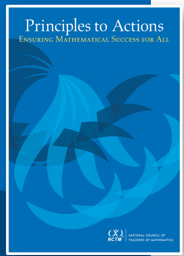
Edward Silver, Margaret S. Smith, Mary Kay Stein



Comparing Two Mathematical Tasks

Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Smith, Stein, Arbaugh, Brown, and Mossgrove, 2004



Comparing Two Mathematical Tasks

Ms. Brown’s class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.

1. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
2. How long would each of the sides of the pen be if they had only 16 feet of fencing?
3. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

Smith, Stein, Arbaugh, Brown, and Mossgrove, 2004

Compare the Two Tasks

- Work each task.
- Share solution strategies.
- Discuss:
  How are Martha's Carpeting Task and the Fencing Task the same and how are they different?

Solution Strategies: Martha's Carpeting Task

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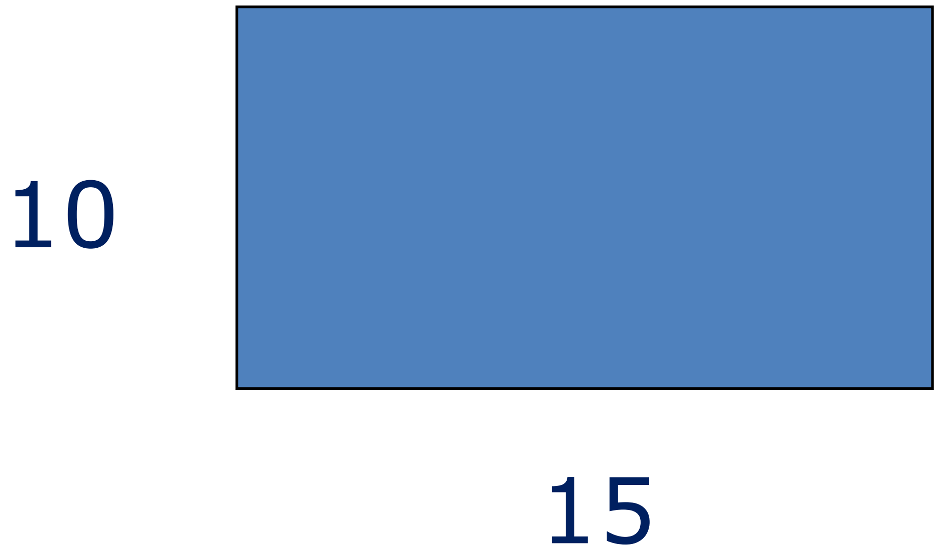
Martha's Carpeting Task Using the Area Formula

A = l x w

A = 15 x 10

A = 150 square feet

Martha's Carpeting Task
Drawing a Picture



Solution Strategies: The Fencing Task

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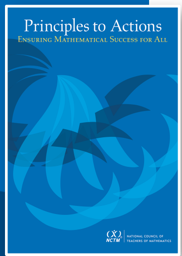
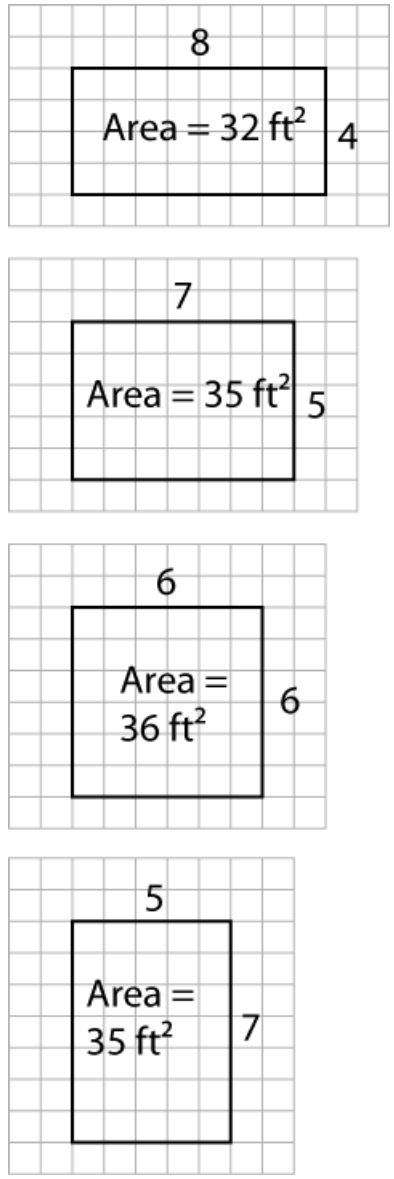


The Fencing Task
Diagrams on Grid Paper



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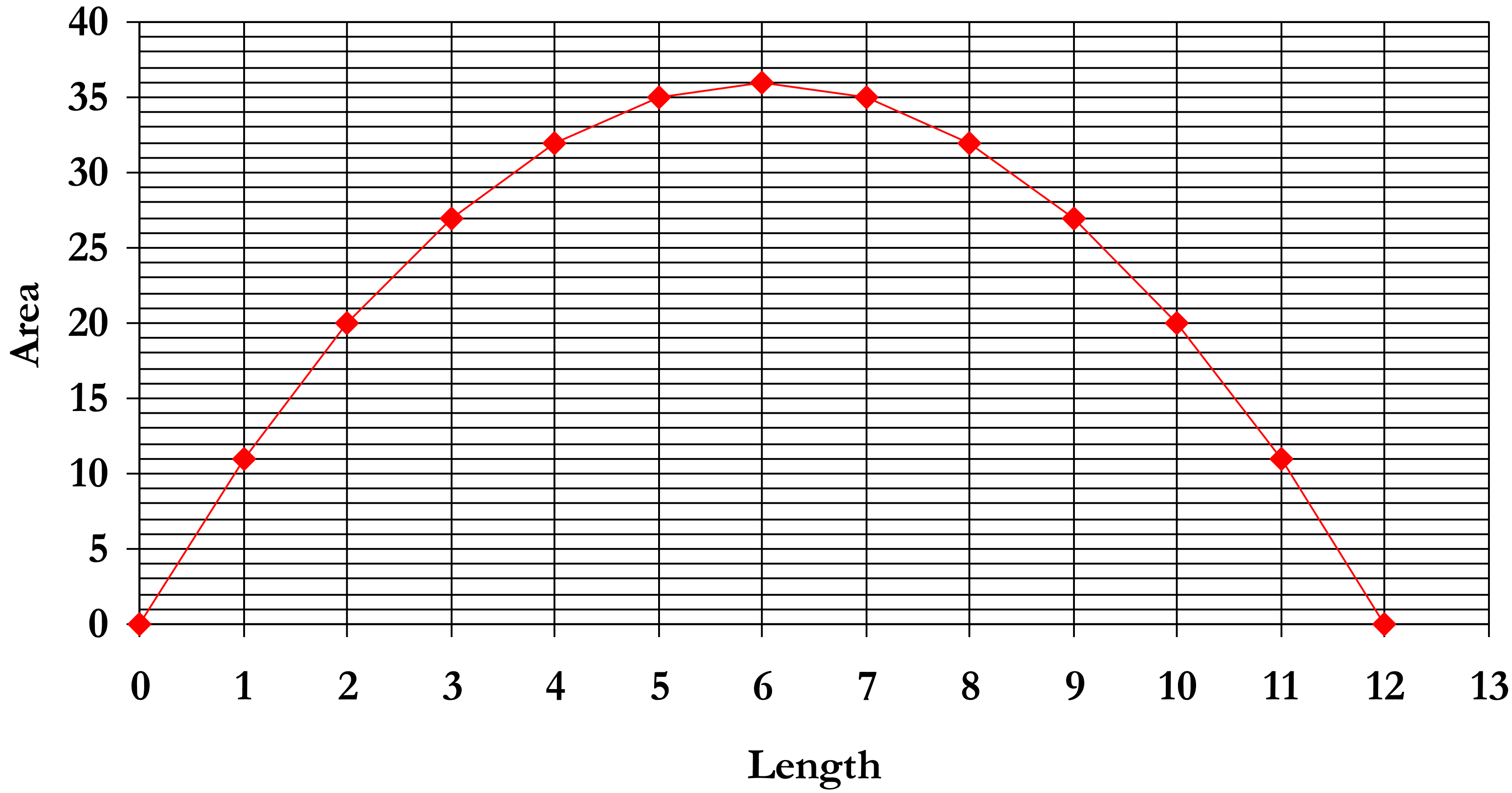
The Fencing Task
Using a Table

Length	Width	Perimeter	Area
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36
7	5	24	35

The Fencing Task
Graph of Length and Area



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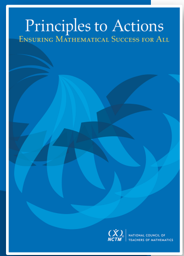


The Fencing Task Using Calculus

$A = lw$

$A = w(12-w)$

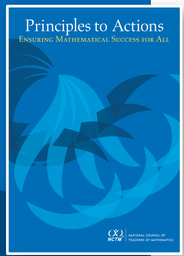
$A = 12w - w^2$

$A' = 12 - 2w$

$0 = 12 - 2w$

$2w = 12$

$w = 6$

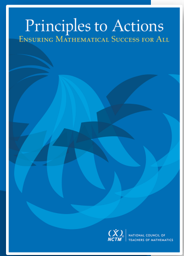


Cognitive Level of Tasks

- Lower-Level Tasks
  (e.g., Martha's Carpeting Task)

- Higher-Level Tasks
  (e.g., The Fencing Task)

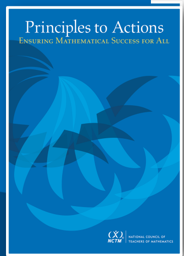
The Quasar Project



Characterizing Tasks

With a Partner:

- Categorize Tasks A – M into two categories: high level cognitive demand and low level cognitive demand.
- Develop a list of criteria that describe the tasks in each category.



