L'Hopital's rule

This rule can be applied to compute limits when:

1. the limit is written as a quotient of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$,
2. $f(x)$ and $g(x)$ are differentiable on a open interval I that contains the point a and $g'(x) \neq 0$ on I except possibly at a,
3. the limit $\lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$ exists.

Then
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

L'Hôpital's rule can be applied several times as long as the quotient is $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and the other conditions hold as well (of course).
It also holds for one-sided limits.

Compute the following limits:

a) $\lim\limits_{x \to 0} \frac{e^x - 1}{\tan x}$

b) $\lim\limits_{x \to \infty} \frac{x}{e^x}$

c) $\lim\limits_{x \to \infty} \frac{x^2}{e^x}$

Let us now consider the following limits. What are their "types"?
What do we need to do to apply L'Hôpital's rule to compute these limits?

d) $\lim\limits_{x\to 0^+} x \ln x$

e) $\lim\limits_{x\to\infty} x^{1/x}$

f) $\lim\limits_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x$

g) A student wants to compute the limit $\lim\limits_{x\to\infty} \dfrac{x+\sin x}{x}$.
To do so, he uses L'Hôpital's rule and gets:

$$\begin{aligned}
&\lim_{x\to\infty} \frac{x+\sin x}{x} \quad (= \text{“}\frac{\infty}{\infty}\text{”, use L'Hôpital's rule)} \\
&= \lim_{x\to\infty} \frac{1+\cos x}{1} \\
&= \lim_{x\to\infty} 1+\cos x.
\end{aligned}$$

As $\cos x$ oscillates when x goes to infinity, he concludes that the limit $\lim\limits_{x\to\infty} \dfrac{x+\sin x}{x}$ does not exist.

Do you agree with this reasoning? If so, explain why you think it is correct. If not, explain where there is a flaw.

h) Explain in words (and without mathematical symbols) what L'Hôpital's rule means in geometrical terms.