

1. (a) Let us start by looking at $\sqrt{10}$. What is a fundamental difference between $\sqrt{9}$ and $\sqrt{10}$?

In one case we know the exact value: $\sqrt{9} = 3$ whereas in the other case we don't have an exact value.

- (b) How could we use what you have done in the pre-class activity to approximate a function?

If nothing comes up, point out step 5., i.e. we know how to compute exact values on the tangent line.

Then introduce the definition of the linearization (or have the students come up with the equation of the tangent line to $f(x)$ at a).

- (c) Approximating $\sqrt{10}$. Keep some space on the right-hand side of the sheet.

The idea here is to do an example on the board, then give a few minutes to the students and to re-read it and write down the different steps involved. We can also then compare the steps with a neighbor.

The steps are:

- i. Determine what the function is.*
- ii. Identify a point a that is "close" to the point we want to approximate and for which we know the exact value.*
- iii. Compute the equation of the tangent line at a .*
- iv. Use the equation of the tangent line to approximate the value of the function by determining the y value of the tangent line at $x = a$.*

2. Approximate $\frac{1}{4.9}$ using an approximate linearization.
Use the steps you have identified in the previous part.

For this activity, one can typically do a Think-Pair-Share: first the students do the exercise alone, then compare with their neighbor, finally the whole class discuss the solution.

If time allows, do the same exercise for another approximation such as $\ln(1.1)$ or $\sin(0.1)$.

3. (a) In the pre-class activity, you looked the tangent lines to \sqrt{x} at both $x = 1$ and $x = 9$. What difference have you noticed?

The tangent line is “closer to the function at 9 than at 1. If no answer comes up, go directly to the computation part.

- (b) Let us approximate $\sqrt{2}$ by using the linearization (i.e. the tangent line). What do you get?

- (c) Using a calculator, compute the error of this approximation (i.e. the difference between this approximation and what you get with your calculator). Also compute the error of the approximation of $\sqrt{10}$ we did before.

- (d) What do you notice? How can we explain this difference? What factors explain this difference?

This points out to the limitation of this method.

The error can become quite big and thus the approximation “bad” depending on the shape of the function around the point on the function we use.

4. We want to compute an approximation of $e^{0.1}$ and $e (= e^1)$ using the tangent line.
- (a) Compute the appropriate linear approximation(s) $L(x)$. What is the function? Around what point(s) can we do that?

- (b) Using this approximation for $e^{0.1}$ and e , are we underestimating or overestimating the actual values of $e^{0.1}$ and e ?

The idea is to look at whether the tangent line are below or above the function. It is a first suggestion about the idea of concavity.

- (c) If we compare the approximations for $e^{0.1}$ and e , which one is closer to the actual values of $e^{0.1}$ and e ? What are your arguments to support your answer?