Higher-Level Tasks

- **Doing mathematics**
  – Candy Jar task
  – Pay It Forward
- **Procedures with connections**
  – Using a 10 x 10 grid, identify the decimal and percent equivalents of 3/5.
  – Explain how the graph of $y = -3x^2 + 7$ compares to the graph of $y = x^2$. Sketch the graph of $y = -3x^2 + 7$.

Smith & Stein, 1998

Lower-Level Tasks

- **Memorization**
  - What are the decimal equivalents for the fractions ½ and ¼?
- **Procedures without connections**
  - Write the fraction 3/8 as a decimal.
  - A rectangular carpet is 12 feet long and 9 feet wide. What is the area of the carpet in square feet?

Smith & Stein, 1998

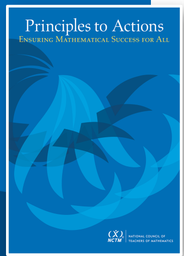
Task Analysis Guide (pp 18-19)

Lower-Level Demands	Higher-Level Demands
<u>Memorization</u> • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	<u>Procedures With Connections</u> • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
<u>Procedures Without Connections</u> • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or` placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focuses solely on describing the procedure that was used.	<u>Doing Mathematics</u> • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Level of Cognitive Demand

Is different than:

- Difficulty
- Importance



Opportunities for *all* students to engage in challenging tasks?

- Examine tasks in your instructional materials:
  - Higher cognitive demand?
  - Lower cognitive demand?
- Where are the challenging tasks?
- Do *all* students have the opportunity to grapple with challenging tasks?
- Examine the tasks in your assessments:
  - Higher cognitive demand?
  - Lower cognitive demand?

Graphing Dice Throws

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

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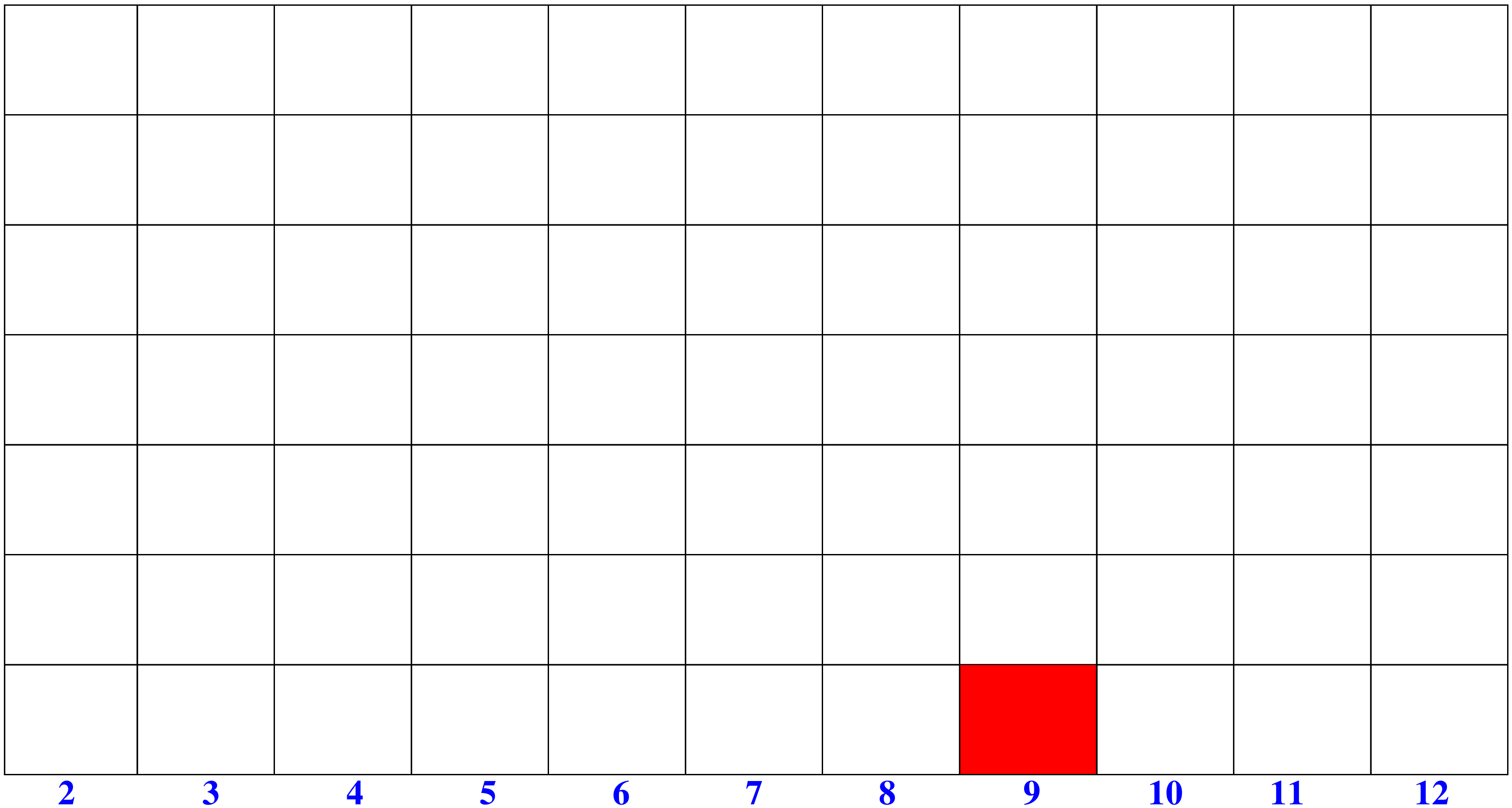
Graphing Dice Throws



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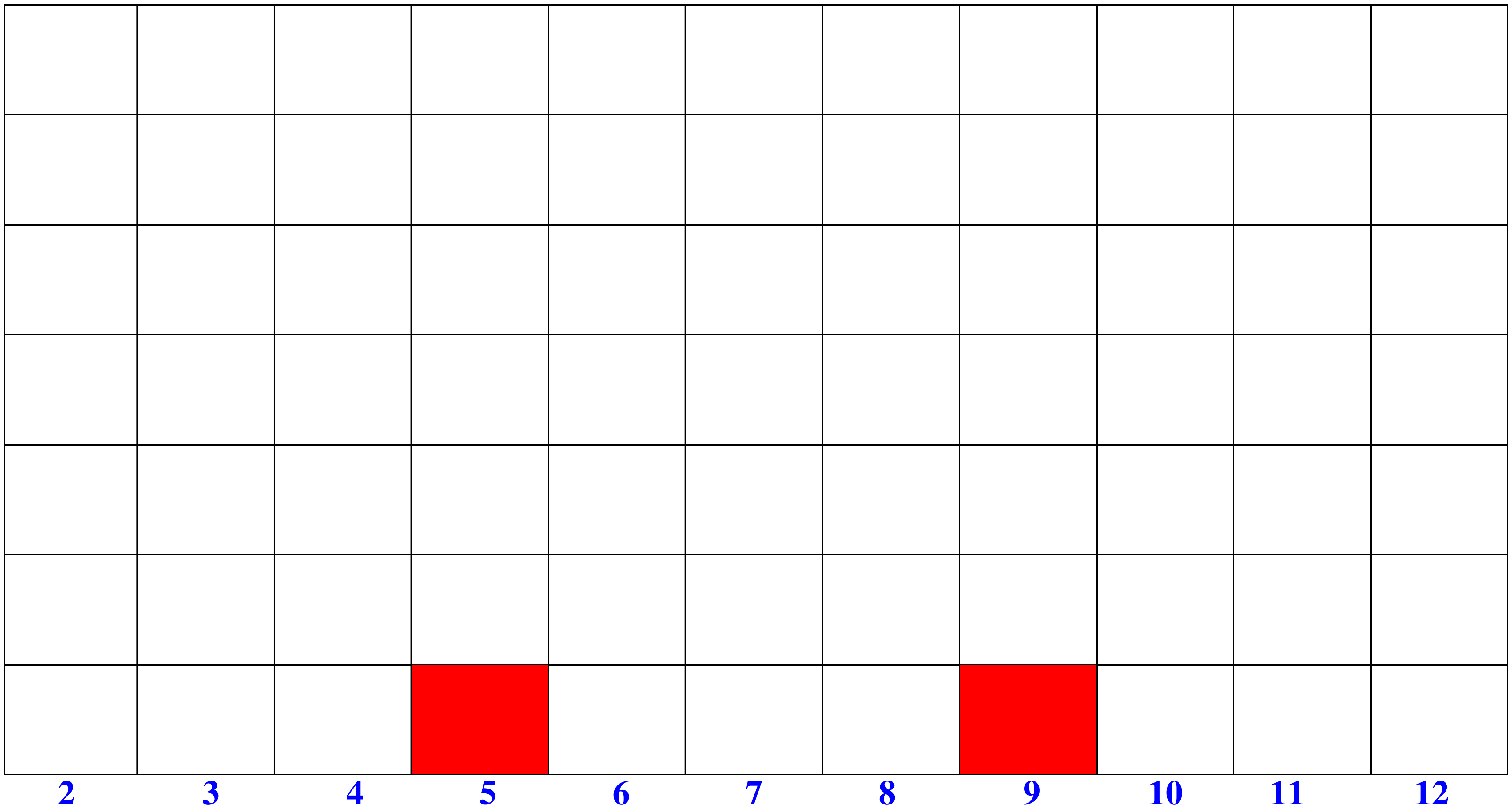
Graphing Dice Throws



