Expected Skills.
At the end of this section, students will be able to:

- list the pieces of information that define a function,
- determine when two functions are equal or not by comparing their algebraic definitions,
- draw the graph of important functions such as $\sin x, \cos x, \tan x$, quadratic functions (i.e. functions with x^2), absolute value of x,
- explain what the inverse function of a given function is,
- correctly use the conventions of dots and empty circles when drawing the graph of a function,
- shift and compress a function vertically or horizontally,
- compute the inverse function of "simple" polynomials.

Pre-Class Activity.
The goal of the pre-class activity is to have students begin to think about the concept of limits through the interpretation of a speedometer reading. Here are some aspects that can be mentioned and discussed in class:

1. How does the speedometer of a car work?
   Underline that the speedometer doesn't give the instantaneous velocity but must give the average velocity over short period of time. This leads naturally to the second and third questions.
   We don't expect students to be able to answer precisely these questions at that point. They will come up again in the course when talking about limits and derivatives.

- Warning: On the subpage entitled "If the speed were constant, we could just say distance = speed × time. Here's the new idea" of the Motivating Examples, the limiting process is not entirely rigorous, especially the explanation in the Solution part – indicate this in case a student ask a question about it.
- On the practical side, to make sure that students understand correctly what they have to read, it may be a good idea to show them the website in class and indicate what pages they have to read.

Worksheet.
This activity follows the following plan:

1. have the students come up and test conditions for two functions being equal,
2. graph and modify the absolute function,
3. investigate indeterminate expressions,
4. determine the inverse of exponential functions, and
5. investigate the distinction between the secant function and the inverse cosine function (Arccosine).

Supplemental Activity.
This activity has students investigate the relationship between three functions: the secant function, the cosine function, and the arccosine function through their graphs. The activity also introduces the principal branch

definition of the arccosine function and has the students investigate the importance of domain and range when working with compositions of functions and their inverses. The activity ends by having students consider reciprocal and inverse functions for other trigonometric functions.

It is suggested that instructors guide groups of students through this activity and frequently debrief the classroom as a whole after each group has completed each problem. Problem 5 of the activity can be adapted as a full class discussion to conclude the activity.

Active Review.
This Active Review is meant to be a session-long sequence of activities emphasizing the importance of mathematical communication in the context of a pre-calculus review. For more information, refer to the first page of the Active Review document.

The goal of this activity is to show you some motivations of why calculus is interesting. Go to the website https://brilliant.org/practice/indeterminate-forms. Read all the subpages on indeterminate Forms (using the right-hand side "navigation bar") and try to answer the questions. Then, do the same for the Motivating Examples page (https://brilliant.org/practice/motivating-example-for-limits).

Note about brillant.org.
The website requires a free sign up if you want to see the answers to the questions and some additional explanations. Whether you decide to sign up or not is up to you. (In case you want some extra explanations on the questions asked, then I would recommend it, and otherwise not).
In all cases, note that we will only use the free part of the website.

Here are some questions to think about when reading these pages:

1. How does the speedometer of a car work?
2. What is the purpose of computing the speed of a car using ever smaller time intervals?
3. What do we need to do to determine the actual *instantaneous* speed of a car (or of another object)?
4. What are the three functions studied in the Motivating Examples page?

We will talk about these questions in class.

1. (a) When are two functions equal? Give several examples and counter-examples (play also with the interval of definition).

   (b) Let f be defined by $f(x) = \dfrac{x^2-4}{x-2}$, g by $g(x) = x+2$, and h by $h(u) = u+2$. Which of the previous three functions are equal?

   Ask the students to complete this after having asked and discussed the two following questions (that can be asked as clicker questions) on whether or not two functions are equal.

2. (a) Draw the graph of the function $f(x) = |x|$ and write down the definition of this function (i.e. $|x| = \dots$).
   Note that this is an example of a function that is defined piecewise and that has a corner (hence non-differentiable at that point).

   (b) How can we "modify" this function to shift it horizontally by 2 units to the right and vertically upward by 3 units (i.e. we want the vertex of the function to lie at $(2,3)$)?

   (c) Let $g(x) = x^2 - 3x - 4$. Is $f(g(x)) = g(f(x))$?

3. Do we have $0 \cdot \infty = 0$, $0 \cdot \infty = \infty$ or something else?
   Underline here that ∞ is not a number and that the above expression can actually give any number.

