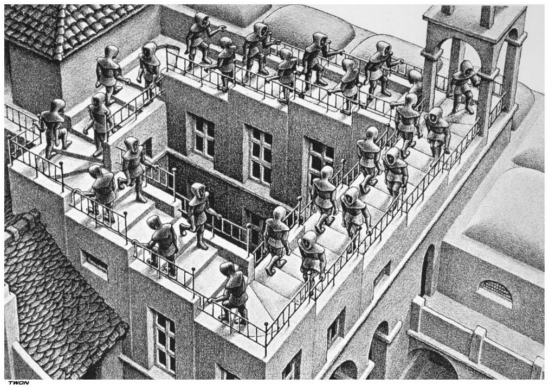
MATH 1300, Mathematical Explorations

Paradoxes

Activity

- Have students discuss the following paradoxes in groups and then as a class.
  - Three boxes each hold two coins. One box has two gold coins, one box has two silver coins, and the last box has one silver and one gold coin. You select a box at random and withdraw a coin at random from that box. The coin you drew is gold. What is the probability that the other coin in the box is gold?
  - Can an omnipotent being create a stone so heavy it cannot lift it?
  - Would you describe a man with one hair on his head as bald? Would you describe a man with two hairs on his head as bald? Would you describe a man with one hundred thousand hairs on his head as bald? Where do you draw the line?
- For each of the following have students discuss: Is it a paradox? Why or why not? How paradoxical is it?
  1. M.C. Esher's *Ascending and Descending*



  2. Proof that $0 = 1$:

$$\begin{aligned} 0 &= (1-1)+(1-1)+(1-1)+\cdots \\ &= 1+(-1+1)+(-1+1)+(-1+1)+\cdots \\ &= 1 \end{aligned}$$

  3. The word "heterological" means "inapplicable to itself." So, for example, the word "German" is heterological because it does not apply to itself, but the word "English" is not heterological because it does apply to itself. Is "heterological" heterological?

4. Problem from *The Surprise Examination or Unexpected Hanging Paradox*:
   "A teacher announces in class that an examination will be held on some day during the following week, and moreover that the examination will be a surprise. The students argue that a surprise exam cannot occur. For suppose the exam were on the last day of the week. Then on the previous night, the students would be able to predict that the exam would occur on the following day, and the exam would not be a surprise. So it is impossible for a surprise exam to occur on the last day. But then a surprise exam cannot occur on the penultimate day, either, for in that case the students, knowing that the last day is an impossible day for a surprise exam, would be able to predict on the night before the exam that the exam would occur on the following day. Similarly, the students argue that a surprise exam cannot occur on any other day of the week either. Confident in this conclusion, they are of course totally surprised when the exam occurs (on Wednesday, say). The announcement is vindicated after all. Where did the students' reasoning go wrong?"
5. $1 = 0.9999\cdots$
6. At 11:59:30 (i.e., with $\frac{1}{2}$ a minute until midnight) you flip a light on. At 11:59:45 (i.e., with $\frac{1}{4}$ of a minute until midnight) you flip the light off. With $\frac{1}{8}$ of a minute until midnight, you flip the light on. With $\frac{1}{16}$ of a minute until midnight, you flip the light off. And so on... Is the light on or off at midnight?

References and resources

Kelsey's Worksheet on Paradoxes

Paper: The Surprise Examination

Follow-on activities

Monty Hall

Zeno's Paradox

Hilbert Hotel