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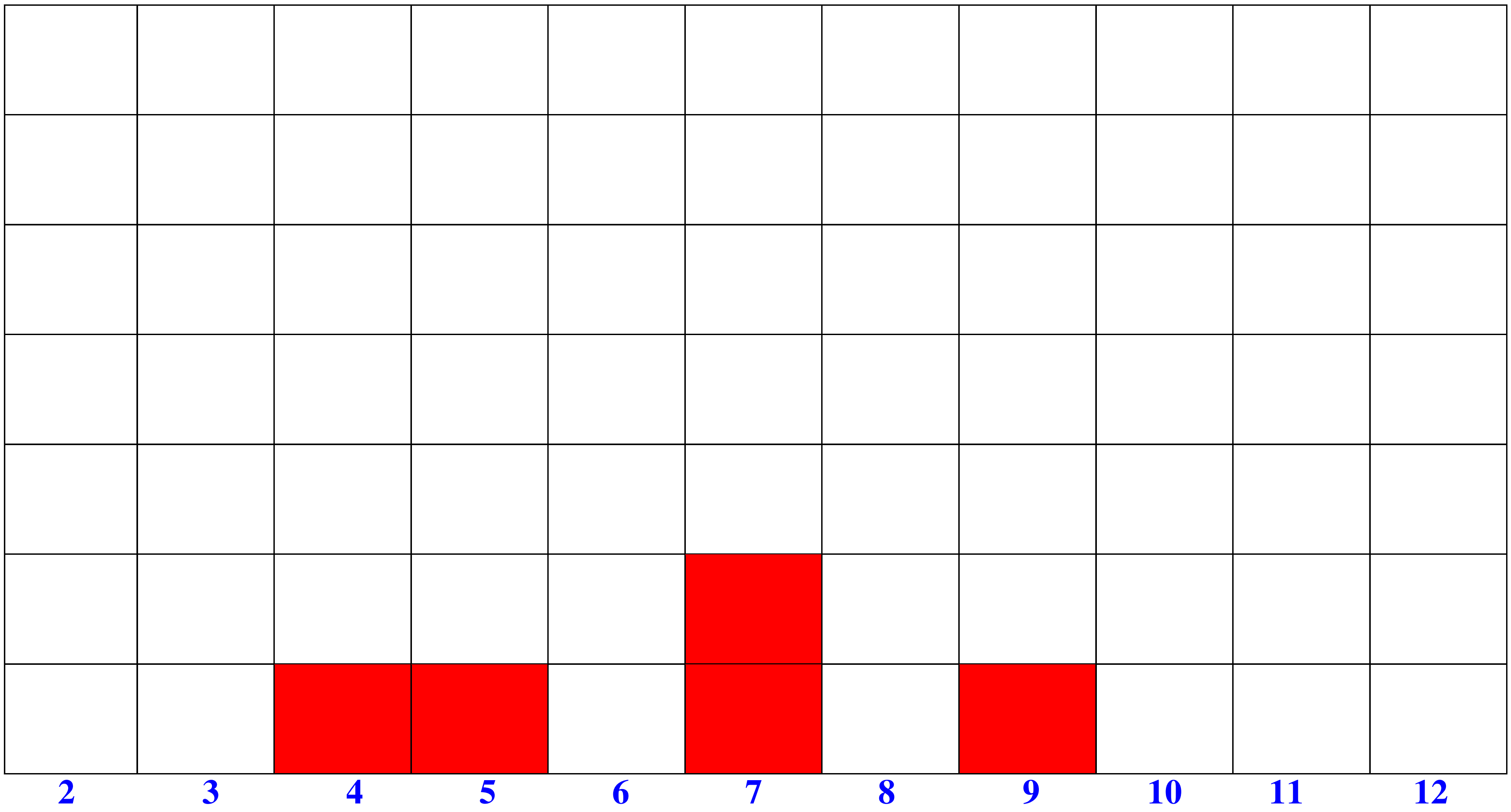

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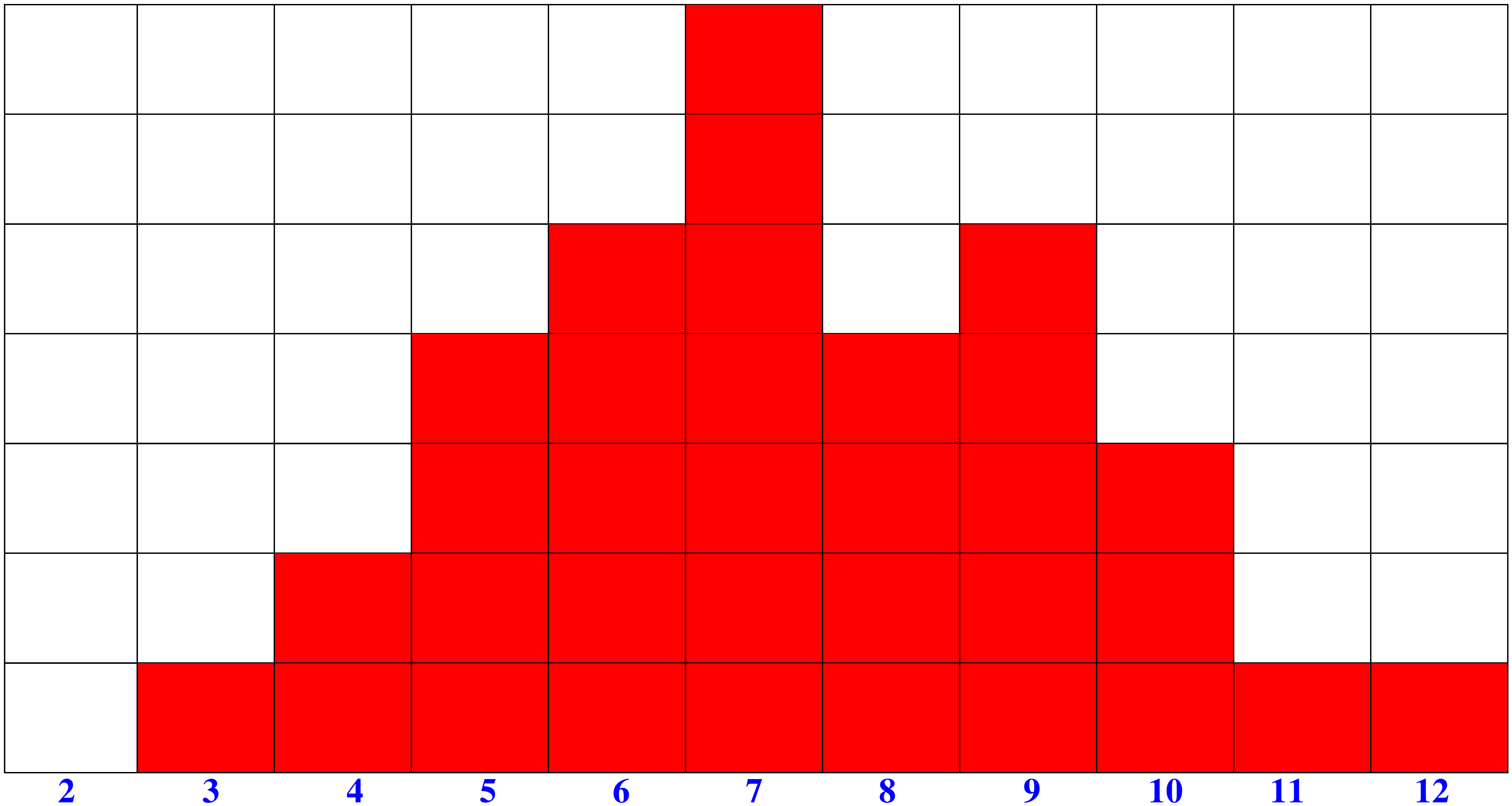

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Graphing Dice Throws

"My children are "low level",

so I do the activity using 1 die."

