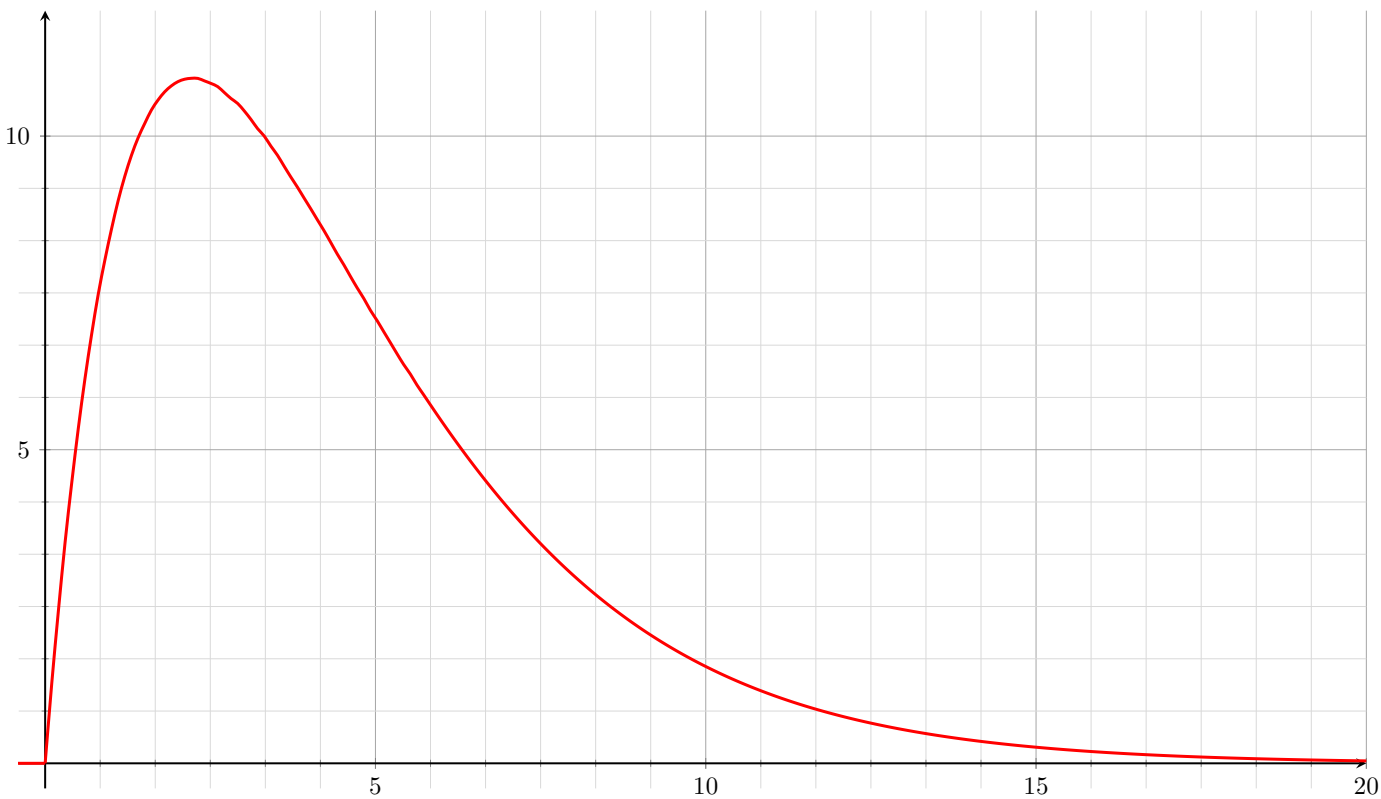
Pharmacokinetics is the process whereby substances (like food and drugs) are ingested into the body and processed. When a nurse first administers a drug, the concentration of the drug in the blood stream is zero. As the drug moves around the body and is metabolized, the concentration of the drug increases. There comes a point when the concentration no longer increases and begins to decline. This is the period when the drug is fully distributed and metabolism is taking place. As time goes on, the drug concentration continues to decrease. The concentration can be modeled by the following piecewise defined function:

$$C(t) = \begin{cases} 0 & t < 0 \\ 133.3 \cdot \left(e^{-0.4t} - e^{-0.5t}\right) & t \geq 0, \end{cases}$$

where t is time measured in hours, and $C(t)$ is the concentration of a single dose in the body, measured in mg/L, t hours after ingestion. This is graphed below.



When administering chemotherapies to treat cancer, for example, doctors must ensure that there is enough drug present to kill the cancerous cells while not overdoing it and killing the patient. The doctor must limit the **total exposure to a drug**. One can then compute what medical researchers describe this as the *time-averaged concentration of drug circulating in the body*. The general question is how can we compute the total exposure to a given drug and then the time-averaged concentration of the drug.

1. Using the graph above, how can we make an over-estimate of the total exposure?

2. Now, how would we make an under-estimate?

3. Given what we might think is a decent estimate, how could we refine our calculation? And then how could we refine even more our calculations?