4. (a) What is the inverse function of the $f(x) = e^x$? Write down $f^{-1}(x)$ as well as its domain and range.

   (b) Using this, find two different expressions of $x = \dots$ that use these functions. When are these expressions true?

   Here we are obviously looking for $e^{\ln x}$ and $\ln(e^x)$.
   Note that the first function is only defined for positive numbers whereas the second is defined for all real numbers.

5. In addition to the basic trigonometric functions of sine, cosine, and tangent, there are reciprocal functions as well. We will consider the secant function: $\sec x = \frac{1}{\cos x}$.

   (a) Draw the graph of the function $f(x) = \sec x$ and write down the definition of this function.

   (b) Let $g(x) = \arccos x$ be the inverse of the cosine function. Is $f(x) = g(x)$?

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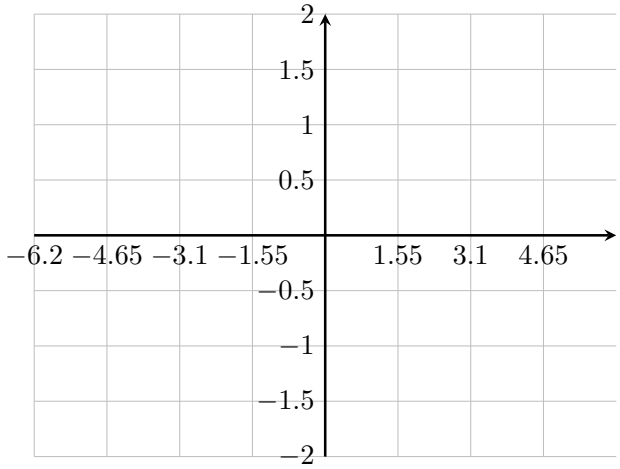
   (a) Draw the graph of the function $f(x) = \sec x$ and write down the definition of this function.

   (b) Let $g(x) = \arccos x$ be the inverse of the cosine function. Is $f(x) = g(x)$?

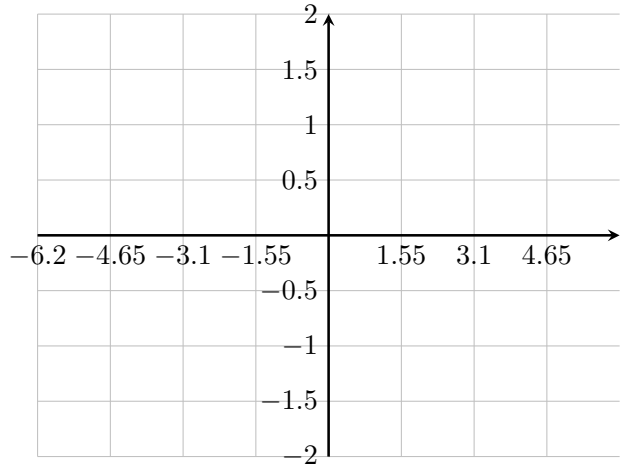
We will be exploring the relationship between the cosine function and the secant function. Let $f(x) = \cos x$ and $g(x) = \sec x = \frac{1}{\cos x}$.

1. Draw the graphs of $f(x)$ and $g(x)$ in the axes below. Write down the definition of $g(x)$ as a piecewise function.

Graph of f

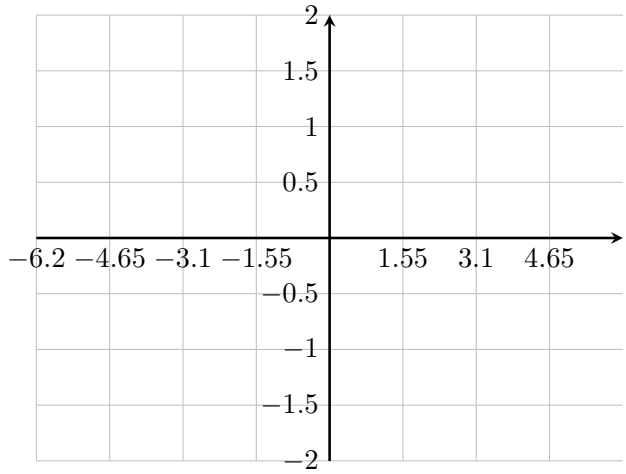


Graph of g



2. Is $g(x) = f^{-1}(x)$? If not, draw the graph of an inverse function for $f(x) = \cos x$ and give a definition for the function.