Graphing Dice Throws

1	2	3	4	5	6

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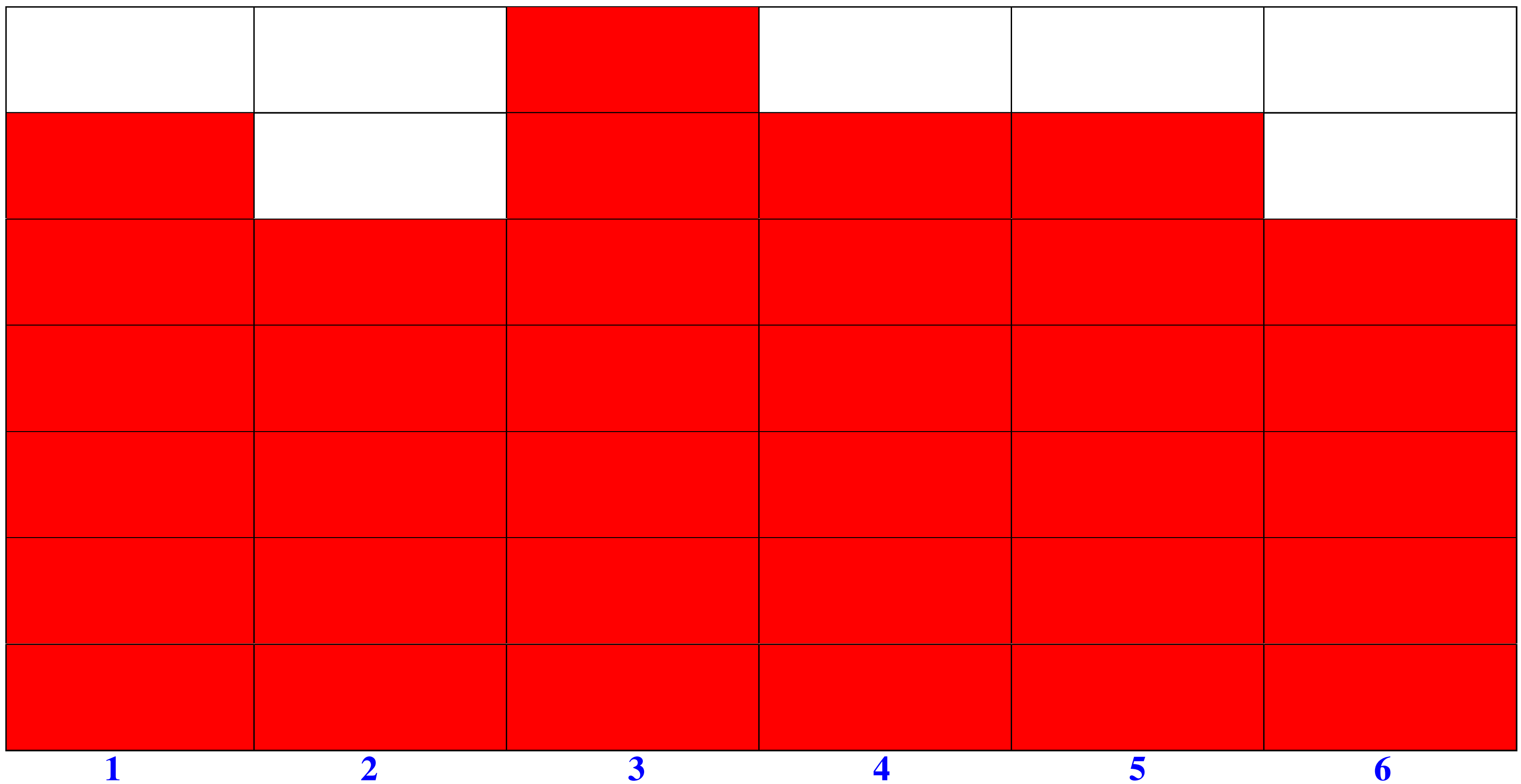
Graphing Dice Throws



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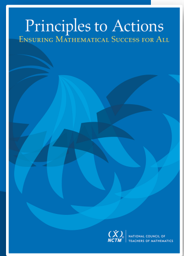


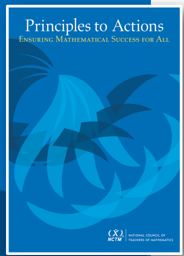
Task Implementation

- Read the *Case of Sandra Pascal* or the *Case of Steven Taylor*. Compare it to the *Case of Mr. Donnelly* or *Case of Ms. Culver.*

Consider:

- How are the implementations in the two classrooms the same and how are they different?
- Which of the effective teaching practices did each of the two teachers engage in?
- In what ways did the use of the effective teaching practices support student learning?

Be sure to use evidence from the case to support your claims.



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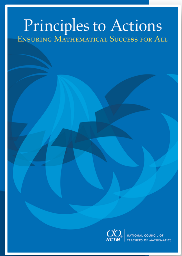

Implement Tasks that Promote Reasoning and Problem Solving

Although selecting tasks that promote reasoning and problem solving is a critical first step, giving the task to students does not guarantee that students will actually engage in the task at a high level.

NCTM, 2014, p.22

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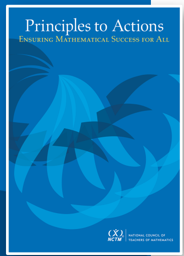
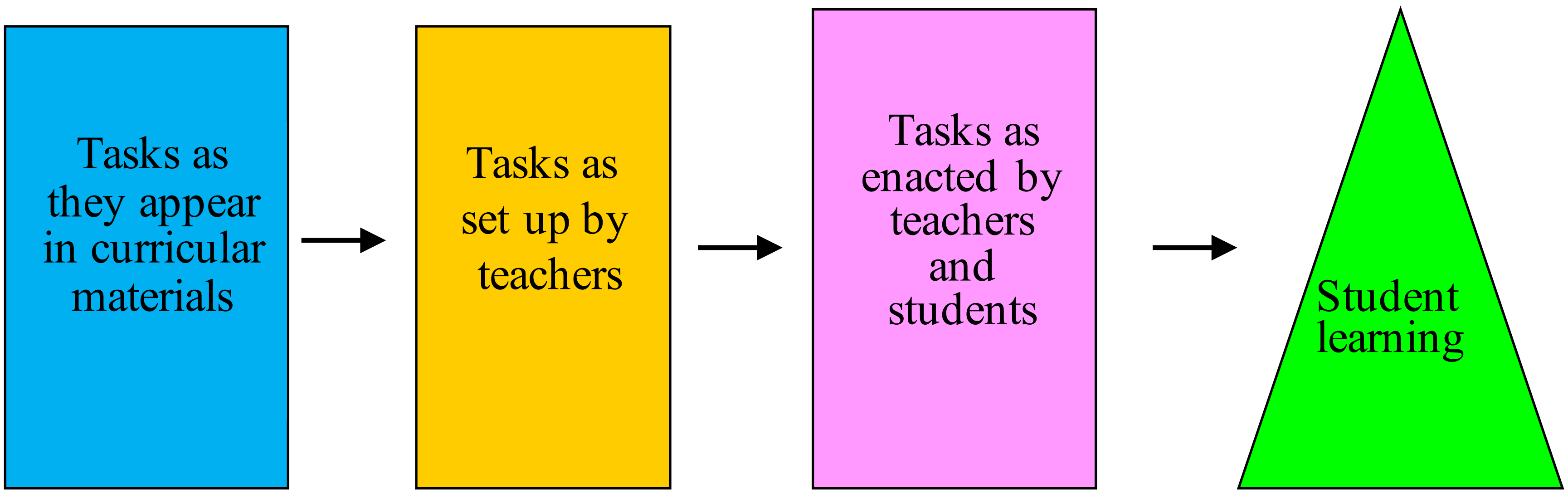


The Mathematical Tasks Framework



Stein, Grover & Henningsen (1996)
Smith & Stein (1998)
Stein, Smith, Henningsen & Silver (2000)

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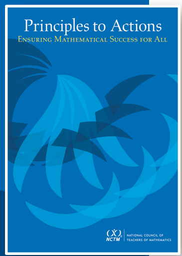
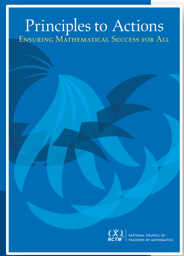


Mathematical Tasks and Student Learning

Students who performed the best on project-based measures of reasoning and problem solving were in classrooms in which tasks were more likely to be set up <u>and</u> enacted at high levels of cognitive demand.

Stein & Lane, 1996; Stein, Lane, & Silver, 1996



NSF Local System Change Project Evaluation Study

While teachers were using the materials more extensively in their classrooms, there was a wide variation in how well they were implementing these materials. Teachers were often content to omit rich activities, skip over steps and jump to higher level concepts, or leave little time for students to 'make sense' of the lessons.

