1. What are Rolle's Theorem and the Mean Value Theorem (MVT)?
   What are their assumptions? How are they related to one another?

2. Here is a concrete application of the Mean Value Theorem.
   In the state of New South Wales, in Australia, the police uses "point-to-point" speed cameras. These cameras evaluate the speed of a given car by measuring how long it takes the car to go from point A to point B (hence the name).

   (a) What "kind" of speed is measured by such speed cameras? What are the advantages and disadvantages of such a system (in terms of measuring speed) compared to a "normal" speed camera?

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   (b) Imagine that using such a point-to-point speed camera, the police fines a driver for driving at 100 kph on a portion of the highway limited at 90 kph. The driver appeals the decision using the following argument. While its average speed was measured as being 100 kph, nothing proves that there was any given point where he drove at that speed. Yet, the law says that one is to be fined if one is observed going over the speed limit *at a given point*.
   The District Attorney calls you as an expert witness as she hopes you can help her show that the fine should be upheld. With your knowledge of calculus, how can you help her?

3. (a) For each of the following cases, draw a sketch for $f(x)$ (and where needed of $g(x)$):

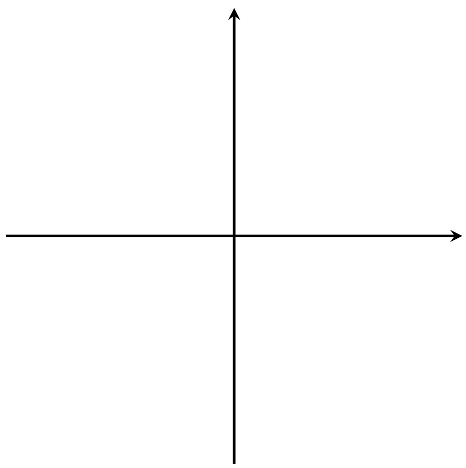
i) $f(x)$ with $f'(x) = 0$,

ii) $f(x)$ and $g(x)$ with $f'(x) = 1/2 = g'(x)$,

iii) $f(x)$ and $g(x)$ with $f'(x) = g'(x)$,

iv) $f(x)$ with $f'(x) > 0$,

i)



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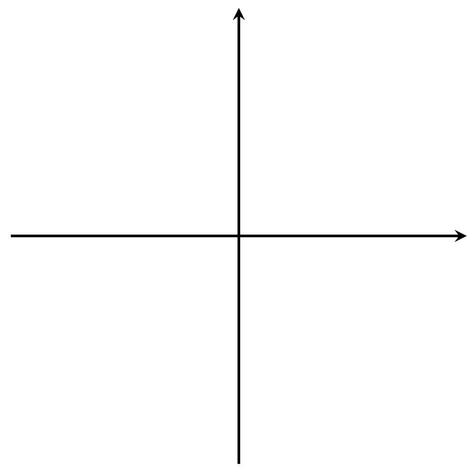

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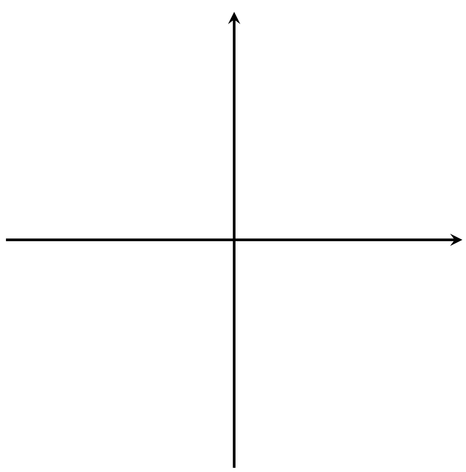

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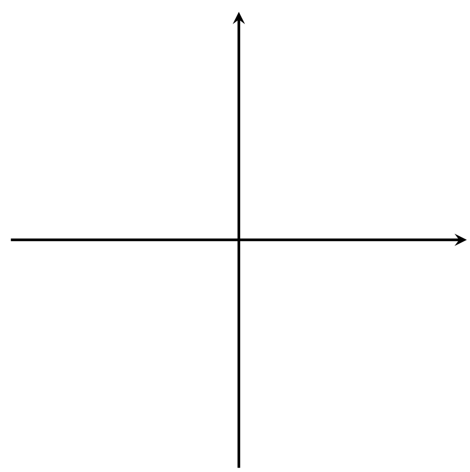


(b) For each graph, justify your sketch using the Mean Value Theorem.
For each case, there are actually infinitely many functions that satisfy the given conditions. Thus identify what the important feature(s) of your sketch is/are.

