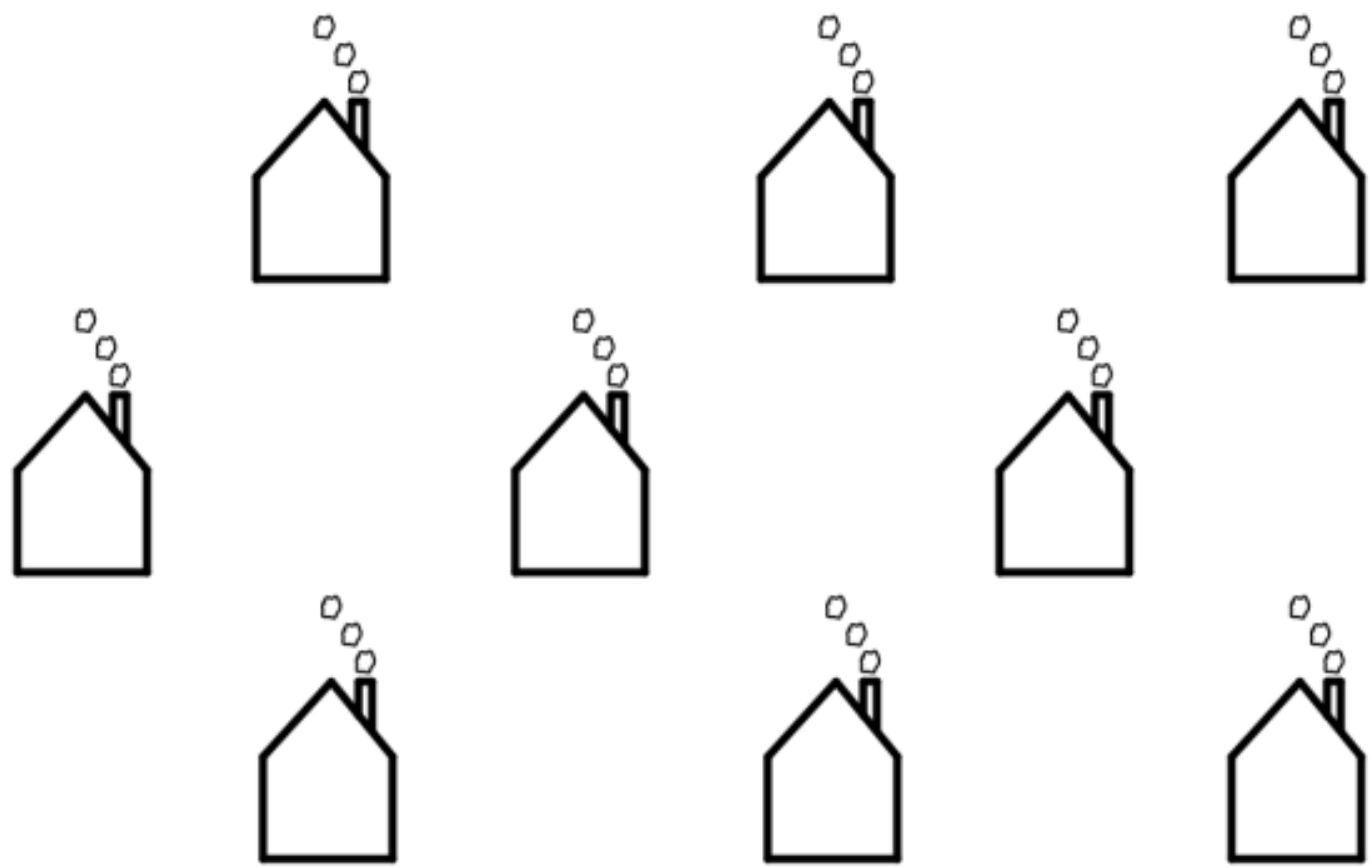
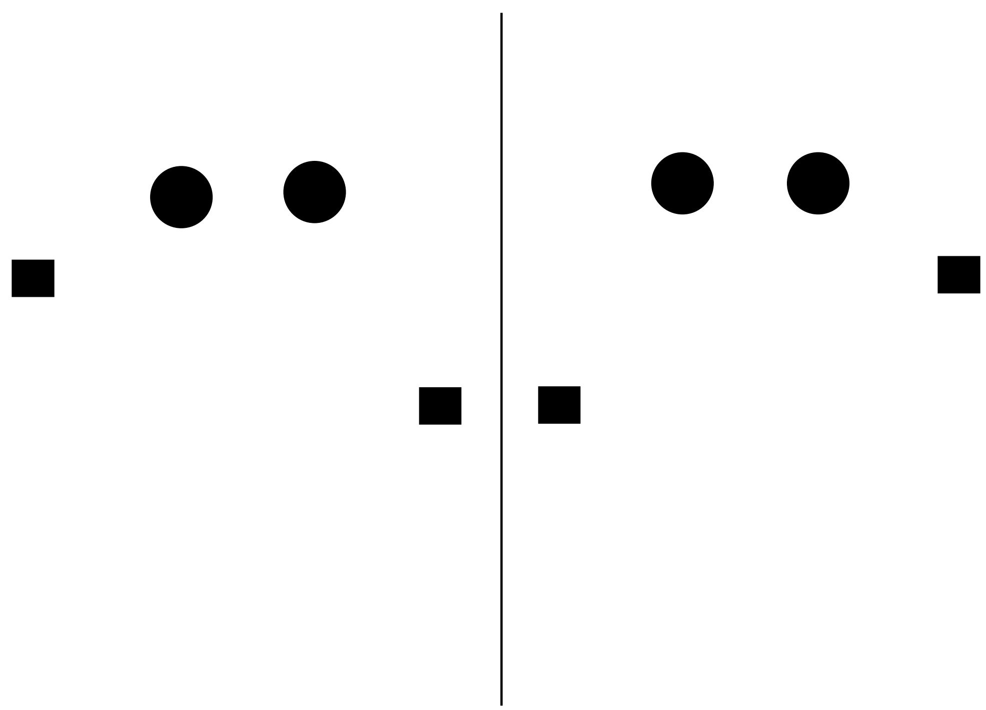
Mathematics of Dancing

