

This activity will focus on special differentiation techniques that arise from various combinations from function composition, the chain rule, and implicit differentiation.

- 1) How does function composition, the chain rule, and implicit differentiation relate to the derivative of the inverse of a function?

- 2) Let $f(x) = e^{x^3}$. Compute the inverse function f^{-1} and its derivative of the inverse f^{-1} . Verify your derivative by computing the derivative in a different way.

- 3) Recall that the canonical inverse function for sine, cosine, and tangent functions are arcsine, arccosine, and arctangent.
- a) Without using the derivative of the inverse function, compute the derivative $(\arctan x)'$. Express your derivative without any trigonometric functions – a triangle diagram may be useful here.
- b) Using the same procedure, compute the derivative $(\arcsin x)'$.
- c) Using the same procedure, compute the derivative $(\arccos x)'$.

4) To compute the derivative of an inverse function, we used a function composition with its corresponding inverse. Note that as long as the outer function of the composition is differentiable, we can use the chain rule and implicit differentiation to ease the algebraic burden of computing the derivative of some functions. Logarithmic differentiation is one such technique using the logarithm as the outer function in the composition.

a) Consider the function $f(x) = \frac{x^2+x+1}{2x^3-x^2+3x-8}$. Use the expression $\ln(f(x))$ to compute the derivative $f'(x)$.

- b) Consider the quotient function $q(x) = \frac{f(x)}{g(x)}$. Compute the derivative $q'(x)$. What assumptions must you make to guarantee this derivative?