Derivative at a Point & The Derivative as a Function (3.1 & 3.2)

Expected Skills.
At the end of this section, students will be able to:

- explain in words what the definition of the derivative means,
- use the definition of the derivative to compute the derivative of a function,
- use the definition of the derivative to compute the slope and equation of the tangent line at a given point,
- given the graph of a function, qualitatively draw the graph of its derivative, and conversely, given two graphs recognize the graph of a function and of its derivative,
- using the definition, determine on which intervals a function is differentiable and on which it is not. This implies being able to compute one-sided derivatives and be able to determine when it does not exist
- list the cases where a function is not differentiable and draw the corresponding graphs,
- recognize on a graph where a function fails to be differentiable.

Pre-Class Activity (ch3-derivatives-1-derivatives-1-pc)**.** The goal of this activity is to have the students think about the ways to approximate the slope of the tangent line at a point. The goal is to "bring" them to the idea of taking secant lines that "become" tangent.
Note that we ask the students to give *as many ways as possible* to do that. The goal is two fold. First, to avoid that those who already know (and hopefully understand) the derivative just say "take the derivative" or "take the limit". On the other hand, the goal is also to have them think about the limitations of each "type" of approximation and to compare them to one another.

Pre-Class Activity (ch3-derivatives-1-derivatives-4-pc)**.** This second pre-class activity can be used in conjunction or instead of the other one (or it could also be used in class).

This activity is more structured in the sense that it asks students to draw and compute the slopes of a series of secant lines. These lines become closer and closer to the tangent line.
The goal of the last two points is to have the students write down a formula that is close to the one used to compute the derivative. (unfortunately, the textbook uses mainly the one with $f(x_0 + h) - f(x_0)$ and not $f(z) - f(x_0)$).

Worksheet (ch3-derivatives-1-derivatives-2-ws)**.** *This activity is not more related to the pre-class activity a than the pre-class activity b*
The goal of this activity is to first have the student evaluate the derivative of a function using the tangent line of the graph. We then ask them to compute the derivative of the function at a point using the definition of the derivative. Finally, we ask the students to do that for any point.
Here we thus try to "connect" the geometric interpretation of the derivative to its definition. We also want

to have the students see how computing the derivative at one point is essentially the same as computing it at any point.

Worksheet (ch3-derivatives-1-derivatives-3-ws)**.** *This activity is not more related to the pre-class activity b than the pre-class activity a*
In this activity we focus on connecting the graph of a function to the graph of its derivative. Using *contrasting cases* of increasing complexity we ask the students to qualitatively draw the graph of the derivative of a function based on the graph of the function.

I guess that the first case (constant function) may actually be difficult for the student.

Worksheet (ch3-derivatives-1-derivatives-5-ws)**.** The goal of this activity is to help the students see when the derivative fails to exist (cf. Thomas [**?**, p. 129]). To do that we connect to prior knowledge about limits and ask the students in which cases a limit can fail to exist. The cases with the cusp as well as the two cases of discontinuity are probably harder to find as they are "embedded" in other cases.
See next activity for something that focuses more on the "geometry".

Worksheet (ch3-derivatives-1-derivatives-6-ws)**.** The goal of this activity is to help the students see when the derivative fails to exist (cf. Thomas [**?**, p. 129]). Here we focus on developing students' intuition about when a derivative fails to exist by looking at graphs of functions.
Only in the second step, do we make the connection with the limit definition.