What is Symmetry?

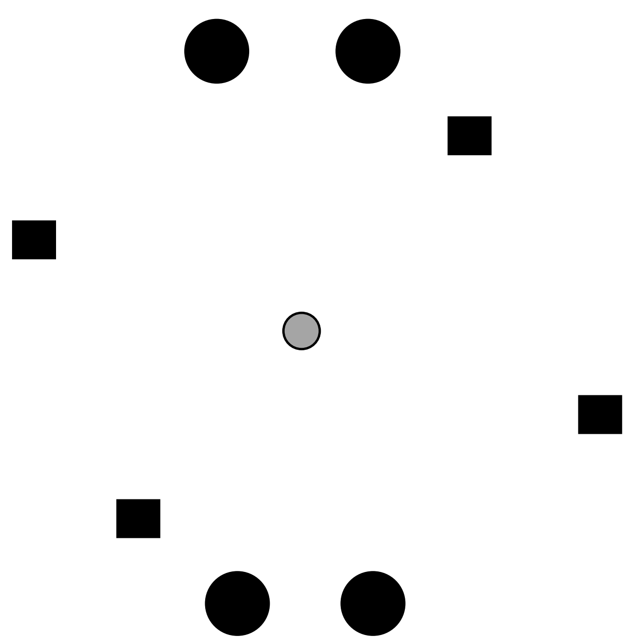
Types of Symmetries:
Translational



Types of Symmetries:
Reflectional



Types of Symmetries:
Rotational



Types of Symmetries: *Glide Reflectional*

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Line Dancing Symmetries

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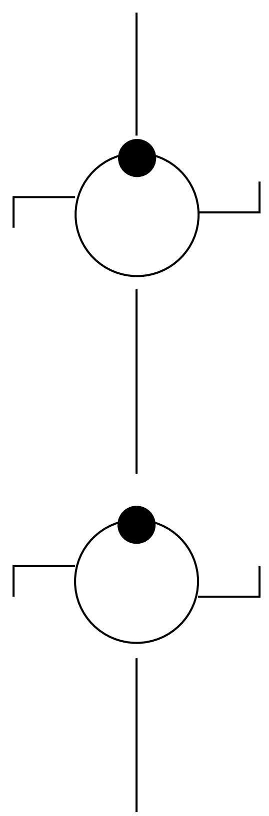
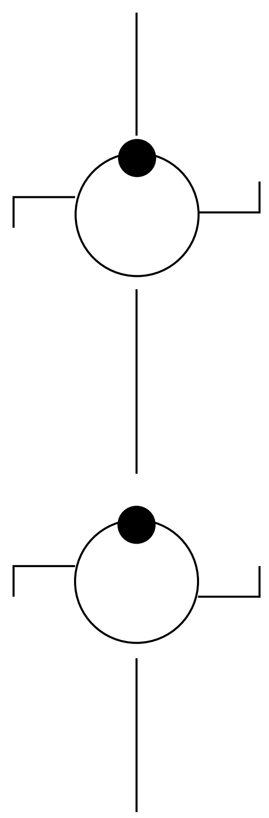
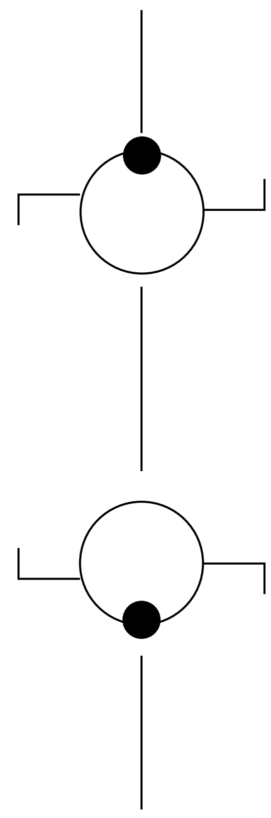


Translational
(T)

