1. (a) Based on your pre-class activity reflections, what should the definition of $\lim_{x\to\infty} f(x)$ "look like"?

   Limit at infinity
   We can think of the limit of $f(x)$ as x approaches infinity *in the following way: choose* any *pre-determined level of precision. Then the limit* $\lim_{x\to\infty} f(x)$ *equals* L *if we can find a number* $N > 0$ *such that for any* $x > N$ *the function* $f(x)$ *approaches* L *with the desired pre-determined level of precision.*
   To make it short, we use the notation $\lim_{x\to a} f(x)$ *for the limit of* $f(x)$ *as* x *approaches* a*.*

   (b) And what about the definition of $\lim_{x\to-\infty} f(x)$?

   Just replace $x > N$ *by* $x < -N$*.*

   (c) What are the key elements of this definition?
   This somewhat *redundant with what was done on the first worksheet on limits but as it is important and a source of misunderstanding for the students, it seems a good idea to repeat it here.*

   - *the limit is a* single number*,*
   - *there are cases where the limit does not exist, have we already encountered such cases?*
   - *we can make the level of prevision as small as we want. Once we have done that, what we need to do if find the number* N *for which all values of* $f(x)$ *are inside this level of precision for* $x > N$ *(resp.* $x < -N$*),*
   - *Directly related to this definition (but part of it): be careful that* ∞ *and* $-\infty$ *are not numbers! We cannot just plug them into the function!*

2. Compute the limits as x goes to infinity and negative infinity for the following functions:

   (a) $f(x) = \dfrac{2x^4 + 2x^2 - 3}{3x^4 + x^3 - 2x^2}$

   Maybe show an example on the blackboard before starting with this example.

   (b) $g(x) = \dfrac{2x^5 + 2x^2 - 3}{3x^4 + x^3 - 2x^2}$

   (c) $h(x) = \dfrac{2x^4 + 2x^2 - 3}{3x^6 + x^3 - 2x^2}$

   What is the general rule one sees from these three cases?

   (d) $j(x) = \dfrac{\sqrt{2x^6 + 2x^2 - 3}}{3x^3 - 2x^2}$

3. Compute the horizontal asymptotes of the following functions:

   (a) $f(x) = \dfrac{\sqrt[3]{x} - 4x + 7}{3x + x^{2/3} - 1}$

   (b) $g(x) = \dfrac{1}{x}\sin x$ (compare your answer with $\lim_{x\to\infty} \sin(\pi x)$ that you computed in the pre-class activity).

   Underline that to compute $\lim_{x\to\infty} \frac{1}{x}\sin x$, one MUST use the squeeze theorem.
   Also underline that this example shows that a function CAN CROSS its horizontal asymptote (maybe show the graph).
   Finally it illustrates that we can have $f(a)$ closer to $\lim_{x\to\infty} f(x)$ than $f(b)$ even though $a < b$, e.g. $f(100) = 0$ whereas $f(100.5) = 1$.

   (c) $f(x) = \dfrac{-x^2 + 5x - 1}{2x + 3}$

   Here the limits do not exists as this function has in fact an oblique asymptote (which we don't explicitly cover).
   One can also point out that the limit goes to $-\infty$ when $x \to \infty$ and to ∞ when $x \to -\infty$.

4. Compute the following limits:

   (a) $\lim_{x\to\infty}(x^2 - x)$

   Underline that we don't have $\infty - \infty = 0$!

   (b) $\lim_{x\to\infty}(\sqrt{x^2+2} - x)$

   (c) $\lim_{x\to\infty}(\sqrt{x^2+ax} - \sqrt{x^2+bx})$, where a and b are constants

   Based on what you have just computed, can you find a limit for which "$\infty - \infty$" gives 3? What about 10? What about any other number?

   The goal here is to show to the students that $\infty - \infty$ can actually be equal to any number.

5. Determine the vertical and horizontal asymptotes of the following functions:

   (a) $f(x) = \dfrac{2x^2 + 1}{3x - 5}$

   (b) $f(x) = \dfrac{2x^2 + 5}{x^2 - 5x}$

   This example also shows clearly that a horizontal asymptote can be crossed by the function (to illustrate this show the graph).