# Area and Estimating with Finite Sums (5.1)

**Expected Skills.**
At the end of this section, students will be able to:

- explain how the process of estimating an area with finite sums work and use it,
- use this process to estimate lengths, speeds and the average value of a function.

**Pre-Class Activity** (ch5-integrals-1-areas-1-pc)**.** In the pre-class activity we take an example from pharmacokinetics to have the students estimate the area under a curve. The goal here is threefold:

- having them realize that they need to compute the area under the curve,
- have them think about how the can overestimate or underestimate the area,
- how they can improve a given estimate.

**Worksheet** (ch5-integrals-1-areas-2-ws)**.** In the class activity, we build up from what we have done in the pre-class activity. Here we want to estimate the area under the line $x = y$. We purposefully take an easy function and an area that students can compute without calculus. The goal of this is that the can compare their answers with the exact value.
First, we ask them to find **as many ways as possible** to estimate the given area. It is important to underline that we want methods that could work for all types of functions, not only lines. It is also important that they should find **as many as possible** and not just 2 or 3. To "gamify" this part, one could form teams and see which teams comes up with the most ways. For each technique, it is important to ask them to determine if they are under- or over-estimating the area (and if the under/overestimation is inherent to the method or not).
Then we ask them to use the "traditional" left-handed, right-handed and midpoint rectangles.
In part 3, we ask students to do something similar but for a half-circle. The goal here is to reinforce what has just been done. The goal is also to push students to think about other ways to approximate an area, in particular using trapezoids.
Finally, in part 6 we have an "application" of the area under the curve by asking the students to compute the distance traveled by a particle for which we know the velocity.

**Supplemental Activity** (ch5-integrals-1-areas-3-sup-rectrules)**.** In the supplemental activity, students practice computing area approximations using various rectangle rules. It is suggested that this activity is prefaced by a discussion about rectangle rules and

- how an interval is subdivided,
- how each subinterval generates a rectangle, and
- how each rectangle contributes to the area estimate.

Given the variety of rectangular rules, the instructor should divide the class into groups with assigned rules. The groups should then complete the handout and bring back to the classroom their findings. The instructor

can then lead a debriefing discussion about patterns that students may see in their work and may challenge the students to think about when particular rules overestimate or underestimate areas.