Line Dancing Symmetries

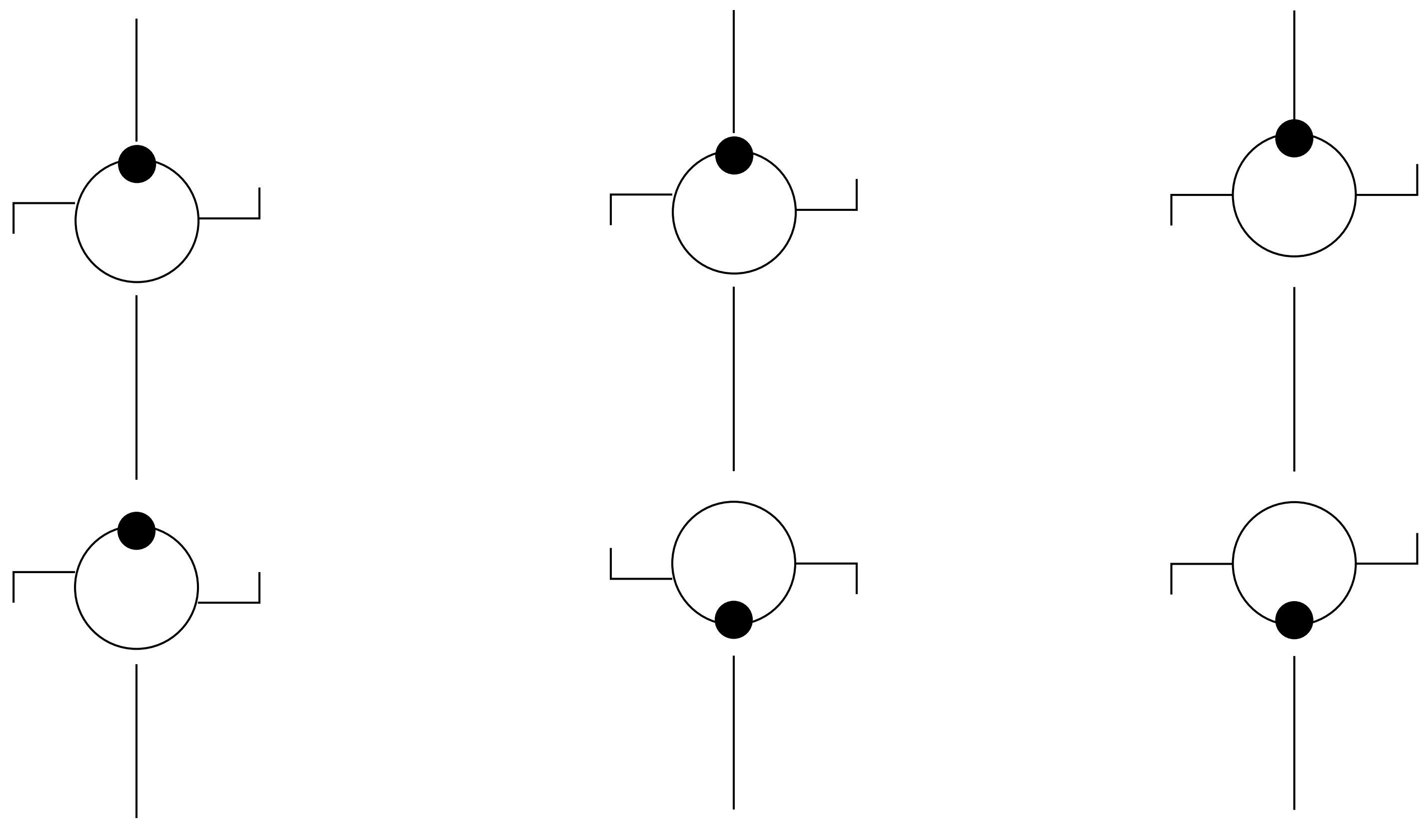


Translational
(T)



Reflectional
(M)

Line Dancing Symmetries

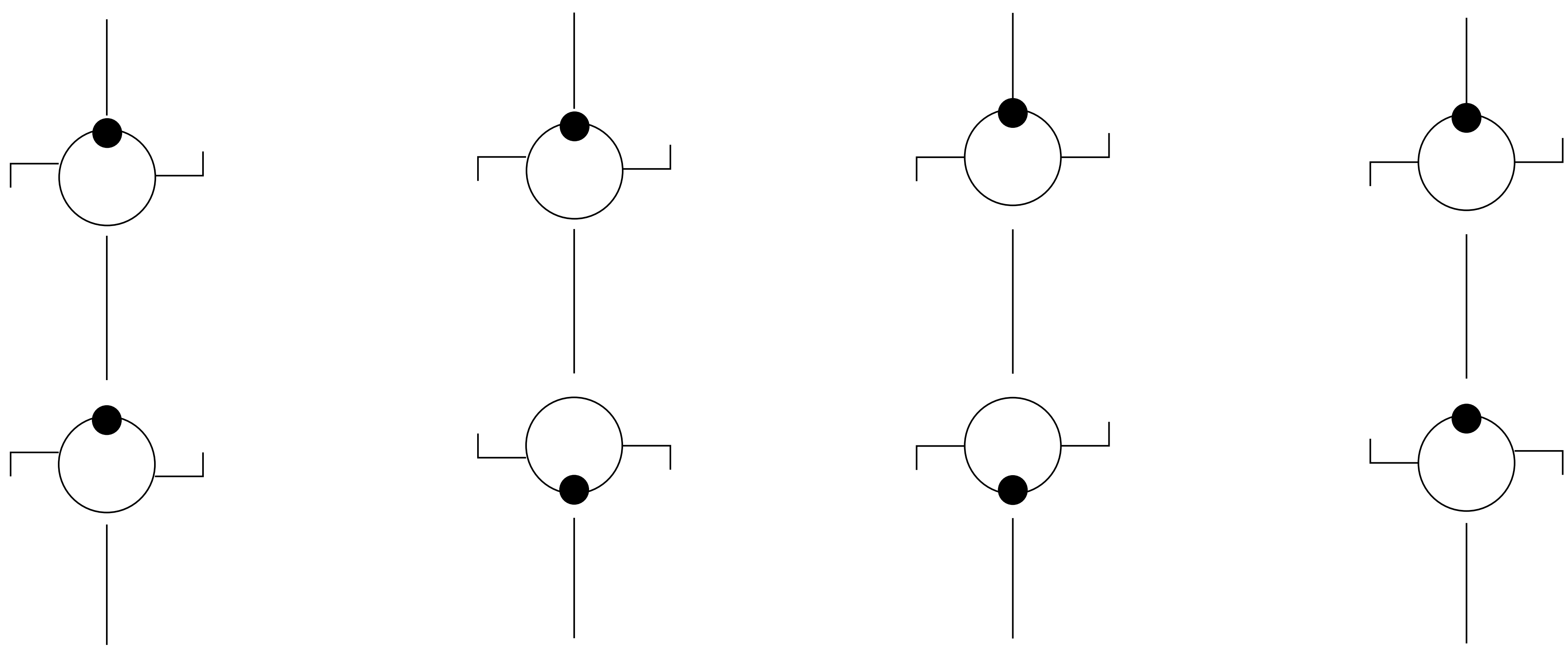


Translational
(T)

Reflectional
(M)

