i) Compute the derivatives of the following functions (using the appropriate rules). Here you do not need to simplify your answer.

(a) $f(t) = \cos(t^2)$

(b) $g(x) = \sqrt{2x^3 + 4x + 2}$

(c) $h(z) = 2e^{z^2+4z} + 5z + 3$

(d) $k(x) = \sin(x) \cdot (x^2 + 5x)^{100}$

(e) $f(x) = \sin^2(x) \cdot (x^2 + 5x)^{100}$

(f) $g(t) = \dfrac{e^t + 1}{\sin(t^4)}$

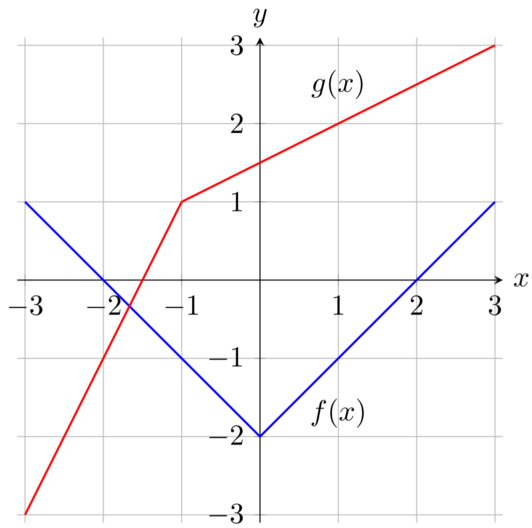
(g) $h(t) = \dfrac{e^{t^3+t} + 1}{\sin(t^4)}$

(h) $k(z) = e^{\cos(z^2)}$

(i) $f(x) = \tan^3\left(\sqrt{x^5 + 2}\right)$

For this exercise, once the students have made their computations, one can also ask them to indicate for each answer which part of the answer correspond to the derivative of what function.

ii) Consider the two functions $f(x)$ and $g(x)$ below.



(a) Let $h(x) = f(g(x))$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

(b) Let $h(x) = g(f(x))$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

(c) Let $h(x) = f(f(x))$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

(d) Let $h(x) = (f(x))^2$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

iii) While revising your course with a friend, this one claims “The chain actually applies all the time!”. What do you think of this claim? To what extent is it correct, or incorrect?

The idea is to have the students realize that the derivative of x is 1 and that we don't write it. In that sense, the chain rule is “always applied”.

Challenge exercises

A. Use the chain rule to find the derivative of $\dfrac{1}{g(x)}$.

B. Use part A., the chain rule and the product rule to prove the quotient rule.