Weiss, et al, 2006

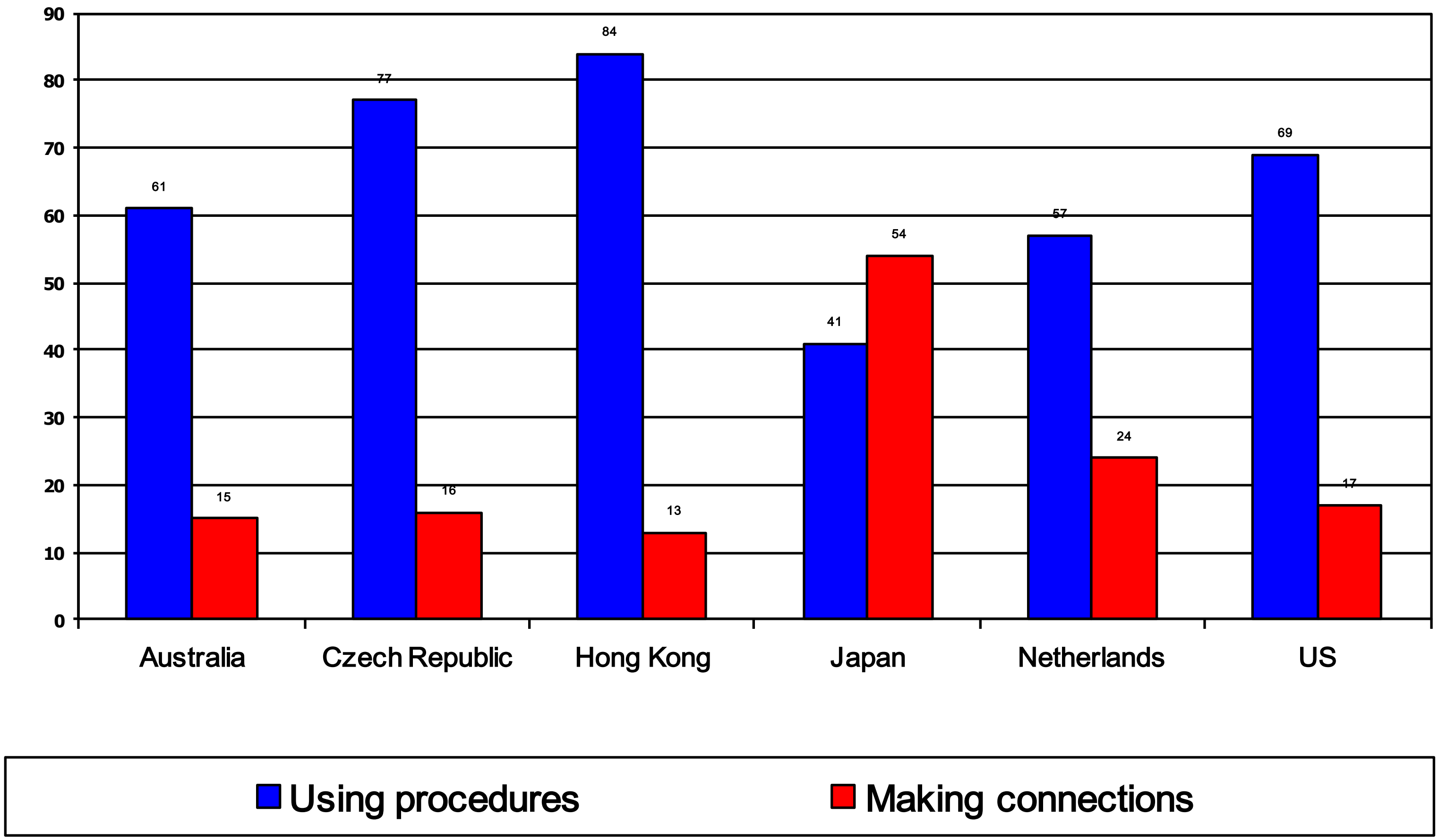
NSF Local System Change Project Evaluation Study

In fact, classroom observations indicated that the lessons taught as the developers intended were more likely to provide students with learning opportunities than those that were "adapted."

Weiss, et al, 2006

Types of Math Problems Presented

1999 TIMSS Video Study



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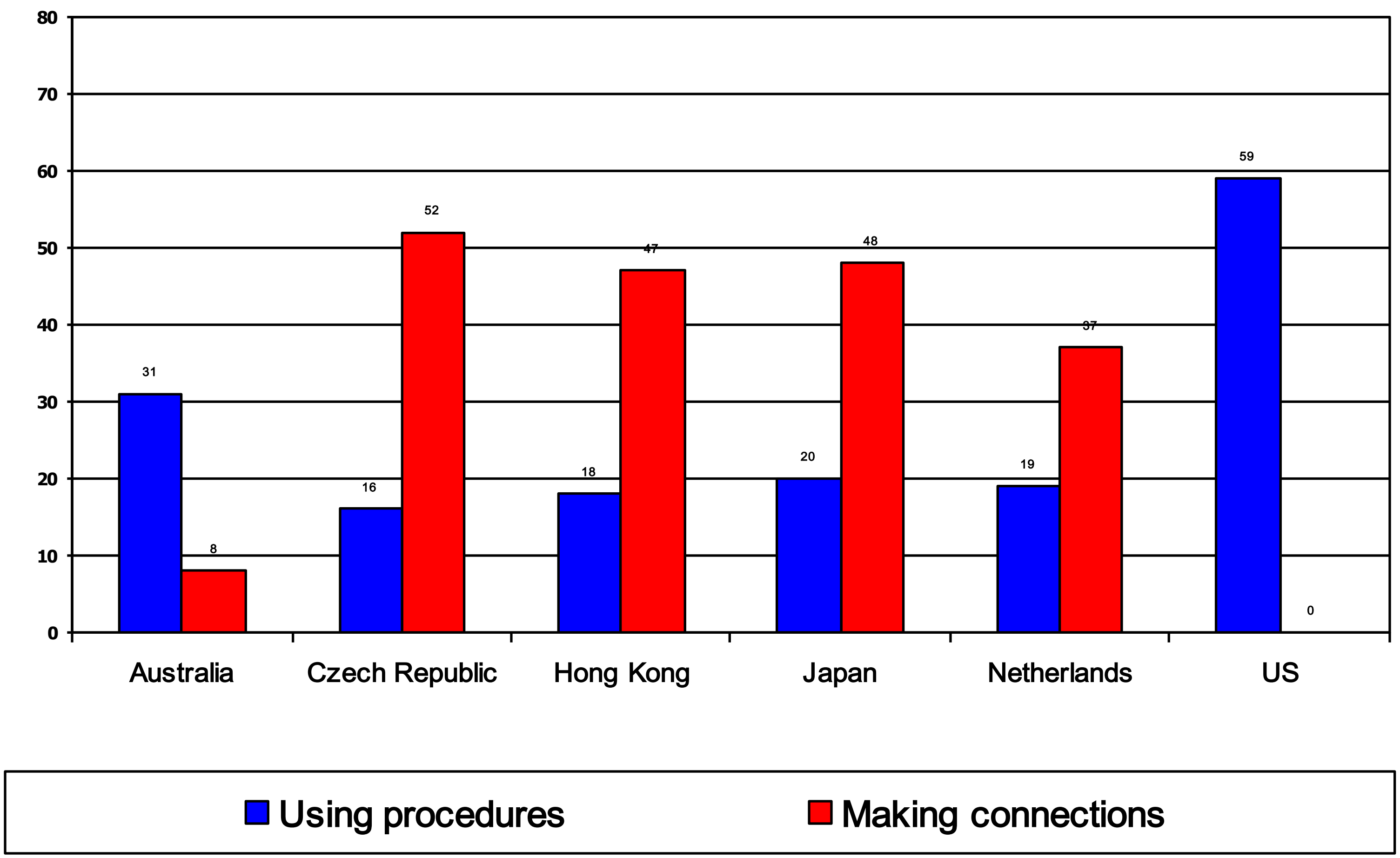
How Teachers Implemented *Making Connections* Math Problems



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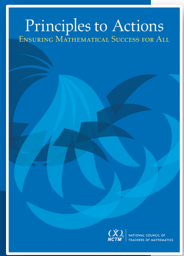
Sharing Brownies

Four friends want to share 7 brownies equally.

How many brownies, or what fractional part of a brownie, will each person get?



Sharing Brownies

1. Individually do the task.
2. Then compare your work with others.
3. Consider each of the following questions and be prepared to share your thinking with the group:
   a) What mathematics *content* does the task involve?
   b) Which mathematical *practices* are needed to complete the task?
   c) What is the *level of demand* of the task?

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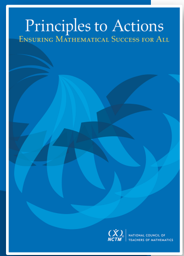

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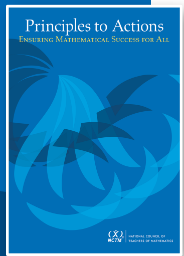
About the Lesson

- Grade 4 classroom.
- The teacher, Ms. Katherine Casey, has been working several years on standards-based mathematics instruction.
- Ms. Lucy West, a mathematics coach, is working with Ms. Casey during this lesson.