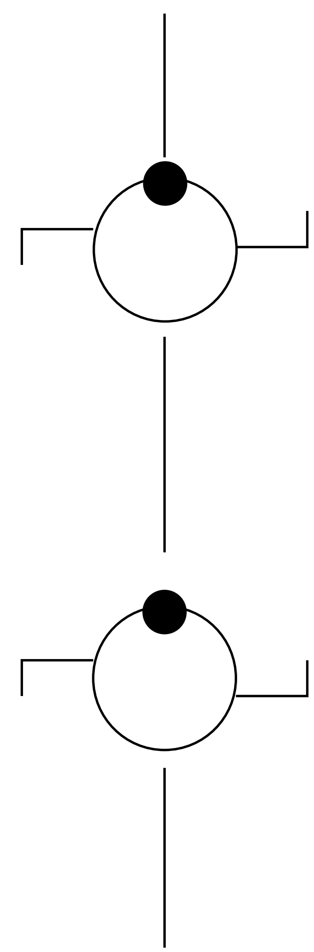
Rotational
(R)

Line Dancing Symmetries



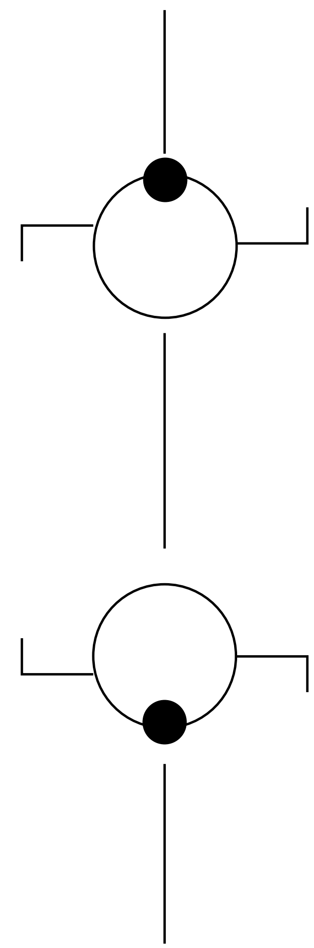
Translational (T) | Reflectional (M) | Rotational (R) | Glide Reflectional (G)

Activity:
Dance with a Partner



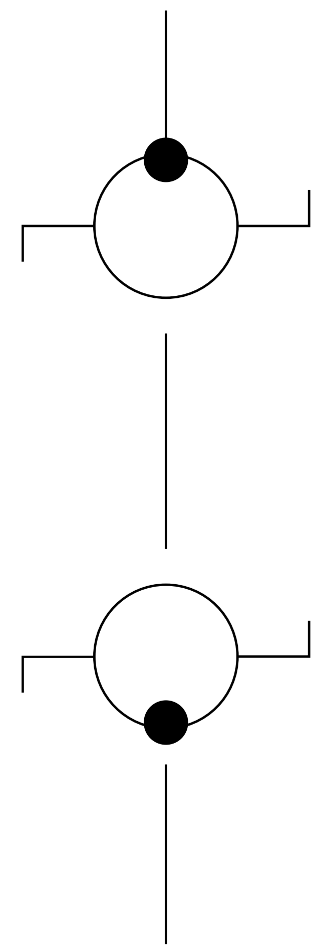
Translational
(T)

Activity:
Dance with a Partner



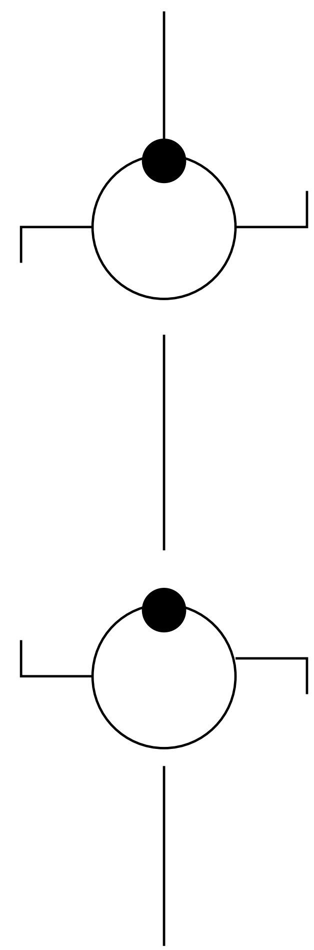
Reflectional
(M)

