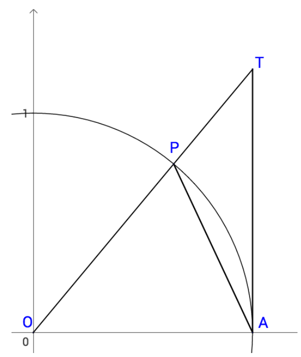
1. Consider the function $f(x) = \begin{cases} x & \text{if } x = \frac{1}{n}, \text{ for } n \text{ non-zero integers,} \\ 0 & \text{if } x \neq \frac{1}{n}, \text{ for } n \text{ non-zero integers.} \end{cases}$ Draw the graph of the function.

   (a) Compute $\lim_{x \to \frac{1}{3}} f(x)$.

   (b) Compute $\lim_{x \to \frac{2}{5}} f(x)$. Compare with your previous answer.

   (c) Compute $\lim_{x \to 0} f(x)$. Justify your answer.

2. The goal of this section is to compute the limit $\lim_{t \to 0} \dfrac{\sin t}{t}$. In order to do so, we will use a geometrical argument (and the figure below).



(a) The figure above represents the unit circle and a given angle t.
Determine the areas of: i) the triangle OPA, ii) the area sector OPA, and iii) the triangle OTA.

(b) "Rank" them in increasing orders (i.e. area 1 $<$ area 2 $<$ area 3). Then multiply the inequalities by $\frac{2}{\sin t}$.

(c) Finally take the reciprocals. What do you get? What can you conclude about $\lim_{t \to 0} \dfrac{\sin t}{t}$? What limit laws or theorem have you used?

(d) Using the previous result. Compute the limits:

i. $\lim_{x\to 0} \frac{\sin(x^2)}{x^2}$ ii. $\lim_{x\to 0} \frac{\sin(2x)}{x}$ iii. $\lim_{x\to 0} \frac{\sin(x^2)}{x}$

3. Find the value of the constant k for the following limits to exist.

(a) $\lim_{x\to 5} \frac{x^2 - k^2}{x - 5}$,

(b) $\lim_{x\to -2} \frac{x^2 + 4x + k}{x + 2}$,

(c) $\lim_{x\to 1} \frac{\sqrt{x^2 + k} - 4}{x - 1}$.