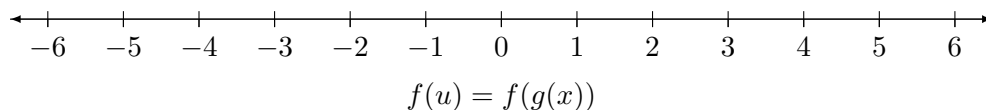
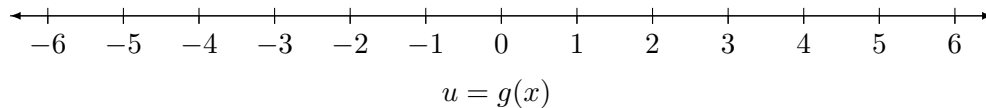
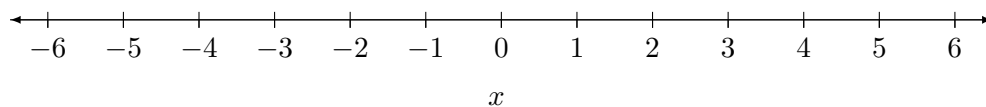


The goal of this exercise is to see why  $f(g(x))' = f'(g(x))g'(x)$ .

- 1) A friend of yours claims that the chain rule is  $f(g(x))' = f'(x)g'(x)$ . Another friend claims that the chain rule is  $f(g(x))' = f'(g'(x))$ . Compute the derivative of  $f(x) = x^6$  using your each friend's differentiation rule, what do you get?

- 2) Why are these alternative answers incorrect? Find a few other functions that can serve as counterexamples to your friends' claims.

- 3) Let us consider a graphical representation of the situation using three number lines. How can you use these number lines to explain the chain rule?



- 4) Let  $p(x)$  be  $p(x) = f(x)g(x)$  with  $f(x)$  and  $g(x)$  differentiable. Write down the limit definition of  $p'(x)$ . Explain in words what this limit represent. (Hint: Try using a rectangle to help you!)