Activity:
Dance with a Partner



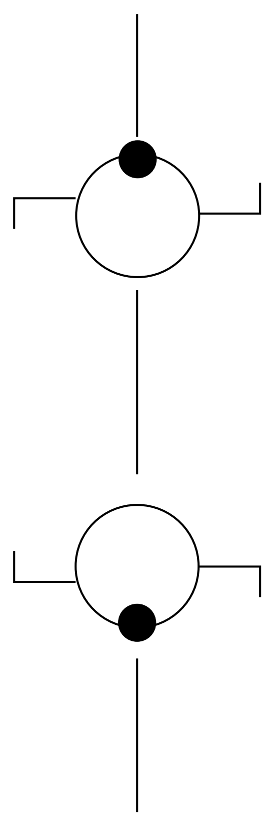
Rotational
(R)

Activity:
Dance with a Partner

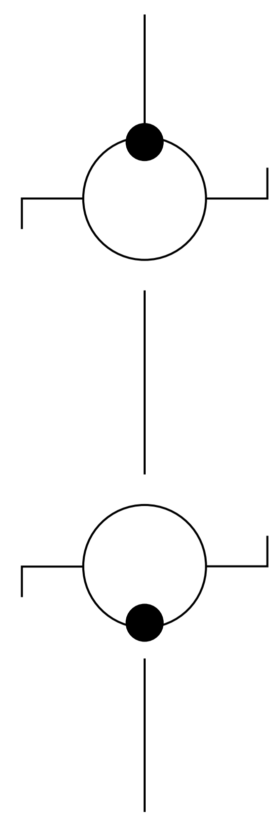


Glide Reflectional
(G)

