The Fundamental Theorem of Calculus – Part 2

If $f(x)$ is continuous on $[a, b]$ and $F(x)$ is any antiderivative on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

1. How do we justify the second part of the Fundamental Theorem of Calculus using the first part of the theorem?

   Here the idea is to connect the two parts of the theorem by having the students be able to give the general idea of this part of the proof.

2. Compute the following definite integrals using the second part of the FTC.

   (a) $\displaystyle\int_0^{\pi} e^x + \cos x - 1\ dx$

   (b) $\displaystyle\int_0^{\pi/4} \frac{1}{\cos^2 x} + \pi - x\ dx$

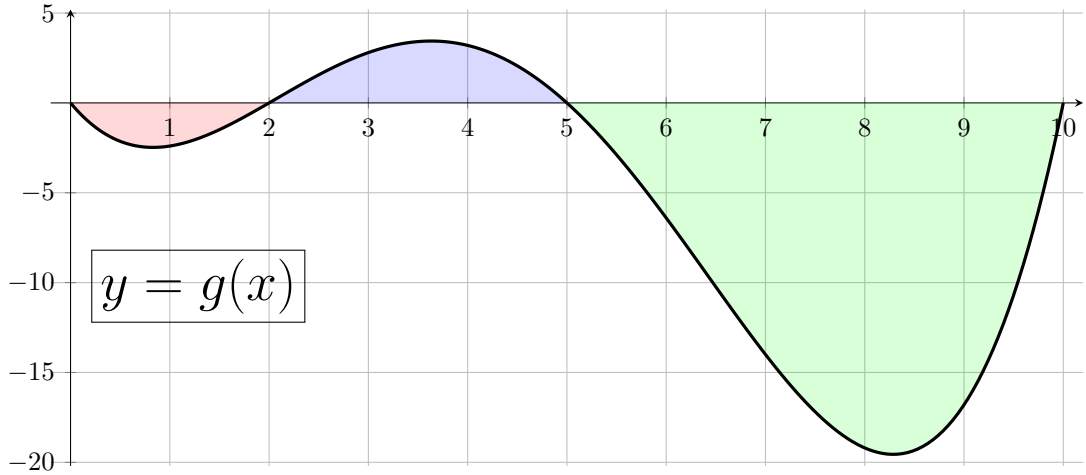
(c) $\displaystyle\int_1^4 \sqrt{x} + \frac{1}{x}\ dx$

(d) $\displaystyle\int_1^{16} x^3 + \sin x + \frac{1}{x^3}\ dx$

(e) $\displaystyle\int_{-4}^{4} |x|\ dx$

(f) $\displaystyle\int_{-1}^{0} \pi^{x-1}\ dx$

3. Consider the graph of the function $y = g(x)$ given below.



(a) What is the relationship between $\int_0^{10} g(x)\,dx$ and the shaded areas in the figure?

(b) What is the relationship between $\int_0^{10} |g(x)|\,dx$ and the shaded areas in the figure?

4. The temperature of a room $T(t)$ in °F at time t is given by

$$T(t) = 85 - 3\sqrt{25 - t} \quad \text{for } 0 \leq t \leq 25.$$

   (a) Find the room's temperature when $t = 0$, $t = 16$ and $t = 25$.

   (b) Find the room's average temperature for $0 \leq t \leq 25$.

5. Archimedes discovered that the area under a parabolic arch is two-thirds the base times the height. Sketch the parabolic arch $y = h - (4h/b^2)x^2$ for $-b/2 \leq x \leq b/2$, assuming that h and b are positive. Then use calculus to find the area of the region enclosed between the arch and the x-axis.