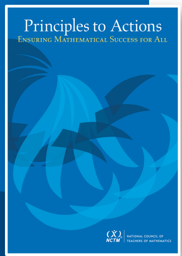
Norms for Watching Video

- Video clips are examples, not exemplars.
  - To spur discussion not criticism
- Video clips are for investigation of teaching and learning, not evaluation of the teacher.
  - To spur inquiry not judgment
- Video clips are snapshots of teaching, not an entire lesson.
  - To focus attention on a particular moment not what came before or after
- Video clips are for examination of a particular interaction.
  - Cite specific examples (evidence) from the video clip, transcript and/or lesson graph.

A Look into a Classroom…

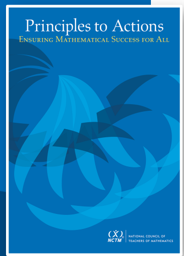
Look for:

- What happened to the cognitive demand when the task was implemented?
- What specific teacher actions affected the cognitive demand and supported/hindered students' engagement reasoning and problem solving?



Sharing Brownies

..\..\..\Brownie lesson\M_E_2407_Brownie_Problem_NCTM.mp4



A Look into a Classroom…

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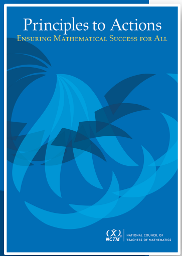
Teacher Actions that Affect Cognitive Demand

- Task set-up
- Supporting students’ exploration of the task
- Orchestrating debriefing discussion



Implement tasks that promote reasoning and problem solving
Teacher and student actions

What are teachers doing?	What are students doing?
Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Selecting tasks that provide multiple entry points through the use of varied tools and representations. Posing tasks on a regular basis that require a high level of cognitive demand. Supporting students in exploring tasks without taking over student thinking. Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	Persevering in exploring and reasoning through tasks. Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Using tools and representations as needed to support their thinking and problem solving. Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Principles to Actions, p. 24

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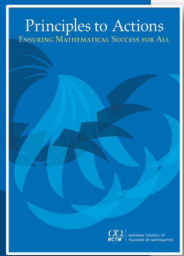

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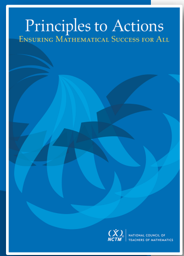

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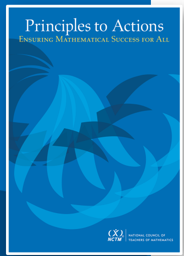


Effective Mathematics Teaching Practices

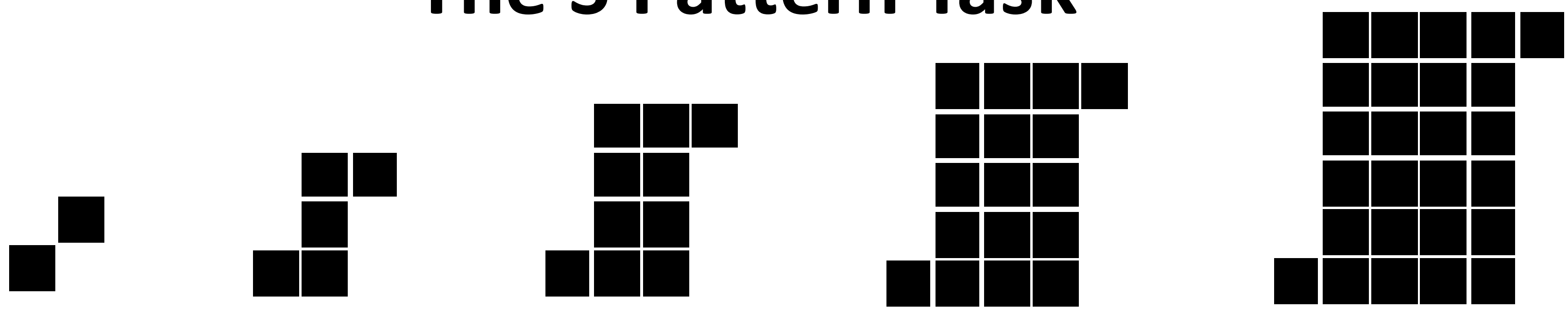
THE CASE OF JEFF ZIEGLER

HIGH SCHOOL

Developed by Margaret Smith and Victoria Bill at the University of Pittsburgh. Video courtesy of Pittsburgh Public Schools and the Institute for Learning.

INSTITUTE for LEARNING

The S Pattern Task[1]



1 2 3 4 5

1. What patterns do you notice in the set of figures?
2. How many square tiles are in figure 7? Write a description that could be used to determine the shape of and total number of square tiles in figure 7. Your description should be clear enough so that another person could read it and use it to think about another figure.
3. Determine an equation for the total number of squares in any figure. Explain your rule and show how it relates to the visual diagram of the figures.
4. Find a second way to describe the pattern and write the equation that matches the description. Compare the two equations and show in the visual representation how one equation is equivalent to the other.
5. If you knew that a figure had 9802 squares tiles in it, how could you determine the figure number? Explain.
6. Does the pattern describe a linear relationship between the figure number and the total number of squares? Why or why not?

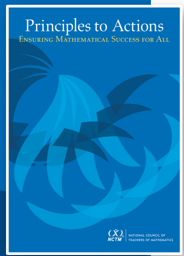
[1] This task was adapted from *Visual Mathematics Course II, Lessons 1-10* published by The Math Learning Center, Salem, OR.

The S-Pattern Task
Video Context

School: Langley High School, Pittsburgh Public Schools
Teacher: Mr. Jeffrey Ziegler
Principal: Linda Baehr
Class: 11th-12th Grade Students

At the time the video was filmed, Jeffery Ziegler was a coach at Langley High School in the Pittsburgh Public School District. The students are 11th and 12th grade and struggle with mathematics.

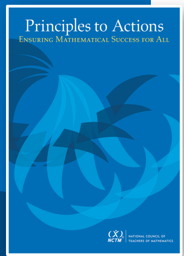
(Jeffrey Ziegler is currently a curriculum supervisor for grades 6-12 mathematics in the Pittsburgh Public School district.)



Mr. Ziegler's *Mathematics* Learning Goals

Students will understand that:

1. An equation can be written that describes the relationship between 2 quantities;
2. Different but equivalent equations can be written that represent the same situation; and
3. The symbolic and pictorial representations can be connected.



Connections to the CCSS Content Standards

Creating Equations★ A-CED

Create equations that describe numbers or relationships.

2. Create equations in two or more variables to represent relationships between quantities~~; graph equations on coordinate axes with labels and scales.~~

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.

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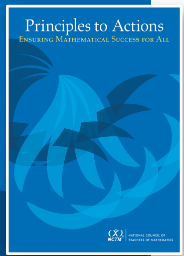
The S-Pattern Task
The Context of Video Clips

The lesson begins with Mr. Ziegler engaging students in a brief discussion of the task. They establish the fact that this was a growth pattern that is growing in two dimensions, getting both "taller" and "bigger". Before they begin their work, Mr. Ziegler tells students:

> "Now there are 6 prompts… Kind of the first one, the second one, third one is to kind of get you started but it is on you guys to work with your groups to come up with a way to find the patterns. You don't necessarily have to word-for-word answer these questions, but they're there to help you maybe get started.

The clip begins as small groups begin to work on the task and Mr. Ziegler visits Groups 1 and 2.



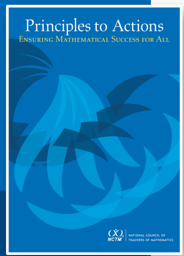
Lens for Watching the Video Clip 1

As you watch the video, make note of what the teacher does as he interacts with groups 1 and 2.

In particular, identify any of the *Effective Mathematics Teaching Practices* that you notice Mr. Ziegler using.

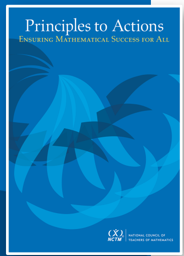
Be prepared to give examples and to cite line numbers from the transcript to support your claims.

