1. What are the 3 or 4 main elements you retain from the pre-class activity?

Let us keep in mind this idea of having a pre-determined level of precision around *one* value. We will investigate a few more examples and then will get to the definition of a limit.

2. Consider the following functions:

$$f(x) = \frac{x}{|x|}, \qquad g(x) = \begin{cases} 1 & \text{if } x = \frac{1}{n}, \text{ for } n \text{ non-zero integers,} \\ 0 & \text{if } x \neq \frac{1}{n}, \text{ for } n \text{ non-zero integers.} \end{cases}$$

(a) What are the domains of definition of these two functions? Sketch their graphs.

(b) Concerning $f(x)$, what value(s) does the function go to if we take points close to $x = 0$?

(c) Can we take a pre-determined level of precision and then find an interval around 0 such that we can assure the function will be in that pre-determined range?

Key point(s) of this example

(d) Let us now look at $g(x)$. What value(s) does the function go to if we take points close to $x = 0$?

(e) Can we take a pre-determined level of precision and then find an interval around 0 such that we can assure the function will be in that pre-determined range?

Key point(s) of this example

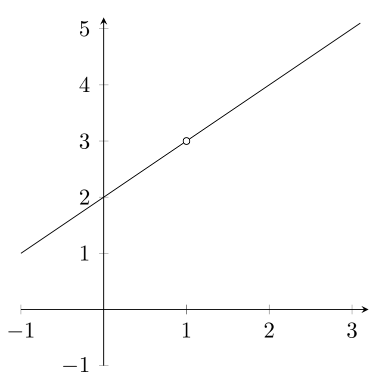
Limits

We can think of the *limit of* $f(x)$ *as* x *approaches* a in the following way: choose *any* pre-determined level of precision. Then the limit $\lim_{x\to a} f(x)$ equals L if we can find an interval around a, such that for any x different from a in this interval, the function $f(x)$ approaches L with the desired pre-determined level of precision. We use the notation $\lim_{x\to a} f(x)$ for the limit of $f(x)$ as x approaches a.

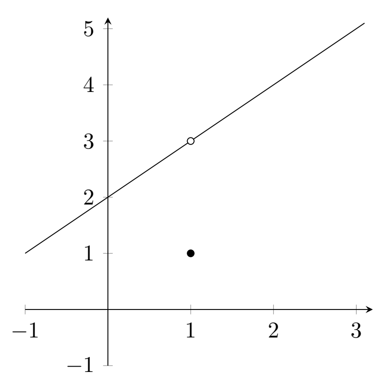
3. What are the key elements of this definition?

4. Let us now look at what this means graphically. For each of the following example, determine $\lim_{x\to 1} f(x)$ as well as $f(1)$.

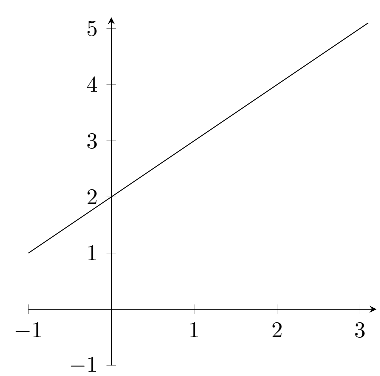
a)



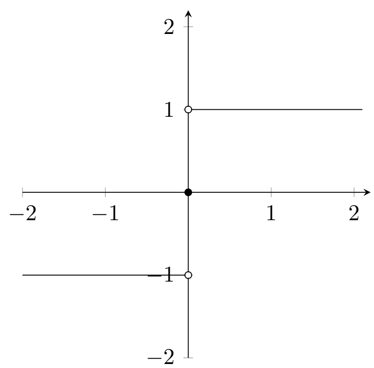
b)

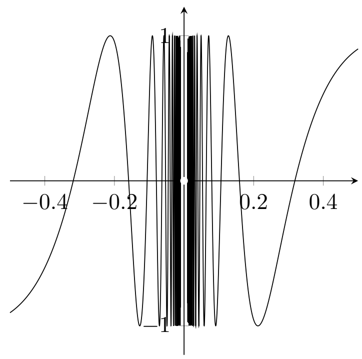


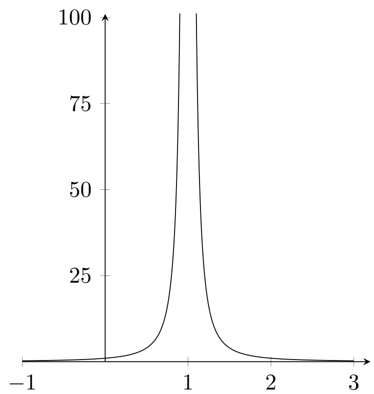
c)



5. Let us now consider the following functions. For i) and ii), determine if the limit $\lim_{x\to 0} f(x)$ exists and if so, what it is. Determine also $f(0)$. For iii), same questions but for $\lim_{x\to 1} f(x)$ and $f(1)$.

i) 

ii) 

iii) 

The first example above motivates the following definition of **one-sided limits**.
For *any* pre-determined level of precision we choose,

- the *limit of $f(x)$ as x approaches a from the left*, written $\lim_{x\to a^-} f(x)$, is the number L that the function $f(x)$ approaches when x is in an open interval (b, a) with $b < a$, in other words with x strictly *smaller* than a.
- the *limit of $f(x)$ as x approaches a from the right*, written $\lim_{x\to a^+} f(x)$, is the number L that the function $f(x)$ approaches when x is in an open interval (a, b) with $a < b$, in other words with x strictly *greater* than a.