

MATH 1300, Mathematical Explorations

Proving the Impossible

Assignments

1. There is a common claim that “you cannot prove a negative.” What kinds of “negative” statements do you think that this refers to? Do you agree or disagree with this claim? (Expected length: 1 paragraph.)
2. Look up the definition of a prime number and some of their properties. Give an example of five prime numbers and two non-prime (i.e. composite) numbers. Explain why mathematicians think of prime numbers as the “atoms” out of which all numbers are built. (Hint: look up “unique prime factorization”)
3. The first major result about prime numbers is Euclid’s proof of the “infinitude of primes.” This is proved by contradiction: by assuming that it is false, and then finding a logical contradiction.
 - (a) Does this method of proof make sense to you? Explain the logic behind this method.
 - (b) When we assume that there are not infinitely many primes, that is the same as assuming there are finitely many. Suppose that there are exactly n primes. We can write a list of all of them: p_1, p_2, \dots, p_n . Now consider the number $p_1 * p_2 * \dots * p_n + 1$. Explain why it isn’t divisible by any of the primes on our list.
 - (c) This means that we’ve discovered a new prime: either this number is prime (and isn’t on our list) or it isn’t prime but it has a prime divisor (which also isn’t on our list). Explain why this is a logical contradiction, and conclude that there are infinitely many primes.
4. This method is often used to prove that certain statements are impossible. What do you think of the idea of proving that something is impossible, or that something doesn’t happen? How does this interact with the idea that “you can’t prove a negative” from the first problem? (Expected length: 1 paragraph.)