Math 6490

LIE GROUPS and LIE ALGEBRAS

Instructor: E. B. DYNKIN

This is an introduction to the theory of Lie groups and algebras and their linear representations — a fundamental part of many branches of Mathematics (algebra, differential and algebraic geometry, topology, harmonic analysis, differential equations...) and an important tool in modern Physics . Only a basic knowledge of mathematical analysis and linear algebra is required. Elements of theory of differentiable manifolds will be introduced as needed.

The following topics will be covered.

1.Lie groups as a tool for treating symmetries in Physics. Applications to the elementary particles theory.

2. The groups of real and complex matrices and their classical subgroups. The corresponding Lie algebras. Exponential mapping.

3. General concept of a Lie algebra. Construction of the corresponding Lie group via the Campbell-Hausdorff-Dynkin formula.

4. Algebras of differential operators and groups of transformations of a differentiable manifold.

5. Universal Lie algebra.

6. Invariant tensor fields on homogeneous spaces. The Laplace-Beltrami operators.

7. Geometric integration theory.

8. Compact Lie algebras.

9. Solvable, simple and semi-simple Lie algebras

10. Structure of semisimple Lie algebras. Root systems. Simple roots. Dynkin and enhanced Dynkin diagrams.

11.The Weyl and Coxeter groups. The core groups of simple Lie algebras. Weyl orbits and nilpotent orbits.

12. Linear representations of semisimple groups. Description of an irreducible representation by the highest weight. Casimir element and weight multiplicities. Weyl's character formula.

Bibliography

1. J.-P. Serre, Lie algebras and Lie groups, Lectures given at Harvard University, Benjamin, New York-Amsterdam, 1965; Springer, New York, 1992.

2. J.-P. Serre, Complex semisimple Lie algebras, Springer, New York 1987

3. J. E. Humphreys, Introduction to Lie algebras and representation theory, Third Printing, Springer, New York, 1960

4. W. Fulton and J. Harris, Representation theory, A first course, Springer, 1991

5. E. B. Dynkin, The structure of semi-simple Lie algebras, Amer. Math. Society Translations, Number 17, New York, 1950

6. E. B. Dynkin, Survey of the basic concepts and facts in the theory of linear representations of semisimple Lie algebras, Supplement in: E. B. Dynkin, Maximal subgroups of the classical groups, Amer. Math. Society Translations, Series 2,

Providence, R. I., 1957 [Reprinted in Selected Papers of E.B.Dynkin with Commentary,Amer.Math.Soc./ International Press, 2000, pp. 111-155.]

7. R. N. Cahn, Semi-simple Lie algebras and their representations, Benjamin/Cummings, Menlo Park, CA, 1984

8. M. R. Sepanski, Compact Lie groups, Springer, 2007