

MATH 1300, Mathematical Explorations

Topology

“In these days the angel of topology and the devil of abstract algebra fight for the soul of every individual discipline of mathematics.” - Weyl

One 75 minute class.

Activity

- Show Kelsey’s Infinite Series video on Topology.
- Reiterate: What is Topology? an area of mathematics concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing. (Wikipedia.) Coffee-cup = doughnut.
- Distribute exercises from Burger-Starbird for students to try at their tables
- Show [this video of a trick performed by Conway](#) and ask for an explanation.
- Then distribute string and ask students to emulate this [trapped cable trick](#).
- Introduce the concept of a surface. (Looks locally like \mathbb{R}^2 .)
- Have students try Tic-Tac-Toe on a torus (Can use Jef Weeks’ [Torus Games](#) software).
- Have students calculate the Euler Characteristic of a torus. What about a figure-eight (a 2-torus) or a pretzel (a 3-torus)? What about a sphere with a cross cap (a disc removed and a Möbius band glued in)? / a Klein bottle.
- Calculate how Euler Characteristic changes when you glue on a handle (it drops by 2) or a Möbius band (it drops by 1). State the Classification of Surfaces.
- Distribute scissors, paper and tape.
- Test the claims about Möbius bands in *Paul Bunyan versus the Conveyor Belt*: Two Möbius bands glued together and cut make either a square or a pair of interlocking hearts, depending on which ways they are twisted.

References and resources

[Infinite Series Video: Topology](#)

Topology Exercises Handout

Topology Handout Answers

[Jef Weeks' Torus Games](#)

Video: [Trapped Cable Trick](#).

Video: [Conway's Trick](#)

Video: [Dave Richeson rod and surface animation](#)

Topologist's Map of the World

Assignments

1. Ask the students to use their phones / computers / iPads to make stop-motion videos like this one: [Dave Richeson rod surface video](#). Or have them look for similar videos online such as coffee-cup to torus.