Fibonacci and his Hypothetical Rabbits

Here are some activities to help you prove the number of rabbit pairs is always a Fibonacci number (assuming that Fibonacci's model is correct, which it actually is not, but let's pretend it is).

1. Start with one juvenile pair xy in the first generation. Make a three-column table showing (1) the number of adult pairs, (2) the number of juvenile pairs, and (3) the total number of pairs. Do this for the first six generations. (Thus, the first row of your table should show the number of adult pairs, juvenile pairs, and total pairs in the first generation. The second row shows the results for the second generation, and so on.)
2. What patterns do you see in the table? (Express them in words, not algebra.)
3. Now rewrite the patterns using variables. Specifically, let A_n and J_n be the number of adult and juvenile pairs in generation n. How do they relate to P_n, the total number of pairs in generation n?
4. Explain why the relationship you just found must always be true, no matter what n is.
5. Look at your table again. How do the numbers in one row determine the numbers in the next row? For example, how is A_{n+1}, the number of adult pairs in the *next* generation, related to A_n and J_n? Why must this be true?
6. How does A_{n+1} relate to P_n? Explain.
7. Explain why the relationships you just found also have to be true for any generation n, not just for the first six generations.
8. What is the equation for J_{n+1} in terms of the variables A_n, J_n, P_n from the preceding generation? Explain.
9. What is the formula for P_{n+1} in terms of A_n, J_n, P_n? Why?
10. Now, what are we trying to prove again? Write the desired result (something about Fibonacci numbers $\ldots$) using the variables A, J, P and the generation counters $n, n+1$ and $n+2$.
11. Prove what we are trying to prove, by using and combining the results you have obtained above.