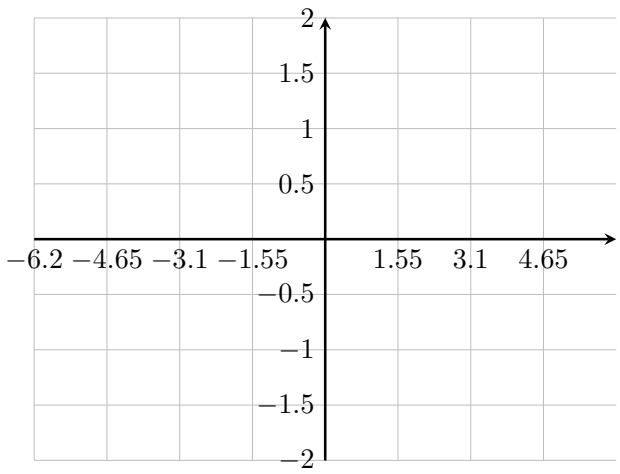
Graph of f^{-1}



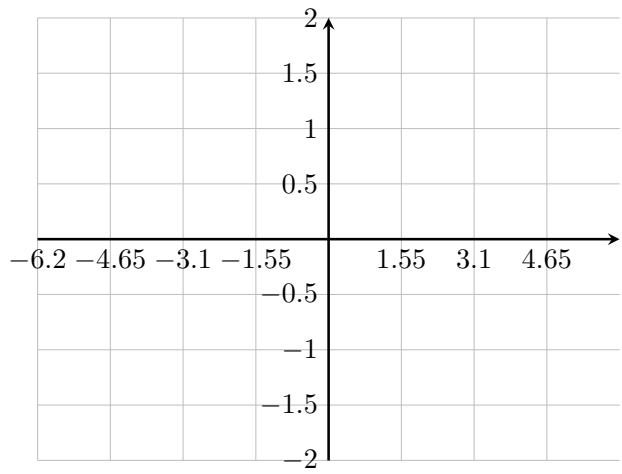
3. The canonical inverse function for $\cos x$ is also known as the arccosine and outputs values between 0 and π. Draw the graph of $h(x) = \arccos x$. How does it compare to the graphs of the inverses drawn in the previous part?

4. Draw the graphs of $f(h(x))$ and $h(f(x))$. Are these graphs the same, why or why not?

Graph of $f \circ h$



Graph of $h \circ f$



5. How would your results extend to the other trigonometric functions $\sin x$ and $\tan x$?

This Active Review is meant as a single-session plan emphasizing the importance of mathematical communication through review materials. The student learning objectives of this Active Review are:

- students can identify and solve algebraic problems involving polynomials, trigonometric functions, exponentials, logarithms, and their function properties.
- students can clearly explain in their own words and diagrams to their peers their solution to a mathematical problem.

The instructor should divide students into groups of 3 and assign problems to students within those groups. The students are then to individually solve the assigned problems and write up a solution. Afterwards, students are to share the solution of the group member to their left to the group. Once all groups have finished sharing their solutions, the instructor should begin a classroom reflection on the activity. The discussion should be led to emphasize the importance of good mathematical communication to others and in solution responses.

- **Solve & Share** (20-30 min):
  - Students are in groups of 3, which can be labeled to facilitate group passing later.
  - Students are assigned a problem and are to solve their assigned problems individually. (10 min)
  - Each group aggregates their solutions and passes it to the next group.
  - Students presents a solution of a problem they did not solve from the solutions passed to their group. (10 min)
  - If there is time, students may grade each solution out of 10 possible points.
- **Classroom Discussion** (15-20 min):
  - What did you notice about the solutions? What was effective in the solution?
  - How did you explain your solution to Problem 1? Were there other explanations or different responses?
  - What was some difficulty/obstacle you needed to overcome in the presentation?
  - What are the goals of writing a mathematical solution?
  - What are some good strategies/checks/heuristics of writing a good solution?

1) Solve for the equation of line parallel to $4x - 3y = 6$ that passes through the origin.

2) Solve for the value(s) of x that satisfy the equation: $e^{\cos^2 x} + e^{-\sin^2 x} = \dfrac{x}{e^{\sin^2 x}}$.

3) If $f(x) = x^2 - 2$, what is the value of $f(f(f(0)))$?