Limits Involving Infinity and Asymptotes (2.6 in Thomas)

Expected Skills.

At the end of this section, students will be able to:

- compute limits at infinity using limit laws and methods studied in class,
- explain in words what an asymptote is,
- compute the equation of a horizontal or vertical asymptote.

Pre-Class Activity (ch2-limits-3-asymptotes-1-pc)**.** The goal of the pre-class activity is to have the students reflect about what the definition of $\lim_{x\to\infty} f(x)$.
We then give them two examples (one where the limit exits and one where it doesn't) to "test" their definition.
Worksheet (ch2-limits-3-asymptotes-2-ws)**.** This activity follows the following plan:

1. have the students come up with the definition of limit at infinity (based on the pre-class activity),
2. compute typical limits of rational functions,
3. compute horizontal asymptotes of functions,
4. compute limits where one gets "$\infty - \infty$"
5. compute vertical and horizontal asymptotes of functions. Note that oblique asymptotes have been "left out".

Supplemental Activity (ch2-limits-3-asymptotes-3-sup-asymptotes)**.** After this activity, students will be able to

- explain in their own words and diagrams what $\lim_{x\to\infty} f(x)$ means
- compute limits of rational functions
- compute horizontal asymptotes and identity vertical asymptotes of functions

The activity begins by asking the students to explain the distinction and connection between $\lim_{x\to\infty} f(x)$ and unbounded limits. The activity then asks the students to connect these limit notions to the graphical features of asymptotes. Finally the activity challenges the students to formulate a procedure to determine horizontal and vertical asymptotes and test this methodology against several functions.

It is suggested that instructors debrief the entire classroom after each group of students has addressed each of the first two problems. Students should be encouraged to collaboratively design a methodology for their group to determine asymptotes. Students should be given enough time to test their methodology against at least the first two given functions. Instructors can conclude the activity with a final classroom debrief where students from different groups present their methodologies on different functions.