4. For each of the following cases, draw a sketch for $f(x)$ where $f(x)$ is defined everywhere and where:

   a) $f'(x) < 0$ on $(-3, -1)$, $f'(-1) = 0$, and $f'(x) < 0$ again on $(-1, 3)$,

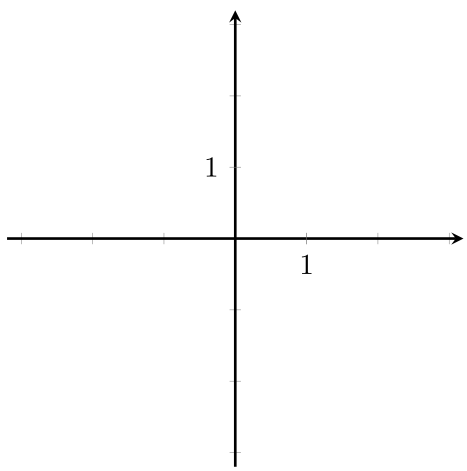
   b) $f'(x) < 0$ on $(-3, -1)$, $f'(-1) = 0$, and $f'(x) > 0$ on $(-1, 3)$,

   c) $f'(x) < 0$ on $(-3, -1)$, $f'(-1)$ does not exist, and $f'(x) < 0$ on $(-1, 3)$,

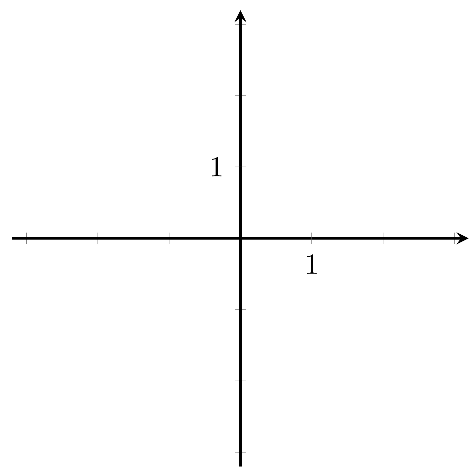
   d) $f(x)$ that is decreasing on $(-3, -1)$, increasing on $(-1, 1)$, neither increasing nor decreasing on $(1, 2)$ and increasing on $(2, 3)$.

   For each of the preceding cases, what "kind" of point is $f(-1)$?

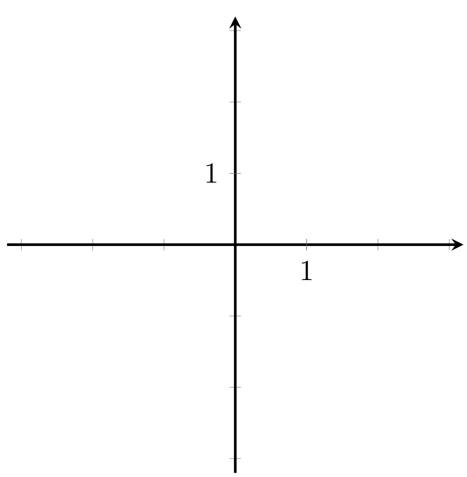
a)



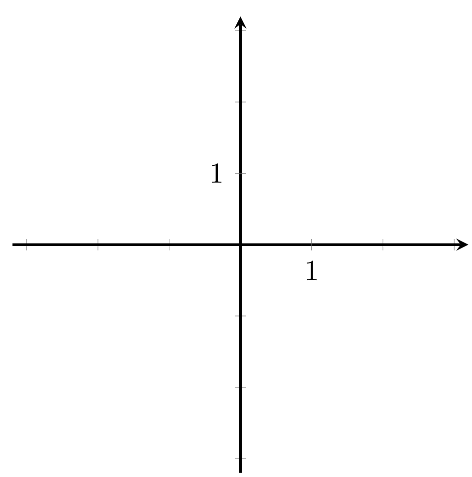
c)



b)



d)



Next time we will see how to use this concretely.