1. Consider the function $f(x) = \begin{cases} x & \text{if } x = \frac{1}{n}, \text{ for } n \text{ non-zero integers,} \\ 0 & \text{if } x \neq \frac{1}{n}, \text{ for } n \text{ non-zero integers.} \end{cases}$ Draw the graph of the function.

   The goal here is to have the students think again about the definition of a limit and also about the difference between a limit and the value of the function.

   (a) Compute $\lim_{x \to \frac{1}{3}} f(x)$.

   Example with $\lim_{x\to a} f(x) \neq f(a)$.

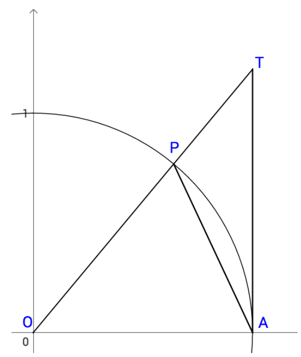
   (b) Compute $\lim_{x \to \frac{2}{5}} f(x)$. Compare with your previous answer.

   Example with $\lim_{x\to a} f(x) = f(a)$.

   (c) Compute $\lim_{x \to 0} f(x)$. Justify your answer.

   $\lim_{x\to 0} f(x) = 0$.

2. The goal of this section is to compute the limit $\lim_{t \to 0} \dfrac{\sin t}{t}$. In order to do so, we will use a geometrical argument (and the figure below).



(a) The figure above represents the unit circle and a given angle t.
Determine the areas of: i) the triangle OPA, ii) the area sector OPA, and iii) the triangle OTA.

The goal of this exercise is to have the students compute the limit $\lim_{t\to 0} \frac{\sin t}{t}$*.*
Looking at the areas of the triangles will also prepare the them for related rates/optimization problems. At the end we want the students to know the result but not to remember how we find the result.

(b) "Rank" them in increasing orders (i.e. area 1 $<$ area 2 $<$ area 3). Then multiply the inequalities by $\frac{2}{\sin t}$.

Underline that here we can multiply by $2/\sin t$ *without changing the inequalities because* $\sin t$ *is positive.*

(c) Finally take the reciprocals. What do you get? What can you conclude about $\lim_{t \to 0} \dfrac{\sin t}{t}$? What limit laws or theorem have you used?

Be sure to underline the fact that we need to use the Sandwich Theorem here (p. 70 in the textbook).

A question that will undoubtedly come up from the students is: "Do we have to know this?".
They don't need to know how to redo the above computations BUT they need to know the result and be able to use it (such as in the following exercise).

These computations are done on pp. 86-7 in the textbook.

(d) Using the previous result. Compute the limits:

$$\text{i. } \lim_{x\to 0} \frac{\sin(x^2)}{x^2} \qquad \text{ii. } \lim_{x\to 0} \frac{\sin(2x)}{x} \qquad \text{iii. } \lim_{x\to 0} \frac{\sin(x^2)}{x}$$

Part ii. can be done either by the change of variable $2x = t$ or using the trig identity $\sin(2x) = 2\sin x \cos x$ (the former is somewhat favored as it doesn't imply knowing any "extra material" such as this identity).

3. Find the value of the constant k for the following limits to exist.
The goal of this exercise to recap the main points and techniques we have used computing limits.

(a) $\displaystyle\lim_{x\to 5} \frac{x^2 - k^2}{x - 5}$,

(b) $\displaystyle\lim_{x\to -2} \frac{x^2 + 4x + k}{x + 2}$,

(c) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x^2 + k} - 4}{x - 1}$.