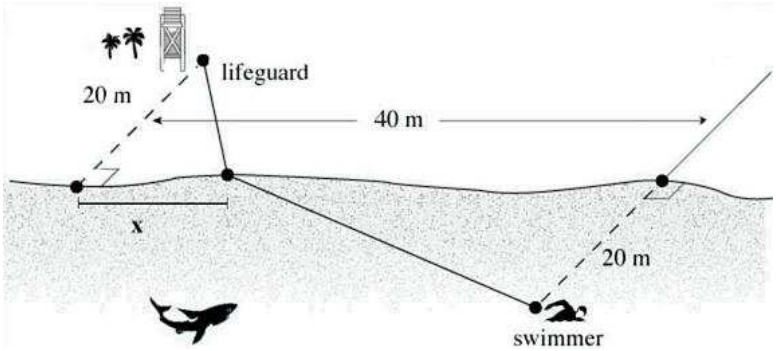
Steps for Solving Optimization Exercises

1. **Identify the quantity to be optimized**.
2. **Draw a picture** representing the situation. Label any part that is relevant.
3. **Introduce variables**. List every relation in the picture and in the problem as an equation or expression, and identify the unknown variables.
4. **Write an equation for the quantity you want to optimize**. Use the relations from the previous step to turn it into a function of a single variable. (This may require considerable manipulation.)
5. **Solve the Exercise**. Determine the domain of your function. Use the first and second derivative tests to identify and classify the critical points. Check critical points and endpoints to find the optimal value.

1. **Soda Can**
   You have to design a new can for a soda company. It has to have a volume of 330 ml (or 330 cm^3). The base and top cost 0.2 dollar per square centimeter to manufacture whereas the side costs 0.05 dollar per square centimeter to manufacture. What are the radius and height that minimize the cost of production?

   One could (as a second step) also ask: what if the top and bottom parts cost 5 times more than the side. (this is basically the same questions, just a bit more abstract).

2. **The Fastest Route**
Sophie, a lifeguard, sees a swimmer in trouble some distance down the beach and wants to reach him in minimum time. On land, Sophie can run at speed $v_L = 6$ meters per second, while in the water she can swim at speed $v_W = 2$ meters per second.



(a) Sketch the situation with a bird-eye view.

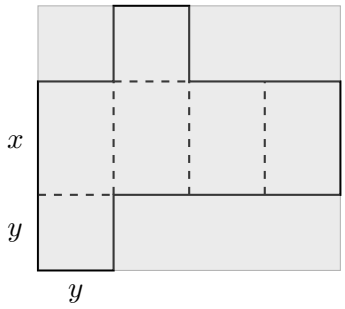
(b) Write a formula for the time required for Sophie to reach the swimmer if she enters the water at a point located x units down the beach from her station. Then compute the derivative of this function and explain how you can find the minimum. You don't need to actually find a value for x.

It turns out the computations to actually find x are complicated, hence we only ask the students to write down the equations need to solve the problem.

3. **U.S. Postal Service Package**
   The U.S. Postal Service only accept packages for standard shipment whose "combined length and girth" has a maximum of 108 inches. The girth is the "distance around" or put more simply, the addition of twice the height and twice the width of the package. If we have a package with a square end, i.e. for which the height equals the width, what are the dimensions that will maximize the volume?

   If students are confused, draw sketch similar to the one of ex. 20a, p. 271 in Thomas.

4. **Minimizing Cardboard**
   Small boxes are sometimes made by folding cardboard shape like this one.



   In our case, we want build a box of 1m^3. If we cut this shape out of a rectangular piece of cardboard (the grey-shaded area), what dimensions of the box will minimize the total area of the piece of cardboard?

5. **Best Picture of the Statue of Liberty**
   You are on a boat cruising around the Statue of Liberty in New York city. You want to take the best possible picture. To do so, you should maximize the angle under which you see the Statue (i.e. the angle formed by: the top of the torch, your camera and the feet of the statue).
   You know that the statue is 92-meter high and stands on a base that is 42-meter high. How far from the base should you be?