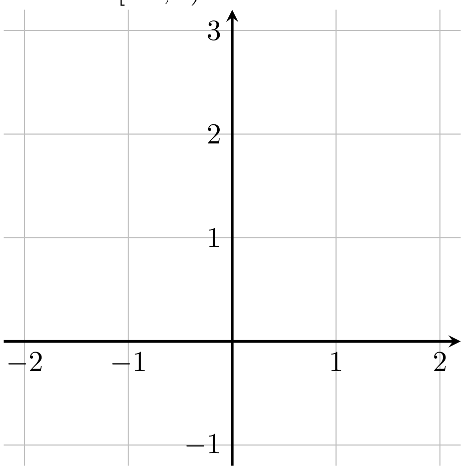
Extreme Value Theorem MATH 1110
Due: DATE Worksheet

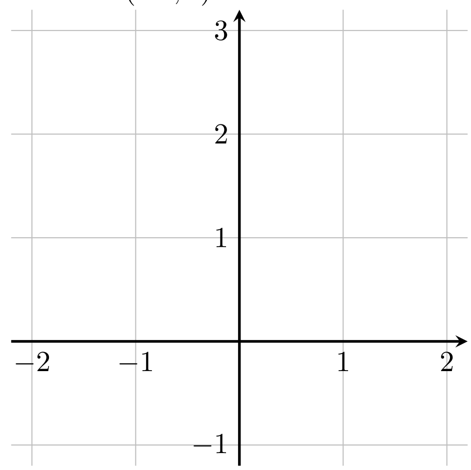
Definition
A function f has an **absolute maximum** (also known as a **global maximum**) at $x = c$ if $f(c)$ is the highest value of f anywhere; more precisely, f has an absolute maximum at $x = c$ if $f(c) \geq f(x)$ for all x in the domain of f. An absolute minimum is defined similarly.

1. If possible, create graphs of functions satisfying each description:

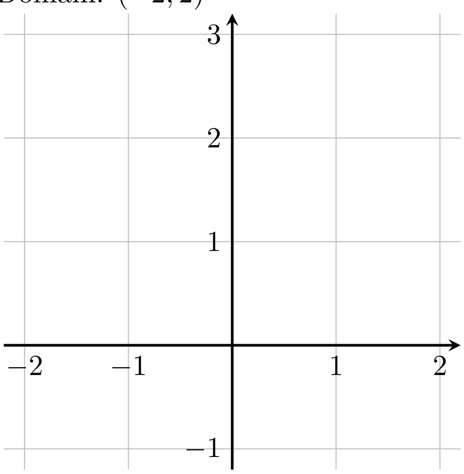
   (a) A continuous function with an absolute maximum of 3 and no absolute minimum.
   Domain: $[-2, 2)$



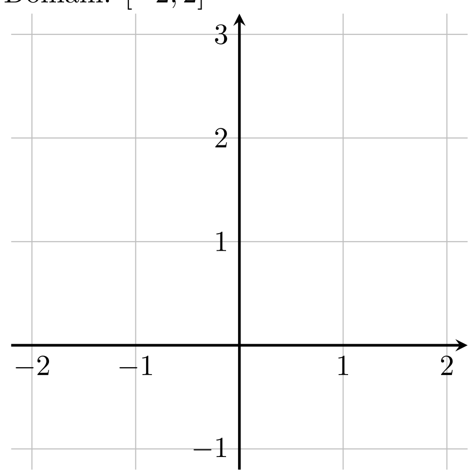
   (c) A continuous function with no absolute maximum and no absolute minimum.
   Domain: $(-2, 2)$



   (b) A continuous function with an absolute maximum of 3 and an absolute minimum of -1.
   Domain: $(-2, 2)$



   (d) A continuous function with no absolute maximum and no absolute minimum.
   Domain: $[-2, 2]$