Compositions of Dance Symmetries



+

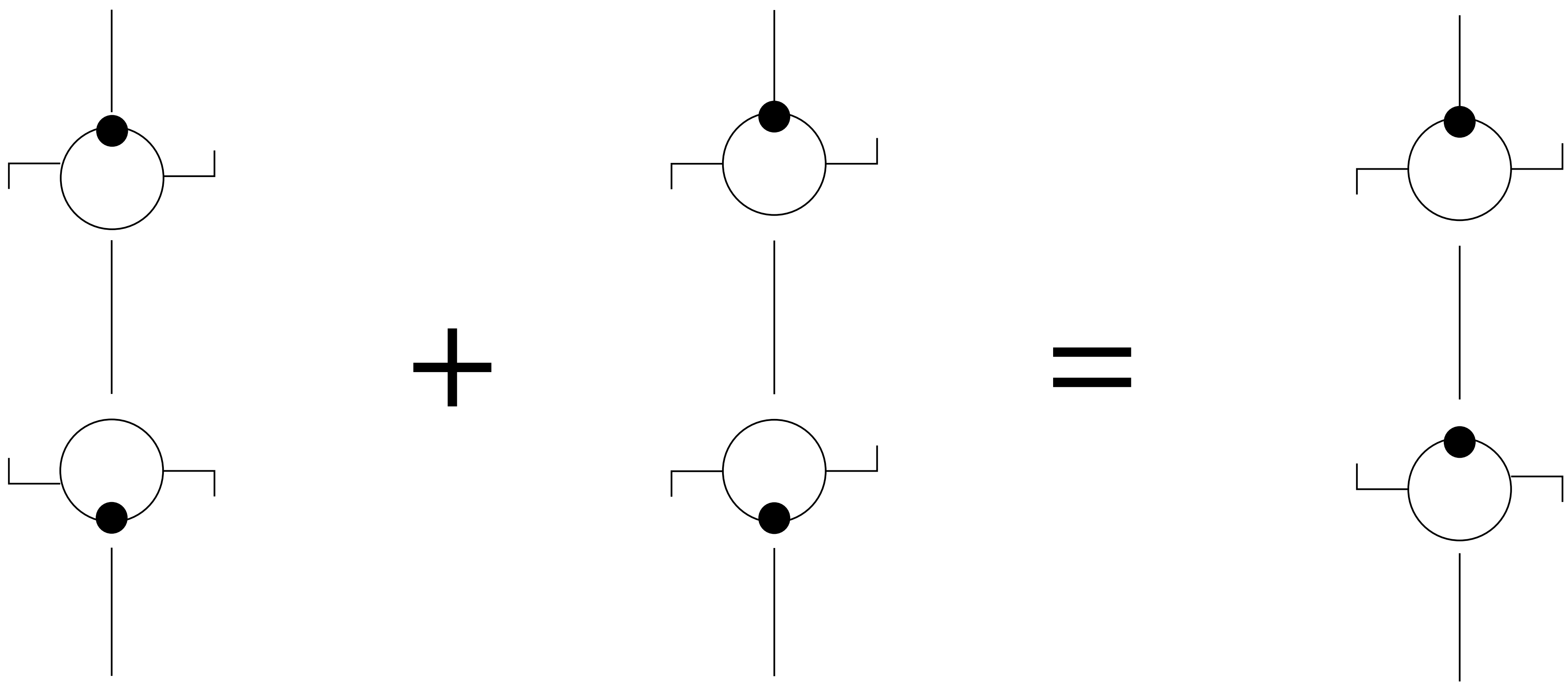


= ???

Reflectional (M)

Rotational (R)

Compositions of Dance Symmetries



Reflectional (M) + Rotational (R) = Glide Reflectional (G)

Activity:

Fill in the Composition Table

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2nd / 1st	T	M	R	G
T				
M				
R				
G				

Activity:

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2nd / 1st	T	M	R	G
T				
M			G	
R				
G				

Activity:
Fill in the Composition Table

1st \ 2nd	T	M	R	G
T	T	M	R	G
M	M	T	G	R
R	R	G	T	M
G	G	R	M	T

What is a group?

A **group** is a *set G* together with an *operation* "•" that satisfies four properties.

2nd / 1st	T	M	R	G
T	T	M	R	G
M	M	T	G	R
R	R	G	T	M
G	G	R	M	T

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(1) The operation • takes elements of *G* to other elements of *G*. (If *x* and *y* are in *G*, so is *x* • *y*.)

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(2) **Identity:** There is a special element e in G (called the *identity*) such that for every x in G, $e \bullet x = x \bullet e = x$.

