

1. The goal of this first part is to make a first “educated guess” of what the derivative of $f(x) = \sin x$ is.
 - (a) Draw the graph of $\sin x$ on the interval $[-2\pi, 2\pi]$ (do that on page 4, leave some room below your graph to draw another graph).
 - (b) On the graph you have drawn, for what point(s) can you determine the slope of the tangent line without any doubt?

Where the tangent line is horizontal.

- (c) What do you think is the value of the derivative at $x = 0$?
Confirm your guess by computing $f'(0)$ (i.e. the derivative of $\sin x$ at $x = 0$) using the definition of the derivative.

- (d) Using the information above as well as the shape of the graph of $\sin x$, draw, below the graph of $\sin x$, the graph of $f'(x) = (\sin x)'$.

2. Let us now prove (or disprove) that your guess is correct by computing the actual derivative of $\sin x$.

(a) Writing the $\lim_{h \rightarrow 0}$ definition of the derivative, write down the definition of $f'(x) = (\sin x)'$.

(b) Let us compute this limit.

To that end, we will need the trigonometric identity $\sin(u + v) = \sin u \cos v + \cos u \sin v$. We will moreover need the limits $\lim_{h \rightarrow 0} \frac{\sin h}{h}$ and $\lim_{h \rightarrow 0} \frac{\cos h - 1}{h}$.

If enough time we can ask which limits laws have been used and why we can use them.

(c) So, the derivative of $\sin x$ is ¹:

$$(\sin x)' =$$

¹The goal of this exercise was to have you compute the derivative of the $\sin x$ using the definition. While you need to know what the derivative of $\sin x$ is, you don't need by heart how to compute it, i.e. you don't need to learn by heart the above computations.

3. Similarly to what we have done above, one can prove that $(\cos x)' = -\sin x$.

Using these two derivatives as well as the definition of $\tan x$, compute the derivative $(\tan x)'$.

4. Compute the derivatives of the following functions.

i) $f(t) = \sin t \cos t$

ii) $g(x) = \sin x + e^x \cos x$

iii) $h(z) = 2 \tan z + \frac{3z}{\cos z}$

iv) $f(x) = \sqrt{x} + \frac{\cos x}{\sin x} + 4$

5. Does the graph of the function $f(t) = t + \cos t$ have a horizontal tangent line on the interval $[0, 2\pi]$?
What about $g(t) = 2t + \cos t$?

Draw your graphs below.