1. The goal of this first part is to compute the derivative of the inverse function $f^{-1}(x)$ of a function $f(x)$ (and this way prove the formula).

   (a) Write down the chain rule.

   The goal of this exercise is to have the students prove the formula for $(f^{-1}(x))'$.

   (b) Let $f(x)$ be a function and $f^{-1}(x)$ its inverse.
   What is $f(f^{-1}(x))$ equal to?

   (c) Take the equation from the preceding point and differentiate both sides with respect to x.
   What do you get? Have you used the chain rule? If so, where?

   (d) Use what you have found to determine the derivative $(f^{-1}(x))'$.

   (e) What do we assume when using the previous formula?

   The existence of $f'(x)$ and $f'(x) \neq 0$.

Let us now apply what we have just seen.

2. We want to compute the derivative of $f(x) = \ln x$. How can we use the preceding formula?

3. Let us now focus on the derivative of $f(x) = a^x$.
   How can we rewrite this function in terms of e^x and $\ln x$?
   Then what is $(a^x)'$?

   One should introduce here the formula for $(\log_a)'$.

4. Compute the derivatives of:

   i) $f(x) = \ln(\sin x)$

   This exercise and the next one are a good opportunity to talk about the "limitations" of the formula of the derivative. I.e. when is the $\ln$ *defined.*

   ii) $g(x) = \dfrac{1}{\ln 3x}$

   iii) $h(t) = 3^{t^2}$

   iv) $f(z) = \log_5 e^z$

v) $g(t) = \ln(e^{3t} \sin^2 t)$

vi) $h(x) = \log_2(2^x e^2)$

vii) What is wrong with the following statement?
The derivative of $f(x) = \ln(\ln x))$ is $f'(x) = \frac{1}{x} \ln x + \ln x \frac{1}{x} = \frac{2 \ln x}{x}$.

One could also ask: how many points should a student who gives this answer get?