1. The goal of this first part is to determine the formal definition of continuity.

   (a) Based on the pre-class activity, what relationship have you observed between the notions of continuity at a point a, limit of the function at a and value of the function at a?

   (b) Based on that, what "should" the formal definition of continuity be?

   (c) What are the three implications of this definition?

2. (a) Which of the following functions are continuous? Drawing their graphs can be helpful. (for $g(x)$, it may be helpful to factor the numerator).

$$f(x) = x + 1, \qquad g(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}, \qquad h(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}, \qquad k(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases},$$

$$l(x) = \frac{1}{x}, \qquad m(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases},$$

(b) Looking at the discontinuities of the previous functions, what "types" of discontinuities do you see? How can you relate these discontinuities with the definition?

3. (a) Consider the function $f(x) = \begin{cases} 1+x & \text{if } x < 2 \\ x^2+3 & \text{if } x \geqslant 2 \end{cases}$. Prove that there is a point where it is discontinuous.

(b) By what constant should you replace the "1" in the $1+x$ part of the definition of $f(x)$ to make the function continuous?

(c) Where is $g(x) = \begin{cases} x^2+3 & \text{if } x \leqslant 0 \\ 1+\sin(3x^2) & \text{if } x > 0 \end{cases}$ discontinuous?

4. (a) Look at the functions $f(x)$ and $g(x)$ on the previous page. For what value(s) of x do we have $f(x) = 4$? And for what value(s) of x do we have $g(x) = 2.5$?

   (b) Would this also happen with the function you wrote down in 3b? Why or why not?

5. Here is an application of the Intermediate Value Theorem (IVT).

   (a) Does the equation $x^3 + 2x^2 - x = 1$ have a solution between $x = 0$ and $x = 1$.

   (b) Show that the equation $-x^2 + 6x - 7 = 0$ has a solution between $x = 0$ and $x = 5$.