In class we will focus on how to "algebraically" compute limits. In this activity, we will look at some functions and computations that we will use in class.

1. Consider the function $f(x) = \dfrac{x^2 - 2x - 3}{x - 3}$. Our goal is to compute $\lim_{x \to 3} f(x)$.

   (a) What is the domain of $f(x)$? What is $f(3)$?

   (b) Can we simplify this fraction (for example by factoring the numerator)? What do you get?

   (c) Let us call this new function $g(x)$. What is $g(3)$? Do we have $f(x) = g(x)$? Why or why not?

   (d) Draw the graphs of $f(x)$ and $g(x)$. What are the similarities and what are the differences? (Warning: if you use a graphing software to help you, be aware of their tendency to "fill holes").

(e) Keeping in mind what we have just observed about $f(x)$ and $g(x)$, what is the relationship between $\lim_{x\to 3} f(x)$ and $\lim_{x\to 3} g(x)$? Why is that so?

2. Let us now look at the function $f(x) = \dfrac{\sqrt{x^2+25}-5}{x^2}$. Our goal will be to compute $\lim_{x\to 0} f(x)$.

   (a) What is the domain of this function? What is $f(0)$?

   (b) Our goal is now to "simplify" this expression. What do you get if you multiply by the conjugate (if you don't know what "multiplying by the conjugate" means, look it up in the textbook or online, for example here: https://youtu.be/WVj284EvgBI).

   (c) Let us call this new expression $g(x)$. What is $g(0)$? Do we have $f(x) = g(x)$? Why or why not?

(d) Draw the graphs of $f(x)$ and $g(x)$. How similar or dissimilar are they? What is the implication of this about $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$?

(e) Something we will discuss in class: why do think in 1. we looked at the limit as x goes to 3 and not as x goes to 0 or 1?