

1. Using the differentiation rules, compute the derivatives of the following functions:

(a)  $f(x) = 3x^4 - 2x^3 + 2x - 5$

*In this first part we ask students to use the differentiation rules for cases of increasing complexity.*

*The goal in this part is “just” to have them use the rules.*

*Adding one or two challenge exercises can be good here as some students will already know this and will finish these exercises very quickly.*

(b)  $f(x) = ax^3 + bx^2 + cx + d$  where  $a, b, c$  and  $d$  are constants

(c)  $f(x) = \frac{x-2}{x+2}$

(d)  $f(t) = \frac{3t+1}{t^2+t-2}$

(e)  $f(x) = 2x^3e^x$

2. Compute the derivatives of the following functions using each time two different methods, once using the quotient rule and once without using the quotient rule. In each case, which method is easier?

*Here we want the students to compare ways of computing derivatives.*

(a)  $f(t) = \frac{t^3 + 5t^2 - 2t}{t}$  and  $g(t) = \frac{t}{t^3 + 5t^2 - 2t}$

(b)  $f(x) = \frac{3}{x^4}$  and  $g(x) = \frac{x^2 + x + 1}{\sqrt{x}}$

- (c) Based on the calculation you have done, when is it easier *not* to use the quotient rule?

3. The goal of this exercise is to see why  $(f(x)g(x))' \neq f'(x)g'(x)$ .
- (a) A friend of yours claims (contrary to what the textbook says) that the product rule *is*  $(f(x)g(x))' = f'(x)g'(x)$ . You want to show him that his claim is wrong. If you compute the derivative of  $f(x) = x^2$  using your friend's differentiation rule, what do you get?

*the next parts of this exercise are probably not "worth" the time they take. One could just stop here and still get the message across.*

- (b) Why is this answer incorrect? Give a graphical argument.

*Give some directions to the students here. We are looking for something similar to the figure on p. 139.*

- (c) Find as many other arguments you could give to your friend to show him his claim is wrong (e.g. by computing the derivative of  $x^2$  using other differentiation rules and see that the answers don't match with the answer obtained above).

*E.g. one can use the definition of the derivative, or with the power rule. There are probably also other ways of showing that the above computation is wrong.*

- (d) Let  $p(x)$  be  $p(x) = f(x)g(x)$  with  $f(x)$  and  $g(x)$  differentiable.

Write down the limit definition of  $p'(x)$ . Explain in words what this limit represent.

- (e) Let us consider a graphical representation of the situation. Let  $f(x)$  be the width of a rectangle and  $g(x)$  its height. Let us suppose that both  $f(x)$  and  $g(x)$  are positive and growing with time. How is this picture you have drawn connected to the limit you have written above? (You can also compare to what you would get using  $f'(x)g'(x)$ ).