Continuity (2.5 in Thomas)

Expected Skills.
At the end of this section, students will be able to:

- explain with their own words the definition of the continuity of a function,
- list the different types of discontinuity shown in the textbook,
- list examples of continuous and discontinuous functions,
- prove the continuity of a function at a given point using the definition and/or the theorems,
- use the Intermediate Value Theorem (IVT) to show the existence of solutions to given equations. This includes being able to:
  - state the theorem,
  - recognize when we can apply the theorem,
  - follow a procedure to show the existence of a root using the theorem.

Pre-Class Activity (ch2-limits-2-continuity-1-pc)**.** in this activity we ask the students look at graphs of functions and determine which ones are continuous (using an intuitive definition of continuity). The goal is to have the students think about their "intuitive" definition of continuity on the one hand and the relationship between the limit and value of the function at that point on the other hand. Looking at that, we will then follow up in class by asking what the formal definition of continuity "should" be.
We also suggest the idea of left-hand and right-hand continuity.

Worksheet (ch2-limits-2-continuity-2-ws)**.** The goal of this worksheet is multifold.
First, using the pre-class activity, we introduce the definition of continuity, ask the students to think about what this definition means and how it is related to "the intuitive definition" of continuity.
We then have the students prove the continuity or discontinuity of functions using the definition.
Then we look at exercises where the students have to properly glue functions together.
Finally, using the previous part we introduce the Intermediate Value Theorem and some applications.

Supplemental Activity (ch2-limits-2-continuity-3-sup-dctsparameter)**.** After this activity, students will be able to

- explain with their own words and diagrams the definition of a function being continuous at a point
- identify points of discontinuity from a function's graph
- give examples of continuous and discontinuous functions
- prove that a function is continuous at a point

The activity has the students investigate instances of continuity and discontinuity for two families of functions parameterized by constants a and b. For each family of functions, students are asked to draw a graph for a

given parameter, prove that the function for the given parameter is discontinuous, and determine a parameter value for which the function is continuous and/or invertible.

It is suggested that instructors assign each student a function (f or g) and have them pair with a student with the same function to work through the four parts of the problem. Students should then form new pairs with students of different functions to share their results to their partner. The instructor can conclude the activity by having 2 students go to the board to present their findings and hold a classroom discussion.

Supplemental Activity (ch2-limits-2-continuity-3-sup-dctstype)**.** After this activity, students will be able to

- explain with their own words and diagrams the definition of a function being continuous at a point
- identify points of discontinuity from a function's graph
- give examples of continuous and discontinuous functions
- prove that a function is continuous at a point

The activity has the students investigate 6 functions for continuity and classify any discontinuities that they find. Students are then asked to formalize their findings using limits. Finally students are asked to determine ways to adjust the given functions to recover continuity.

It is suggested that instructors periodically debrief the classroom after each group has completed a problem. Problem 4 can be used as a way to conclude the activity by having the classroom brainstorm ways to change the given functions so that their altered forms would be continuous.

Supplemental Activity (ch2-limits-2-continuity-3-sup-ivt)**.** After this activity, students will be able to

- explain the IVT in their own words and figures
- determine where the IVT can be applied
- apply the IVT to determine roots of functions

The activity has the students address the Intermediate Value Theorem by considering the logic of the statement, expressing the statement in their own words, and applying it to find an intersection point of two graphs.

It is suggested that instructors write the IVT in formal terms on the board to introduce the activity. Students should be working through Problem 1 in groups. For Problem 2, instructors should make it clear that students should take the time to write their individual response. Students then will have to share/explain their neighbor's response to the group.