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(3) **Inverses:** every x in G has an *inverse*, an element y in G for which $x \bullet y = y \bullet x = e$.

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M	M	T	G	R
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(4) **Associativity**: for all x, y, and z in G, $(x \bullet y) \bullet z = x \bullet (y \bullet z)$.

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(1) The operation • takes elements of G to other elements of G. (If x and y are in G, so is $x \bullet y$.)

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Then Z is a group with operation “+”.

(a) The sum of two integers is an integer.

(b) **Identity:** the number 0 is the identity: $0 + n = n + 0 = n$ for any n.

(c) **Inverses:** the inverse of an integer n is $-n$.

Mattress Flipping Problem

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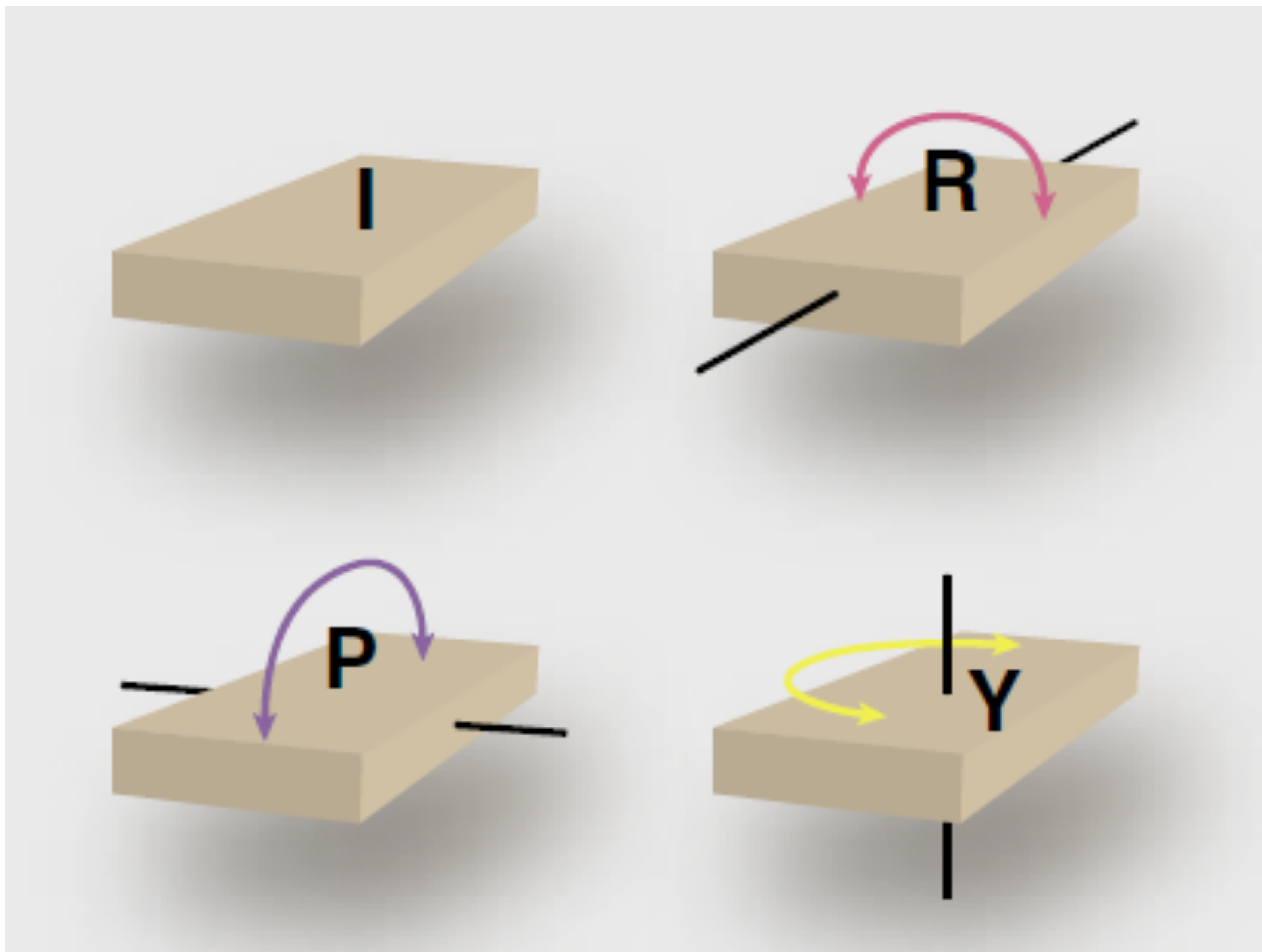
Goal: Find an “move” that we can do to a mattress to move it through every possible position.