Also, record any questions that you have (wonderings)



Video Clip 1

http://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Jeffrey-Ziegler-and-the-S-Pattern-Task/



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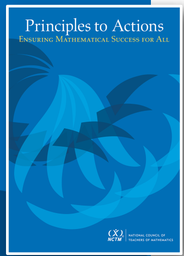
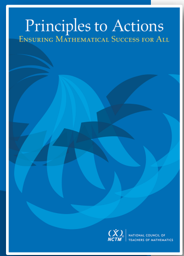


Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

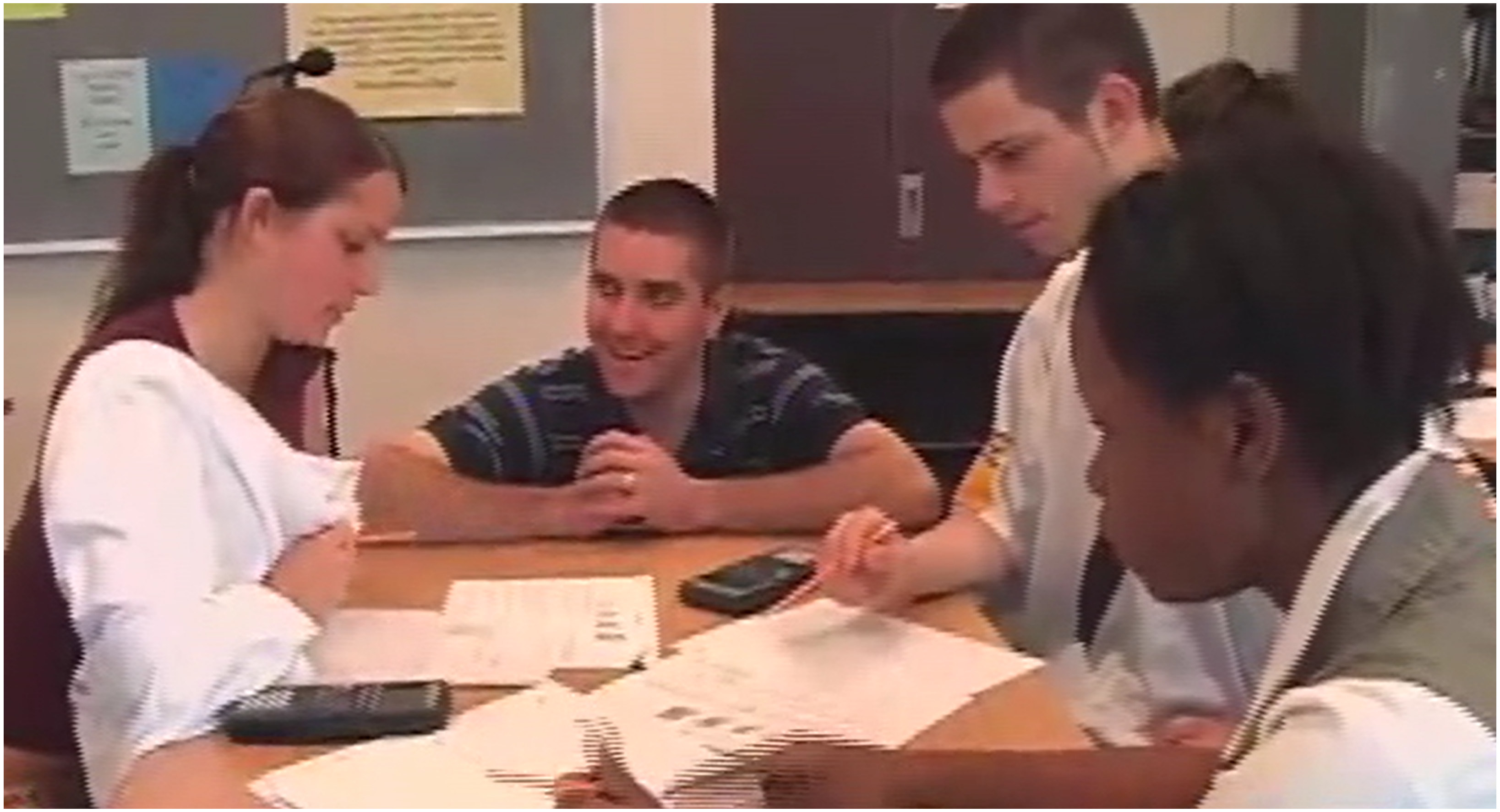


Lens for Watching the Video Clip 2

In the second video clip Mr. Ziegler visits Groups 1 and 2 for a second time.

Considering the teacher's actions and interactions with Groups 1 and 2 in both clips, identify what the teacher does to support his student's productive struggle.

Video Clip 2



http://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit
The-Case-of-Jeffrey-Ziegler-and-the-S-Pattern-Task/

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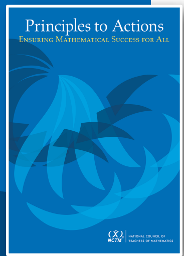

Effective
Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. ***Support productive struggle in learning mathematics.***
8. **Elicit and use evidence** of student thinking.

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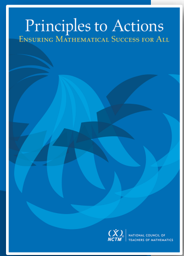

Support Productive Struggle In Learning Mathematics: Teacher and Student Actions

What are teachers doing?

- Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.
- Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.
- Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.
- Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.

What are students doing?

- Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
- Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
- Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
- Helping one another without telling their classmates what the answer is or how to solve the problem.

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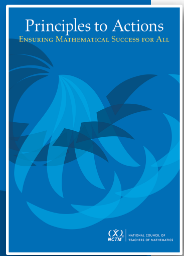

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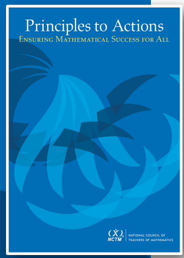

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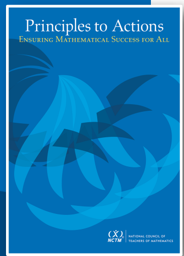
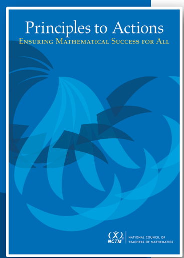


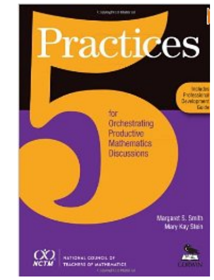
Facilitate Meaningful Mathematical Discourse

Mathematical Discourse should:

- Build on and honor students' thinking;
- Provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and
- Advance the mathematical learning of the whole class.



Five Practices for Orchestrating Productive Discussions



- ***Anticipating*** likely student responses
- ***Monitoring*** students' actual responses
- ***Selecting*** particular students to present their mathematical work during the whole class discussion
- ***Sequencing*** the student responses
- ***Connecting*** different students' responses—to each other and to key mathematical ideas.

Smith & Stein, 2011

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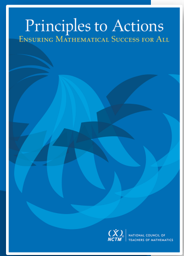
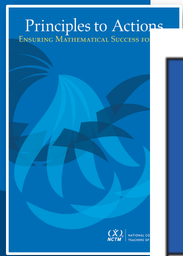


S-Pattern Task

Analyze the student solutions.

- Which students would you have present their solutions?
- In what order would you have them present?
- Why?
- What questions would you want to be sure to ask?



Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

Principles to Actions, p. 35

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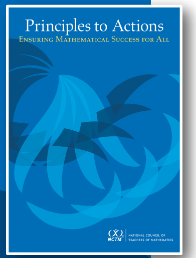

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Your Feelings Looking Ahead?

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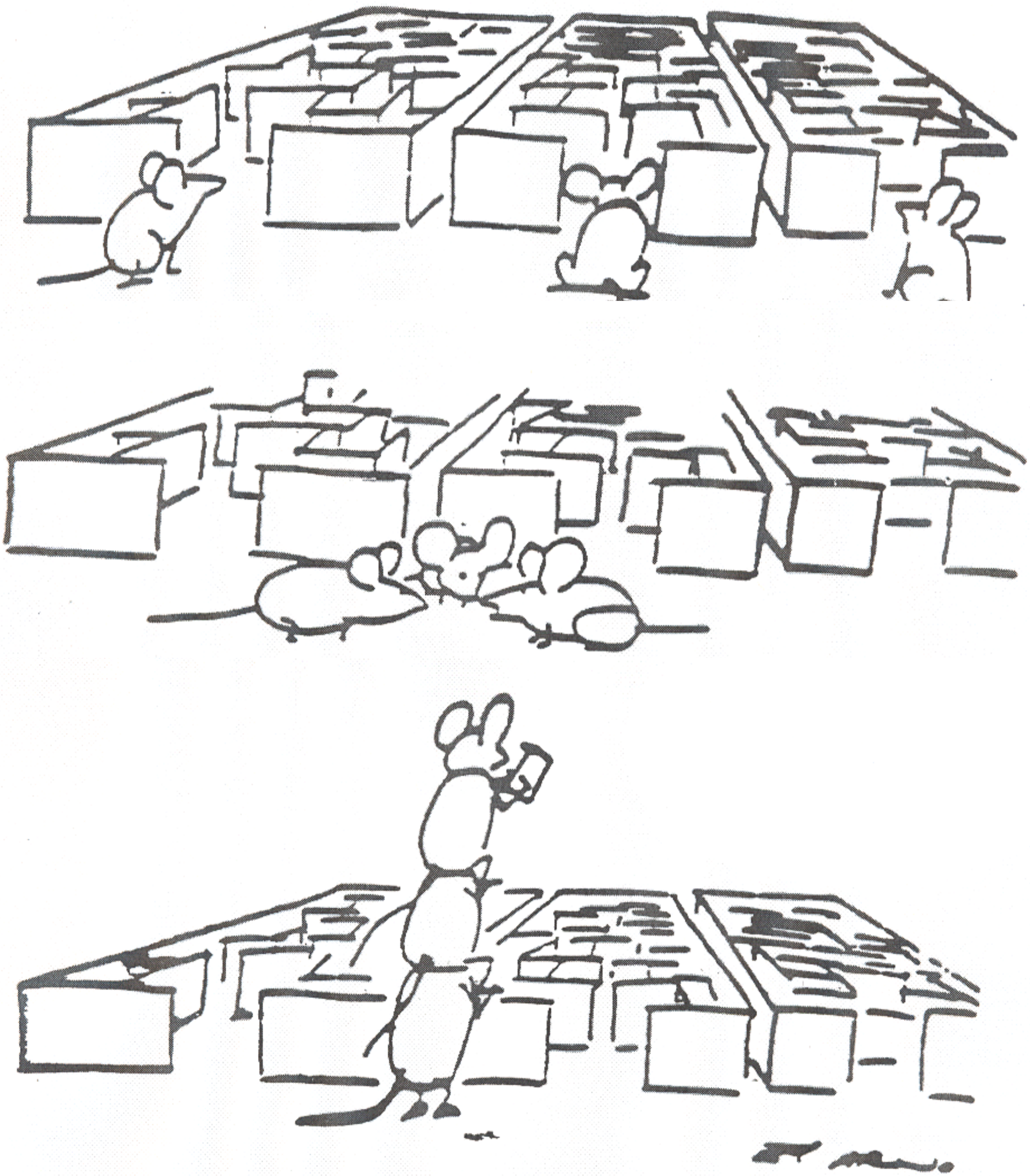
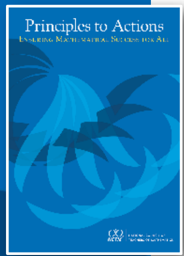


