In calculus, we do a lot of adding. Since mathematicians tend to be lazy (or efficient, depending your point of view) there is a specific notation for sums, it is called the **sigma notation**.

To what sum does the following notation correspond to?

$$\sum_{k=1}^{1110} k$$

This notation is important because it is very often used in math!As it is often used in math. Our goal here is to be able to "go back and forth" between the notation with the dots and plus signs and the notation with the sigma. We will also compute some "simple" sums.

1. Rewrite the following sums using "+" signs and dots. Then compute these sums.

   (a) $\displaystyle\sum_{k=1}^{10}(k+1)$

   (b) $\displaystyle\sum_{k=7}^{10}(k+1)$

   (c) $\displaystyle\sum_{k=1}^{5}(k+1)$

   (d) $\displaystyle\sum_{n=10}^{10}(k+1)$

2. Compute the following sums.

   (a) $\displaystyle\sum_{k=6}^{9}(3k+2)$

   (b) $\displaystyle\sum_{k=10}^{12}(k+1)^2$

   (c) $\displaystyle\sum_{k=1}^{5}k$

   (d) $\displaystyle\sum_{k=1}^{10}k$

   (e) $\displaystyle\sum_{k=1}^{100}k$

3. Let us consider the function $f(x) = \frac{1}{x}$. We want to estimate the area under this function between $x = 2$ and $x = 5$. Our goal is to make the connection between the sums below and various estimations of this area.
   Draw the graph of $f(x) = \frac{1}{x}$. Then for each of the following sums explain in words and pictures what they correspond to.

   (a) $\displaystyle\sum_{k=0}^{2} f(2+k) \cdot 1$

   (b) $\displaystyle\sum_{k=1}^{3} f(2+k) \cdot 1$

   (c) $\displaystyle\sum_{k=1}^{3} f\left(2 + k\frac{5-2}{3}\right) \cdot \left(\frac{5-2}{3}\right)$

   (d) $\displaystyle\sum_{k=1}^{3} f\left(1.5 + k\frac{5-2}{3}\right) \cdot \left(\frac{5-2}{3}\right)$

(e) $\displaystyle\sum_{k=1}^{6} f\left(2 + k\frac{5-2}{6}\right) \cdot \left(\frac{5-2}{6}\right)$

(f) Write down the sum for an approximation of the area under $f(x) = 1/x$ between $x = 2$ and $x = 5$ using 10 right-handed rectangles.

4. Same question but for 1000 rectangles. Then, how can we write thus sum for "infinitely many" rectangles?

Challenge Exercises on Sums (only if you are **very** comfortable with the above material).
One can prove that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. Use this formula as well as the formula for the sum $\displaystyle\sum_{k=0}^{n} k$ found above to compute the following sums.

i) $\displaystyle\sum_{k=1}^{21} (3+k) =$

ii) $\displaystyle\sum_{k=14}^{74} k(k+1) =$