Hilbert Hotel

Activity

- Cardinality worksheet and review

- Bijections worksheet and review

  - Looking at things that all have cardinality $\mathbb{N}$: $\mathbb{Q}$, $2\mathbb{N}$, $10^{\mathbb{N}}$, $\mathbb{N}^2$.
  - Looking at things that all have cardinality $\mathbb{R}$: any interval.

- Diagonalization Goal: We want to prove that card($\mathbb{N}$)$\neq$card($\mathbb{R}$). Show that there is no bijection between the natural numbers ($\{1, 2, 3, ...\}$) and the real numbers (all decimal numbers). To do this, we will assume there is a matching then derive a contradiction.

- Hilbert Hotel activity - see Strogatz's video or Math Explorer's Club worksheets for possible questions for students to explore.

- Conclude with statement of Continuum Hypothesis.

References and resources

Math Explorers Club Spring 2016 Module: Cardinality and Bijections Worksheet

Math Explorer's Club Spring 2017 and Fall 2019: Infinity and Paradoxes

Strogatz's Hilbert Hotel

Follow-on activities

Koch Snowflake

Zeno's Paradox