Infinity and Paradoxes

Activity

- Warm-up: What's the biggest number you can think of?

- Introduce the idea of infinity - maybe give a brief history (Greek's ignored it; People used it lightly; Paradoxes arose; Cantor fixed it...see Wikipedia).

- *Paradoxes of Infinity.* Write the following on the board and ask students to come up with answers at their tables:

    - Is ∞ even or odd?
    - What's $\infty - 1$?
    - What's $\infty - \infty$?
    - What's $0 \cdot \infty$?
    - What's $2 \cdot \infty$?
    - What's ∞ / ∞?

- Have students work with their groups through sections 2.2 and 2.3 in Art of Mathematics: Infinity.

- Discuss Paradoxes of Infinity. Takeaway: Infinity is not a number like $2, 106, \pi$, or $3/4$. We need a different set of rules and ideas about how the concept of infinity works. It's a mistake to confuse ∞ with a number. Asking if $\infty \cdot 0 = 0$ is like asking if *triangle* $\cdot 0 = 0$; it's a type error... Most people say "Anything times zero is zero" but they are neglecting the implicit quantification - any number times zero is zero - and infinity is not a number.

Assignments

1. Read Pages 5 - 21 in David Foster Wallace's "Everything and More".

2. DFW defines the 'abstract' as that which is "removed from or transcending concrete particularity, sensuous experience... [E]ssential to math is the sense in which abstracting something can mean reducing it to its absolute skeletal essence." Mathematics, in particular, requires abstractions to be rigorous, to obviously participate in the rules of formal logic. The goal of our class (and, arguably, DFW's book) is to tour the difficulties of defining 'infinity' and the tensions between precision, abstraction and intuition inherent in that project. In one(ish) page describe a specific example of abstraction (or the theory of something abstract) within your field of study. How does your example relate to DFW's interpretation of the abstract and the mathematical project of understanding the abstract infinity?

References and resources

Wikipedia: Infinity

The Art of Mathematics: The Infinite

Math Explorers Club Spring 2016 Module: Big Numbers and Infinity

Follow-on activities

Hilbert Hotel

Koch Snowflake

Zeno's Paradox