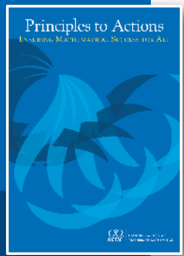


The Title is Principles to *Actions*

The Teaching and Learning Principle

Teacher Actions:

- Consistently implement the eight Mathematics Teaching Practices.
- Elicit, value, and celebrate varied approaches and solution paths that students take to solve mathematics problems, explain their thinking, and critique the arguments of others.



The Title is Principles to *Actions*

The Teaching and Learning Principle

Teacher Actions:

- Give priority to the mathematical practices, including problem solving, reasoning, and constructing viable arguments in every aspect of classroom practice—including teaching, assessment, curriculum decisions, and the use of tools and technology.
- Plan and implement units and lessons that promote positive dispositions toward the study of mathematics, including curiosity, self-confidence, flexibility, and perseverance.

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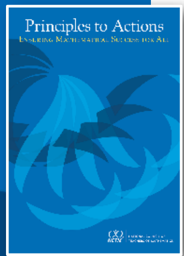
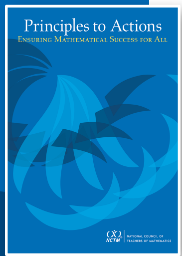


The Title is Principles to *Actions*

Principals, Coaches, Specialists, and Other School Leaders

- Make the eight Mathematics Teaching Practices a schoolwide focus that is expected for all teachers to
- Strengthen learning and teaching for all students, and provide professional development, training, and coaching to make the implementation of these practices a priority;
- Maintain a schoolwide culture with high expectations and a growth mindset; allocate time for teachers to collaborate in professional learning communities;
- Support improvement with multifaceted assessments used to monitor progress and inform changes to instruction;
- Make the mathematical success of every student a nonnegotiable priority.



http://www.nctm.org/PtA/

Classroom Resources | Publications | Standards & Positions | Research & Advocacy | Conferences & Professiona

Principles to Actions

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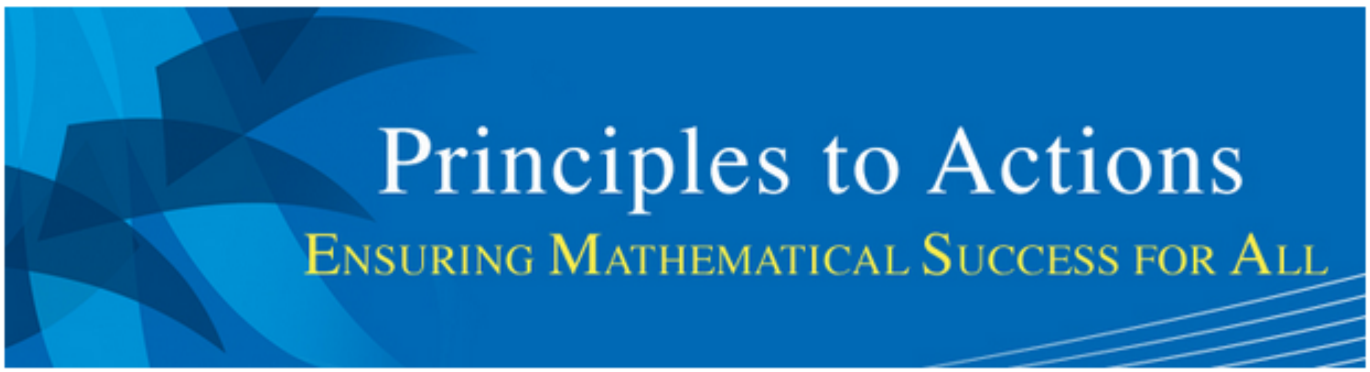
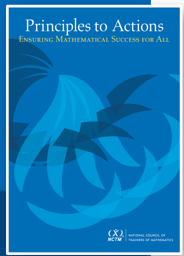


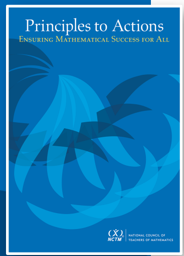
Specific, research-based teaching practices that are essential for a high-quality mathematics education for all students are combined with core principles to build a successful mathematics program at all levels.

Principles to Actions offers guidance to teachers, mathematics coaches, administrators, parents, and policymakers.



Principles to Actions Resources

- *Principles to Actions* Executive Summary (in English and Spanish)
- *Principles to Actions* overview presentation
- *Principles to Actions* professional development guide (Reflection Guide)
- Mathematics Teaching Practices presentations
  - Elementary case, multiplication (Mr. Harris)
  - Middle school case, proportional reasoning (Mr. Donnelly) (in English and Spanish)
  - High school case, exponential functions (Ms. Culver)
- *Principles to Actions* Spanish translation



http://www.nctm.org/PtAToolkit/

Principles to Actions Professional Learning Toolkit

13

These grade-band specific professional learning modules are focused on the Effective Teaching Practices and Guiding Principles from *Principles to Actions: Ensuring Mathematical Success for All*.

Presentation, presenter notes, and required materials are provided in each module to support professional learning with teachers through analyzing artifacts of teaching (e.g., mathematical tasks, narrative and video cases, student work samples, vignettes) and abstracting from the specific examples general ideas about how to effectively support student learning.

The Teaching and Learning Modules were developed in collaboration with the Institute for Learning (IFL) at the University of Pittsburgh.

Learning modules are available exclusively to NCTM members. Limited open examples are provided for each grade level (denoted with *).

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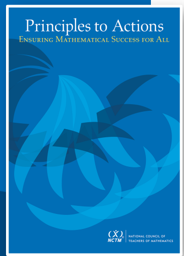


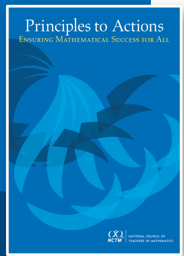
http://www.nctm.org/PtAToolkit/

Effective Teaching Practices

To access module materials, use links on left column of table below.

Primary emphasis | Secondary emphasis

	Establish math goals to focus learning	Implement tasks that promote reasoning and problem solving	Use and connect math representa-tions	Facilitate meaningful math discourse	Pose purposeful Questions	Build procedural fluency from conceptual under-standing	Support productive struggle in learning math	Elicit and use evidence of student thinking
Middle School								
Candy Jar*	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary
Calling Plans 2	Secondary	Secondary			Primary			
Hexagon	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary	Primary	Secondary
Counting Cubes*	Secondary	Secondary	Primary	Primary				
Two Tanks	Secondary	Secondary	Primary		Primary			Primary
Triangle	Secondary	Primary				Primary		
High School								
Pay It Forward*	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary	Secondary
S-Pattern*	Secondary	Secondary	Secondary		Secondary		Primary	Secondary
Playground	Secondary	Secondary			Primary		Primary	
Bike and Truck	Secondary	Secondary		Primary	Primary			
Calling Plans 1	Secondary	Secondary	Primary		Primary			
Sine Waves	Secondary	Secondary			Primary		Primary	



Teaching and Learning Mathematics Videos

http://www.nctm.org/Standards-and-Positions/Common-Core-State-Standards/Teaching-and-Learning-Mathematics-with-the-Common-Core/

- Building Conceptual Understanding for Mathematics
- Mathematics in the Early Grades
- Developing Mathematical Skills in Upper Elementary Grades
- Mathematical Foundations for Success in Algebra
- Preparation for Higher Level Mathematics
- Standards for Mathematical Practice
- Parents Supporting Mathematics Learning

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The Title Is Principles to *Actions*

Your Actions?

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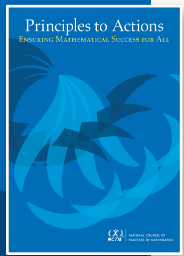

Thank You!

Diane Briars

dbriars@nctm.org

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