

$$V \approx 1.20 \times 10^{79} m^3.$$

How many molecules could we pack into such a volume? What we know about the distribution of matter in the universe suggests that such a volume packed with lead would safely overestimate the true number of molecules in the universe.<sup>4</sup>

So we need to figure out how many molecules there are in a cubic meter of lead. The requisite data for lead is a molecular mass of  $207 \frac{g}{mole}$  and a density of  $11.3 \frac{g}{cm^3}$ . Combined with *Avagadro's number*,  $6.022 \times 10^{23} \frac{molecules}{mole}$ , we calculate:

$$\frac{(11.3 \frac{g}{cm^3})(6.022 \times 10^{23} \frac{molecules}{mole})}{207 \frac{g}{mole}} \approx 3.29 \times 10^{22} \frac{molecules}{cm^3} = 3.29 \times 10^{28} \frac{molecules}{m^3}.$$

In other words, what this last calculation shows, is that each cubic meter of lead contains about  $3.29 \times 10^{28}$  molecules. We have about  $1.20 \times 10^{79}$  cubic meters to fill up. So if we filled up the entire observable universe we've been considering with lead, the total number of molecules would be about

$$(1.20 \times 10^{79} m^3) \left( 3.29 \times 10^{28} \frac{molecules}{m^3} \right) \approx 3.95 \times 10^{107} molecules.$$

This is not "much more" than a googol of molecules.

## 2. Large Numbers and Everyday Things

To get a better sense of the fairly large, let us begin with something we believe we know well: time.

In the next five problems you are asked to estimate various quantities. First reactions are what is desired, so you should make these estimates without performing any calculations.

1. Quickly estimate how long you think one thousand seconds are in a more appropriate measure of time.
2. Quickly estimate how long you think one million seconds are in a more appropriate measure of time.
3. Quickly estimate how long you think one billion seconds are in a more appropriate measure of time.
4. Quickly estimate how long you think one trillion seconds are in a more appropriate measure of time.
5. Quickly estimate the number of seconds you have been alive.
6. Precisely determine how long one thousand seconds are in a more appropriate measure of time.
7. Precisely determine how long one million seconds are in a more appropriate measure of time.
8. Precisely determine how long one billion seconds are in a more appropriate measure of time.
9. Precisely determine how long one trillion seconds are in a more appropriate measure of time.
10. What do Investigations 6-9 tell you about the relative size differences between thousands, millions, billions and trillions?
11. Compute the number of seconds in one day.
12. Compute the number of seconds in one year.
13. Use Investigation 11 and Investigation 12 to precisely determine, within a hundred thousand seconds or so, the number of seconds you have been alive.
14. How close were your estimates in Investigations 1-4 to the actual results in problems Investigations 6-9 and Investigation 13?
15. What does your answer to problem Investigation 14 tell you about your fluency with large numbers?

<sup>4</sup>In contrast to our intuition, a cubic meter of aluminum consists of more than twenty times as many molecules as a cubic meter of lead. Can you find out why? And what is the most "molecular" element so we have an honest upper bound?

16. Show that the speed of light in meters per second is, as stated in the text:

$$\text{Speed of light} \approx 9.45 \times 10^{15} \frac{m}{year}.$$

17. Show that the volume of sphere of radius  $r \approx 1.42 \times 10^{26} m$  is given by

$$V \approx 1.20 \times 10^{79} m^3.$$

Money is something else we know fairly well, for good or ill. Let us consider some examples of large sums of money.

Using newspaper reports, televised news reports, the Internet, or other source in your library if necessary, find several specific items whose costs or budgets are:

18. ... in the thousands of dollars.
19. ... in the millions of dollars.
20. ... in the billions of dollars.
21. ... in the trillions of dollars.

### 3. A Trillion Dollars in Human Terms

Here's another example of large numbers, one with obvious human implications. It was developed by **Lou Jean Fleron** (American Labor Educator; 1940 - ), mother of the first author, in the 1980's. Those with an interest in the topic should feel free to update the numbers and/or change the geographical regions to suit their purpose.

