

- 1) The goal of this first part is to make a first “educated guess” of what the derivative of $f(x) = \cos x$ is.
 - (a) Draw the graph of $\cos x$ on the interval $[-2\pi, 2\pi]$ (do that on page 2, leave some room below your graph to draw another graph).
 - (b) On the graph you have drawn, for what point(s) can you determine the slope of the tangent line without any doubt?

- (c) What do you think is the value of the derivative at $x = \pi/2$? Confirm your guess by computing $f'(\pi/2)$ (i.e. the derivative of $\cos x$ at $x = \pi/2$) using the definition of the derivative.

- (d) Using the information above as well as the shape of the graph of $\cos x$, draw the graph of $f'(x) = (\sin x)'$.

Draw your graphs below.

2) Let us now prove (or disprove) that your guess is correct by computing the actual derivative of $\cos x$.

(a) Writing the $\lim_{h \rightarrow 0}$ definition of the derivative, write down the definition of $f'(x) = (\cos x)'$.

(b) Let us compute this limit.

To that end, we will need the trigonometric identity $\cos(u + v) = \cos u \cos v - \sin u \sin v$. We will moreover need the limits $\lim_{h \rightarrow 0} \frac{\sin h}{h}$ and $\lim_{h \rightarrow 0} \frac{\cos h - 1}{h}$.

(c) So, the derivative of $\cos x$ is ¹:

$(\cos x)' =$

¹The goal of this exercise was to have you compute the derivative of the $\cos x$ using the definition. While you need to know what the derivative of $\cos x$ is, you don't need by heart how to compute it, i.e. you don't need to learn by heart the above computations.