Here we will look at the concept of a limit from a "general point of view". In class we will see more precisely what the definition of a limit is and what its various subtleties are.

Consider the following functions:

$$f(x) = \frac{x^2 + x - 2}{x - 1}, \qquad g(x) = \frac{\sqrt{x^2 + 4} - 2}{x^2}, \qquad h(x) = \frac{\sin x}{x}.$$

1. For each function, determine its domain and list the point at which it is *not* defined.

2. If a function $f(x)$ is undefined at a point $x = a$, it means that we cannot plug in this point into the function. In other words, we cannot compute $f(a)$. Nevertheless, we *can* compute the value of the function for points that are close (and even very close) to a.

   For each function above, compute the value of the function for points that very close to the point where it is undefined. In each case, what do you notice? Do you see a "pattern"?

A conclusion we can draw from the previous points is that even though these functions are not defined everywhere, if we plug in points that are close to the point where they are undefined (in this case $x = 1$ for $f(x)$ and $x = 0$ for both $g(x)$ and $h(x)$), we see that these functions do go to a specific value.
We will investigate this aspect more precisely now.

3. Use Geogebra or Desmos to graph the above functions. Do the graphs correspond to your computations? Are the graphs correct (warning: graphing softwares have the bad tendency of "filling" holes)?

4. For the function $f(x)$ there is no point a such that $f(a) = 3$ (in other words, $f(x)$ never gives the value 3). But then, one important question is the following: even though $f(x)$ never gives the value 3, can the function get as close as we want to 3? Or put differently, can we pre-determine a level of precision for which the function will "attain" 3?
Let's say we want $f(x)$ to "attain" the value 3 with a level of precision of 0.1. Find several values of x for which $2.9 < f(x) < 3.1$ (i.e. for which $f(x)$ is between 2.9 and 3.1).

5. Can you find an interval around $x = 1$ for which you can assure that $f(x)$ is going to be between 2.9 and 3.1?

6. Let us now do the same with a level of precision of 0.01, in other words, we now want $f(x)$ to be between 2.99 and 3.01. Can you find an interval around $x = 1$ for which you can assure that $f(x)$ is going to be between 2.99 and 3.01? Does the interval you found at the previous point also work here?

7. Let us finally inquire the same question with the function $h(x) = \dfrac{\sin x}{x}$. By computing the value of $h(x)$ for points that are close to $x = 0$, we see that $h(x)$ goes to 1. Let's say we want a precision of 0.1, i.e. we want $h(x)$ to be between 0.9 and 1.1.
Try to find an interval around $x = 0$ for which $h(x)$ is in that range. Does the interval you found for $f(x)$ for a precision of 0.1 work here? Would a bigger interval also work?