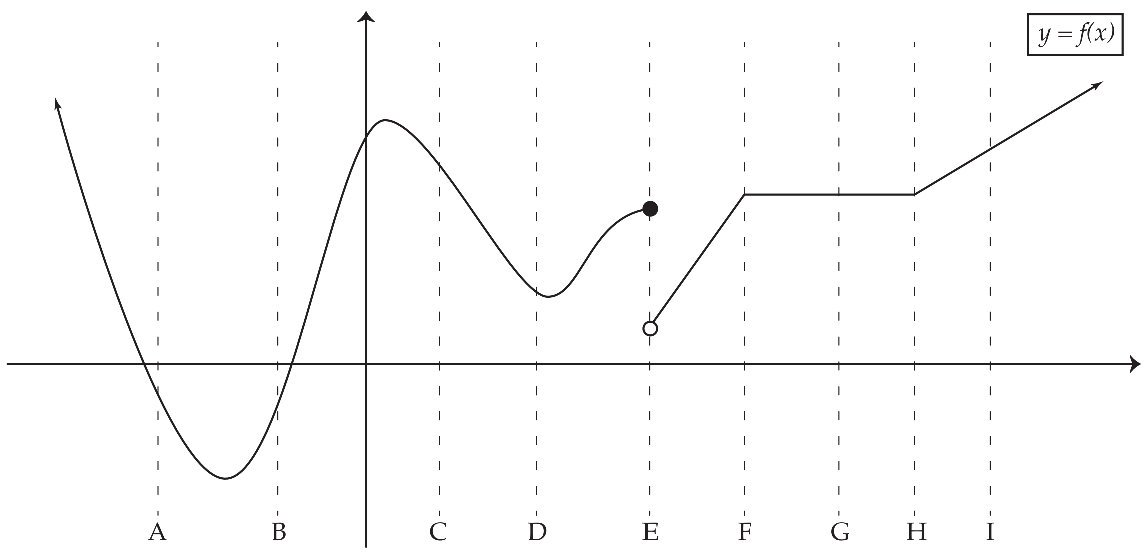


2. Based on the previous exercise and on the pre-class activity, what conditions must a function $f(x)$ satisfy to guarantee that it has an absolute maximum and an absolute minimum?

   Can have a "debate" here, or a poll (people writing down their answer on a piece of paper, or voting with Pingo)

   Introduce the Extreme Value Theorem and explain why it is interesting.

3. A portion of the graph of the function $f(x)$ is shown in the figure below.



For each of the questions below, circle **ALL** of the available correct answers.

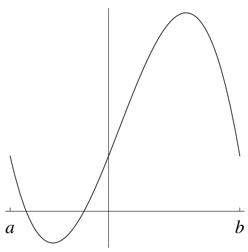
(a) On which intervals does $f(x)$ satisfy the hypotheses of the **Extreme Value Theorem**?

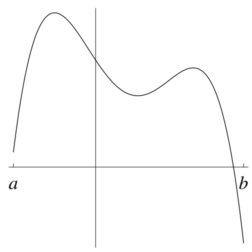
$[A, C]$ $[A, F]$ $[B, E]$ $[D, F]$ $(G, I]$ None

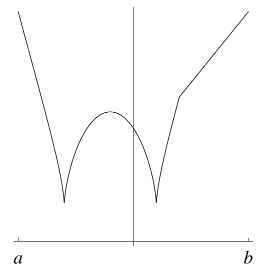
(b) On which intervals does $f(x)$ satisfy the conclusion of the **Extreme Value Theorem**?

$[A, C]$ $[A, F]$ $[B, E]$ $[D, F]$ $(G, I]$ None

4. We now know a continuous function defined on a closed interval will have an absolute maximum and an absolute minimum. Often, our goal is to find the absolute minimum or maximum (if it exists) of a function on a given interval.
   Look at the graphs below and think about a strategy. **How would you go about identifying absolute maximum and absolute minimum on** $[a, b]$?







*Define **critical points** as points where $f'(x) = 0$ or what $f'(x)$ doesn't exist. Have someone explain why we need to include $f'(x)$ doesn't exist.*

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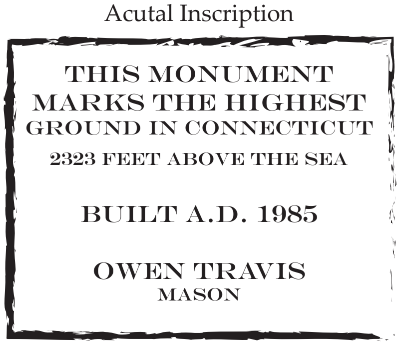
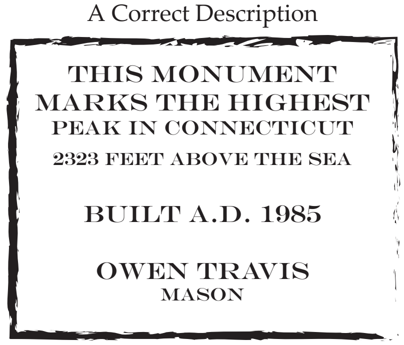


A Correct Description



Point to remember:

Always check boundary points!! This is also a good point to discuss the difference between local and global min/max).

5. Suppose $f(x)$ is a continuous function defined on a closed interval. Is every critical point an absolute maximum or an absolute minimum? Give examples.

   In addition to the peak in Connecticut, have students come up with other functions or graphs.

6. (a) Does $f(r) = 2\pi r^2 + \frac{256\pi}{r}$ have an absolute maximum and absolute minimum on $[1, 8]$? If so, where? (Give the values of r at which the absolute minimum and absolute maximum occur.)

   For this question and the following one, underline the Extreme Value Theorem and what it says.

   (b) Does f have an absolute minimum and absolute maximum on $(0, \infty)$? If so, where?

7. A soda company wants its aluminum soft drink cans to have a volume of 128π cubic centimeters. The company's factory can only manufacture cans that are at least 2 cm tall and have a radius of at least 1 cm. In order to conserve resources, the company wants to minimize the amount of aluminum needed for a single can. What dimensions should they make their cans?

   It turns out that the function one gets is the same as the function studied in part (a). So this question is really about modeling.
   Based on time, decide if you want to do it.