The population of several states is given in the table below:

Kansas	2,486,000
Missouri	5,138,000
Nebraska	1,585,000
Oklahoma	3,158,000
Iowa	2,787,000

You probably have friends and/or relatives who live in these states.

For each of the following questions find the desired quantities and give a single value for the entirety of these five states considered together.

22. If we consider the average family size to be 4, how many total families are there in these states?
23. What would the total cost be to build a \$100,000 house for each of the families in these states?
24. What would the total cost be to give each of the families in these states a \$15,000 car?
25. What would the total cost be to build a \$5 million library and \$15 million hospital in a total of 250 cities in these states?
26. What would the total cost be to build a \$10 million school in a total of 500 cities in these states?
27. If you started with \$1 trillion dollars and did all of the things described in Investigations 23-26, how much money, if any, would be left?
28. If you put the left over balance calculated in Investigation 27 in the bank and earned 3% interest per year, compounded yearly, how much interest would be earned each year? [Assume the interest is not reinvested.]
29. If you used the yearly interest from Investigation 28 to pay doctors, nurses, teachers, police officers, and firefighters each \$40,000 a year salaries, how many such public servants could you hire with the interest alone?
30. Using Investigation 29, how many doctors, nurses, teachers, police officers, and firefighters would the interest provide for each of 250 different cities in these states?

All of the costs you considered in Investigations 23-29 cost less than \$1 trillion dollars. In comparison, in each of the years 2002 - 2010 the United State national debt grew at least one-half a trillion dollars.

At 4:06:52 p.m. EST on 1/20/2010, as this was being typeset, the debt was \$12,325,889,768,889.98. Less than *one minute later*, the debt had grown by another \$2,499,876!<sup>5</sup>

31. Find some information on the U.S. national debt currently and relate what you find to the discussion of large numbers in a brief essay of a few paragraphs.

#### 4. Innumeracy

In 1988 the book *Innumeracy: Mathematical Illiteracy and Its Consequences* by John Allen Paulos (American Mathematician; 1945 - ) was released. The book was widely read and widely praised, spending more than 5 months on the *New York Times* Best Seller list and rising as high as #5 on this list. Translated into a dozen languages, the book continues to be influential.

As the title suggests, *innumeracy* is a quantitative analogue of illiteracy. In other words, it is the inability to make sense of and reason with numbers and numerical quantities. Many believe that this is a serious problem in modern society. As Paulos writes:

At least part of the motivation for any book is anger, and this book is no exception. I'm distressed by a society which depends so completely on mathematics and science and yet seems so indifferent to the innumeracy and scientific illiteracy of so many of its citizens.

This short book contains hundreds of ways in which innumeracy misleads us into poor decision making in regards to personal risk, financial decisions, assessment of statistics, and interpretation of data. It is a powerful book.

Having discussed large numbers in several different contexts, it is appropriate for you to reflect on the potential implications of innumeracy. Some suggested avenues for investigation are as follows:

32. INDEPENDENT INVESTIGATION: Examples from *Innumeracy* - Find the book *Innumeracy* and read a few sections. Pick out an example that is particularly interesting to you. In a brief essay, describe this example and consider the implications of such innumeracy.

33. INDEPENDENT INVESTIGATION: Innumeracy Experiment - Survey a collection of your friends, family, and peers to see how prevalent innumeracy is. Do this by describing your work with large numbers and then telling them about the figures you found in Investigations 18-21. However, when you report your figures, interchange some of the terms millions, billions and trillions so the figures are not correct. See what proportion of your audience catches on. Describe your results and then discuss the societal impact of a populace who generally cannot distinguish the words million, billion, and trillion when used in context.

34. INDEPENDENT INVESTIGATION: Original Example - Create, develop, and/or find your own example that illustrates innumeracy. Describe the example, its relevance, and its potential impact on society.

35. INDEPENDENT INVESTIGATION: Original Essay - Develop and answer your own essay prompt that addresses the issue of innumeracy in a way that you find compelling, important, and/or related to interests of yours.

---

<sup>5</sup>There are a number of debt clocks online, including [http://www.brillig.com/debt\\_clock/](http://www.brillig.com/debt_clock/).