

In class, we will inquire the following question: giving a function $f(x)$ and its derivative $f'(x)$, can we compute the derivative $(f^{-1}(x))'$ of the inverse function? And if so, what is the relationship between the two derivatives $f'(x)$ and $(f^{-1}(x))'$? (This result will enable us to compute the derivatives of functions such as $\log_a(x)$, $\ln x$ or $\arctan x$.)

The goal here is to develop an intuition of what the answers to the above questions may be.

1. Let us consider the function $f(x) = x^2 + 1$. What is its inverse function $f^{-1}(x)$?

2. Is it always true? Or more precisely, on what interval(s) is this true?

3. Draw the graph of $f(x)$. Then, draw the graph of $f^{-1}(x)$.

How do you get the graph of the inverse $f^{-1}(x)$ from the graph of $f(x)$?

5. Draw the tangent lines at $(x_0, f(x_0))$ and $(f^{-1}(x_0), x_0)$. What do you notice about the slopes of these two tangent lines (you can approximately measure them)?