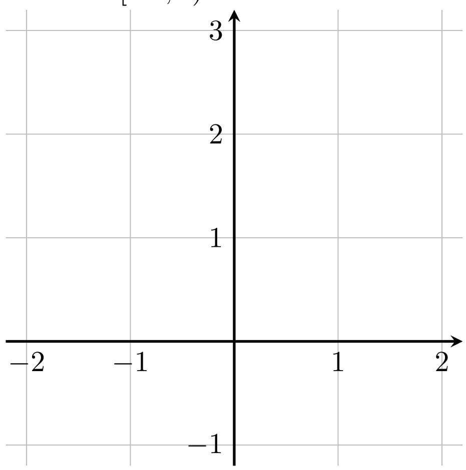
Definition

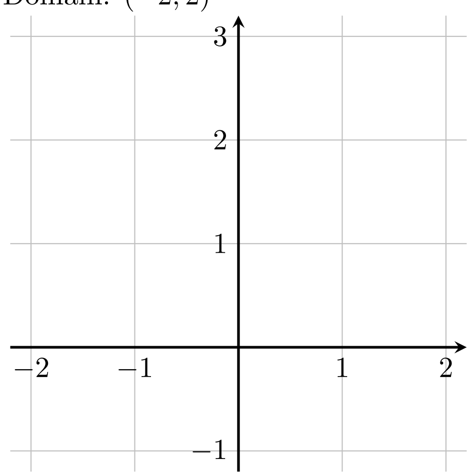
A function f has an **absolute maximum** (also known as a **global maximum**) at $x = c$ if $f(c)$ is the highest value of f anywhere; more precisely, f has an absolute maximum at $x = c$ if $f(c) \geq f(x)$ for all x in the domain of f. An absolute minimum is defined similarly.

1. If possible, create graphs of functions satisfying each description:

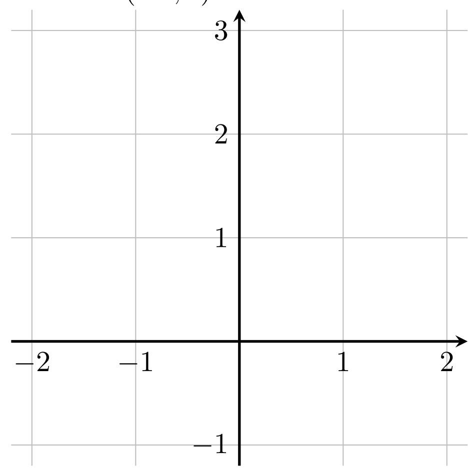
   (a) A continuous function with an absolute maximum of 3 and no absolute minimum.
   Domain: $[-2, 2)$

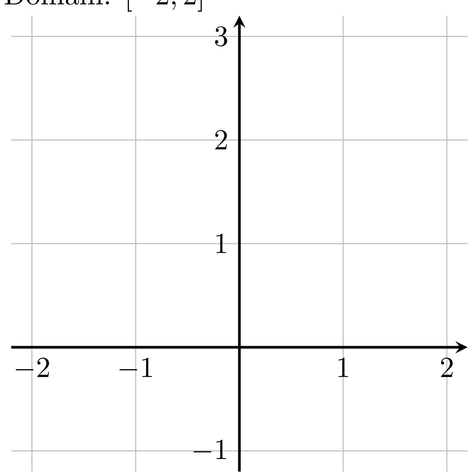
   (b) A continuous function with an absolute maximum of 3 and an absolute minimum of -1.
   Domain: $(-2, 2)$

   (c) A continuous function with no absolute maximum and no absolute minimum.
   Domain: $(-2, 2)$