Mattress Moves



Activity:

Fill in the Mattress Move Table

2nd / 1st	I	R	P	Y
I				
R				
P				
Y				

Activity:

Fill in the Mattress Move Table

2nd / 1st	I	R	P	Y
I	I	R	P	Y
R	R	I	Y	P
P	P	Y	I	R
Y	Y	P	R	I

Activity:
Fill in the Mattress Move Table

1st \ 2nd	I	R	P	Y
I	I	R	P	Y
R	R	I	Y	P
P	P	Y	I	R
Y	Y	P	R	I

So why can't we apply a single move to put the mattress into all possible positions?

What’s the Difference?

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Line Dancing Moves

1st \ 2nd	T	M	R	G
T	T	M	R	G
M	M	T	G	R
R	R	G	T	M
G	G	R	M	T

What’s the Difference?

Line Dancing Moves

1st \ 2nd	T	M	R	G
T	T	M	R	G
M	M	T	G	R
R	R	G	T	M
G	G	R	M	T

Mattress Flipping Moves

1st \ 2nd	I	R	P	Y
I	I	R	P	Y
R	R	I	Y	P
P	P	Y	I	R
Y	Y	P	R	I

What's the Difference?

Line Dancing Moves

1st \ 2nd	T	M	R	G
T	T	M	R	G
M	M	T	G	R
R	R	G	T	M
G	G	R	M	T

Mattress Flipping Moves

1st \ 2nd	`I`	`R`	`P`	`Y`
`I`	`I`	`R`	`P`	`Y`
`R`	`R`	`I`	`Y`	`P`
`P`	`P`	`Y`	`I`	`R`
`Y`	`Y`	`P`	`R`	`I`

{T,M,R,G} and {`I`,`R`,`P`,`Y`} are **isomorphic** groups.

Summary

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- Composing symmetries naturally leads to the definition of a group.
- Groups arise from “real life” as well as from pure mathematics (like the integers Z).
- Different objects can have the same (isomorphic!) groups of symmetries
- Studying the group of symmetries of an object can help to obtain non-obvious properties of this object. (Like the impossibility of constructing a simple mattress flipping schedule.)

Thank you!

For More Information

Here is the link to a book about the mathematics of dancing:

https://www.artofmathematics.org/books/dance

Here is a link to lecture notes by P. Etingof, which is a fantastic introduction to group theory:

http://www-math.mit.edu/~etingof/groups.pdf