Definition
A differentiable function $f(x)$ is *concave up* on an interval (a, b):

when the derivative is increasing (or equivalently when the tangent lines are below the graph of the function)

and *concave down* on (a, b) when:

when the derivative is decreasing (or equivalently when the tangent lines are above the graph of the function)

1. How can we test the concavity (up or down) of a function $f(x)$ that is twice differentiable?

   Introduce the second derivative test (p. 243 in Thomas).

2. (a) Consider the function $f(x) = x^3 + 3x^2 - 1$. On what intervals is the function concave up, respectively concave down?

   At the end of this example, introduce the notion of inflection point.

   Definition
   A point $(a, f(a))$ on a function is an *inflection point* if

   What was the inflection point in the previous example?

(b) Compute the inflection points for the following functions:

ii. $f(x) = x^3$ *$f''(0) = 0$ and $x = 0$ is an inflection point*

iii. $f(x) = \sqrt[3]{x}$ *$f''(0)$ doesn't exist and $x = 0$ is an inflection point*

iv. $f(x) = x^4$ *$f''(0) = 0$ but $x = 0$ is not an inflection point*

(c) Look at the four functions of part 2. For each function, compute the value of the second derivative at the inflection point. What can we conclude from this?

For $f(x) = x^3 + 3x^2 - 1$: inflection point at $x = -1$ and $f''(-1) = 0$.

One really needs to check that $f''(x)$ changes its sign. Just having $f''(a) = 0$ or $f''(x0$ does not exist at $x = a$ is not enough.

3. We now have all we need to sketch functions (or to check that the graph given by a graphing software is correct). List the pieces of information one needs to do a "good" sketch ? *adapted from Thomas p. 248*

    1. *Identify the domain of the function* $f(x)$

    2. *Compute* $f'(x)$ *and* $f''(x)$*)*

    3. *Find the critical points and the behavior of the function at these points*

    4. *Determine the intervals where the function is increasing/decreasing*

    5. *Determine the points of inflection and the concavity of the function*

    6. *Identify the possible asymptotes (if any)*
    Could add: plot the points identified in the previous steps

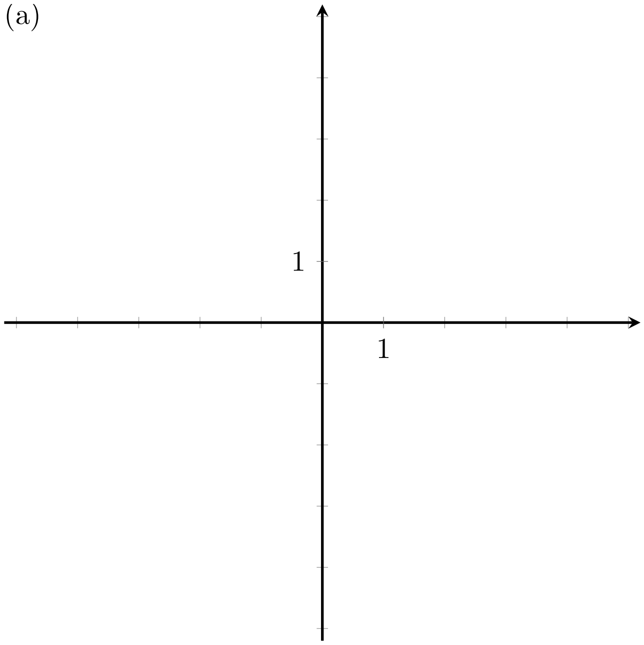
4. Using the procedure described above, sketch the following functions on the following page:

    (a) $f(x) = x\sqrt{9 - x^2}$ *underline here that an inflection point is not necessarily a critical point!*

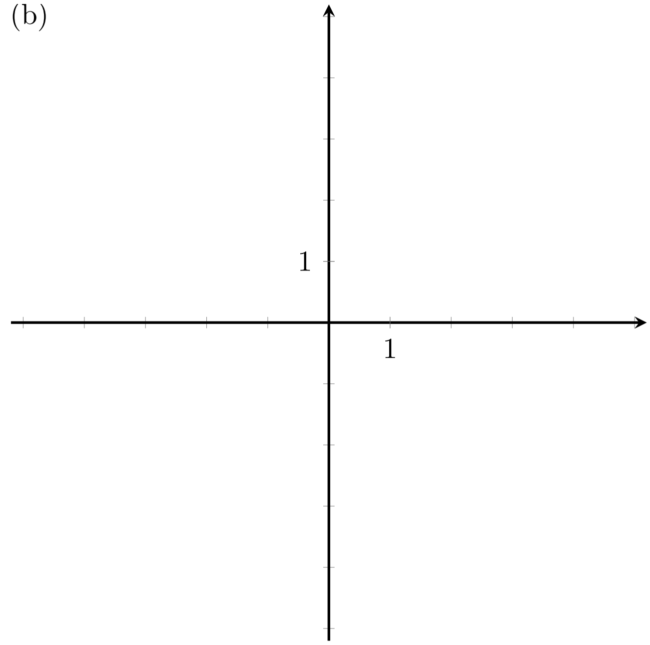
    (b) $f(x) = \dfrac{x^2 - 3}{x^2 - 4}$ *has vertical and horizontal asymptotes*

    (c) $f(x) = \sqrt{|x|}$ $f''(x)$ *doesn't exists at* $x = 0$*,this doesn't mean necessarily there is a change in concavity*

(a)



(b)



(c)