

The goal of this exercise is to explore when a function fails to have a derivative at a point.

We have seen that the definition of the derivative at a point x is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{z \rightarrow x} \frac{f(z) - f(x)}{z - x}. \quad (1)$$

- i) Explain in terms geometrical terms (more precisely, in terms of the graph of the function $f(x)$, as well as secant and tangent lines) what the limits in (1) “measure”.

- ii) List the cases for which a limit may fail to exist.

- iii) For each of the cases you have listed at the previous point, what does it mean in terms of the slope of the tangent line that the limit fails to exist? For each case, how does the function look like (you draw a sketch for each case)? Why does such a shape prevent the derivative to exist?

Warning: some cases actually contain subcases.