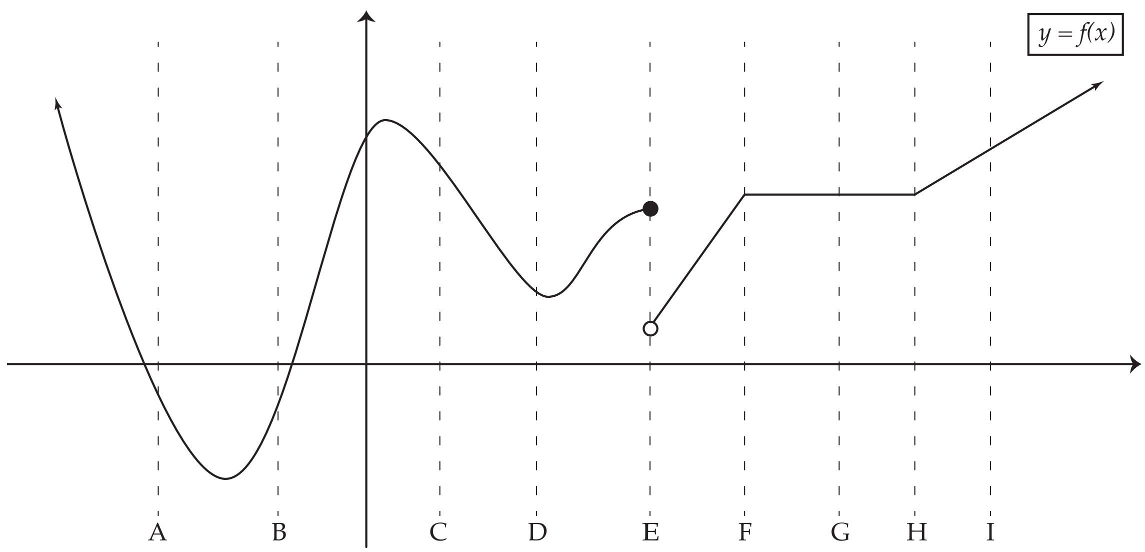
   

   (d) A continuous function with no absolute maximum and no absolute minimum.
   Domain: $[-2, 2]$

2. Based on the previous exercise and on the pre-class activity, what conditions must a function $f(x)$ satisfy to guarantee that it has an absolute maximum and an absolute minimum?

3. A portion of the graph of the function $f(x)$ is shown in the figure below.



For each of the questions below, circle **ALL** of the available correct answers.

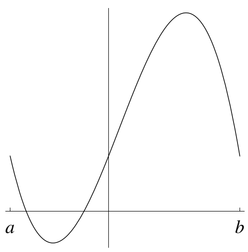
(a) On which intervals does $f(x)$ satisfy the hypotheses of the **Extreme Value Theorem**?

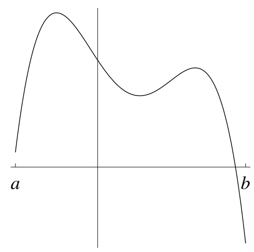
$[A, C]$ $[A, F]$ $[B, E]$ $[D, F]$ $(G, I]$ None

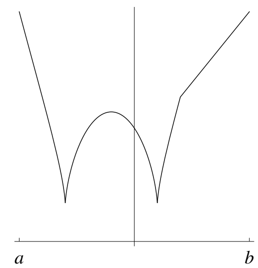
(b) On which intervals does $f(x)$ satisfy the conclusion of the **Extreme Value Theorem**?

$[A, C]$ $[A, F]$ $[B, E]$ $[D, F]$ $(G, I]$ None

4. We now know a continuous function defined on a closed interval will have an absolute maximum and an absolute minimum. Often, our goal is to find the absolute minimum or maximum (if it exists) of a function on a given interval.
   Look at the graphs below and think about a strategy. **How would you go about identifying absolute maximum and absolute minimum on** $[a, b]$?







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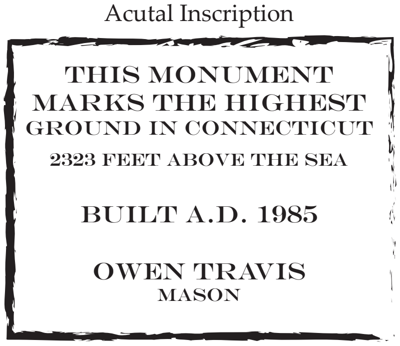
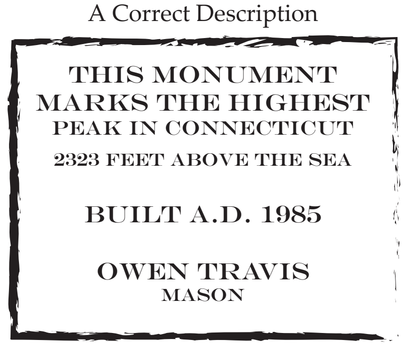


A Correct Description



Point to remember:

5. Suppose $f(x)$ is a continuous function defined on a closed interval. Is every critical point an absolute maximum or an absolute minimum? Give examples.

6. (a) Does $f(r) = 2\pi r^2 + \frac{256\pi}{r}$ have an absolute maximum and absolute minimum on $[1, 8]$? If so, where? (Give the values of r at which the absolute minimum and absolute maximum occur.)

   (b) Does f have an absolute minimum and absolute maximum on $(0, \infty)$? If so, where?

7. A soda company wants its aluminum soft drink cans to have a volume of 128π cubic centimeters. The company's factory can only manufacture cans that are at least 2 cm tall and have a radius of at least 1 cm. In order to conserve resources, the company wants to minimize the amount of aluminum needed for a single can. What dimensions should they make their cans?