MATH 1300, Mathematical Explorations

Game Theory

Activity

- Bring stash of M&Ms and halloween candy to class.
- Highest number n wins $100/n$ M&Ms game. Each table is a team. Play twice: first time just a discussion at the table. Second time, have the tables nominate one person to negotiate for them. (30 seconds each.)
- Show Golden Balls video (standard example).
- Have four pairs of students (one student from each table) play the Golden Balls game for halloween candy for their table.
- Discuss non-cooperative games and Nash Equilibria — situations in which one player cannot improve the outcome for themselves unilaterally (Nash equilibria in bold below). Explain Golden Balls example. Then explain the other two games and ask students to identify the Nash Equilibria in them.

  Golden Balls:

	Split	Steal
Split	\$5,000, \$5000	**\$0, \$10,000**
Steal	**\$10,000, \$0**	**\$0, \$0**

  Prisoner's Dilemma:

	Snitch	Silent
Snitch	**2 years, 2 years**	3 years, 0 years
Silent	0 years, 3 years	1 years, 1 years

  Bach or Stravinsky (or battle of the sexes):

	Bach	Stravinsky
Bach	**ecstatic, happy**	miserable, miserable
Stravinsky	miserable, miserable	**happy, ecstatic**

- Nash's theorem (for which he won a Nobel Prize for Economics) is that in noncooperative games there is always a *mixed* Nash Equilibrium — that is, if the game is such that each player chooses a strategy whereby she will act in a certain way with a certain probability – then there is a strategy for all the players such that no player can unilaterally improve their expected outcome. The proof uses a fixed point theorem from topology.

- End first class with game theory clip from A Beautiful Mind, and Golden Balls amazing example.
- Have an iterated Golden Balls or prisoners dilema tournament. Each table is a team. Give each table a die for probability. They discuss a strategy: it must be a fixed, but it can depend on what happened in the previous rounds, and on a coin toss / dice role.
- Possibly run tournament twice so students have a chance to adjust their strategy.
- Discussion of the life of John Nash (this can serve as an introduction to the poster project) and Brouwer's Fixed Point Theorem: state the theorem. State examples (you can do the first one or two in class with strips of ruled paper and a print out of a numbered grid):
  - 1 dim: folding a ruler onto itself, we can always find a fixed point.
  - 2 dim: folding a piece of paper onto itself, we can always find a fixed point. (What if we cut a hole in the paper?)
  - 3 dim: after stirring a cup of coffee, there's always point in the liquid that is where it started.

Notes

- Students may already be familiar with this material from Economics classes.
- Might be good to incorporate more mathematics
- Running the tournament with teams getting knocked out each round doesn't incentivize a mixed strategy - maybe have groups pair off and play iterated games in pair - then tally final scores at the end to choose a winner.

Assignments

1. Find and discuss a 'real world situation' in which a non-cooperative game occurs. Identify the Nash Equilibria (if any). Discuss a strategy.
2. In the tournament: Why did you do what you did? On reflection, what do you now think is a good strategy? Read the two articles listed below and answer the questions following them.
   - Does it play to be nice - part I
   - Does it play to be nice -part II

   Does it pay to be nice?

   Discuss 'real-world' situations that this game and our insights apply to.

References and resources

Golden Balls video

Golden Balls video - unusual strategy

A Beautiful Mind Clip

Does it play to be nice - part I

Does it play to be nice -part II