MATH 1300, Mathematical Explorations

Knot Theory

Two 75 minute classes

Activity

- Bring string and scissors to class.
- Ask students for examples of who uses knots in real life (sailors, weavers, mountain climbers, fishermen, etc.) and to brainstorm how the mathematical approach might compare.
- Have students work through Unlinking strings - Sec 7.1 of Knot Theory Book. Have class pair up, make the 'manacles' and try the exercise. Do you think it is possible to *unlink* the participants?
- The Human Knot game. Sec 7.2. Demonstrate with someone that two people, can't make a knot. Have them try it with three people. They untangle every time. Then try four or five or more people:

  I. If your group has an even number of people, each person should grasp the right hand of a person not their neighbor. If your group has an odd number of people, everyone except for one should grasp the right hand of a person who is not their neighbor. The remaining person should use their right hand to grasp the left hand of another person.

  II. Now everybody else should grasp the left hand of another person different from the person whose right hand they grasped.

  III. Now try to untangle the resulting knot.

- What can you get? Tables may join and make much bigger knots. Ask them to take photos of their knots on someones cellphone.
- Pass out Mladen Bestvina's 2003 handout and tell them about knot tables. Ask them to use the pictures of their knots to try and find them in the knot tables.
- Have them make a human trefoil knot. Maybe then figure 8 knots (warning: it's a bit intimate). Have them make string models of a trefoil and a figure 8 knot.
- Finish class 1 by raising the issue of how to tell when a knot is knotted/when two knots are different.
- Continue with Bestvina's notes in class 2.
- Mention that this approach is very limited. Why for example are left- and right-trefoils different?

If time:

- Unknotting number $u(K)$ is the least n such that there is a diagram for K that can be transformed to the unknot by reversing n crossings. Note that it is always possible to transform a knot diagram to a diagram for the unknot by reversing some of the crossings. (why?). Examples: $u(7_2) = 1,\ u(5_1) = 1,\ u(7_4) = 1,\ u(7_1) = 3$. These are easy to guess, but hard to prove.)
- Give them some background on knot theory and knots in science. Kelvin & Tait's vortex atom theory: atoms and knots in the ether. Knots in DNA. Knots in string theory. Knots as an intro to topology.
- Discuss Borromean rings. Can mention their curious properties, similarity with braids. Ask students to consider: Can you do the same thing with more components?

References and resources

Bestvina's Notes

Kelvin & Tait's Vortex Atom Theory

The Art of Mathematics: Knot Theory