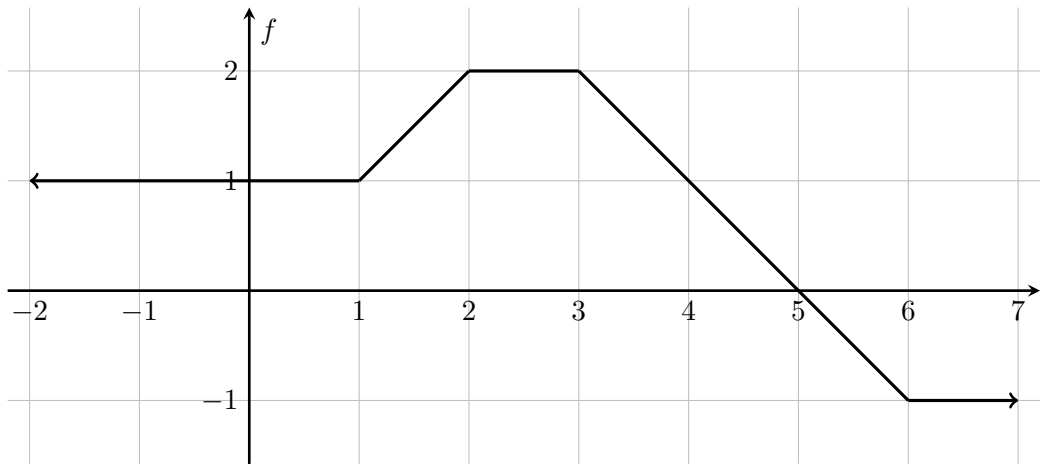
At this point, there are two general questions we would like to answer:

i) Does every continuous function have an antiderivative?

ii) Are the notions of integration and differentiation related and if so, how?
The answer to the first question will lead to to the answer of the second.

1. **Area Accumulation Functions**
Consider the function $f(x)$ whose graph is given below.



(a) Define $F(x) = \int_0^x f(t)\,dt$. What does this function represent?

(b) Evaluate the following:

$F(0) =$ *0* $\qquad$ $F(2) =$ *2.5*
$F(1) =$ *1* $\qquad$ $F(-1) =$ *-1*

(c) Shade in and find the area represented by $F(3) - F(1)$.

It is the area under the curve between $x = 1$ and $x = 3$, an area of 3.5.

(d) Find a formula for $F(x)$ between $x = 0$ and $x = 1$

x

(e) Give two values at which $F(x) = 0$. (Hint: assume the graph continues to the right.)

$x = 0$ *and* $x = 12$

(f) Which is larger: $F(3)$ or $F(4)$? Explain.

$F(4)$ is larger because $F(x)$ accumulates area and all the area between $x = 3$ and $x = 4$ counts as positive since $f(x)$ is positive there.

(g) Which is larger: $F(5)$ or $F(6)$? Explain.

$F(5)$ is larger because the area accumulated between $x = 5$ and $x = 6$ counts negatively, since $f(x)$ is negative there.

(h) Give the open intervals on which $F(x)$ is increasing and decreasing. Explain.

$F(x)$ is increasing from $x = 0$ to $x = 5$. On this interval the area is all positive, so as it accumulates, the value of $F(x)$ must increase. To the left of $x = 0$, $F(x)$ is also increasing because the value of the integrals becomes less negative as the value of x moves to the right.
Then it is decreasing on $(5, \infty)$. Underline that actually $f(5) = 0$.

2. Let us consider the function $Z(x) = \int_0^x 2\,dt$.

(a) Sketch a graph of $f(t) = 2$ for $t \geq 0$.

(b) Use geometry to fill in the chart below and generalize it for an arbitrary value of x.

x	0	1	2	3	...	x
$Z(x)$						

(c) Using the values in your table, sketch a graph of Z. What is the slope? The y-intercept?

(d) How would we change the definition of $Z(x)$ given above so that the slope would be 3?

3. **Fundamental Theorem of Calculus (Part 1)**

Let $f(t)$ be continuous function on $[a, b]$. Let $F(x) = \int_a^x f(t)dt$.

(a) Sketch the quantities $F(x)$ and $F(x + h)$ where h is a small positive number. To make it simple, take a function $f(t) > 0$.

The idea is to give of motivation of the FTC. Cf. p. 331 in Thomas

(b) Approximate $F(x + h) - F(x)$ by the area of a rectangle.

(c) This motivates the Fundamental Theorem of Calculus.

Give the first part of the Theorem p. 332.
This answers our questions at the beginning of the worksheet.

4. For the following questions, use the First Fundamental Theorem of Calculus to find $F'(x)$.

   (a) $F(x) = \int_1^x \sqrt[4]{t}\, dt$

   (b) $F(x) = \int_0^x \sec^2 t\, dt$

   (c) $F'(x)$ for $F(x) = \int_3^{5x^2+8} \cos(t)\, dt$ (Don't forget about the chain rule!)

   (d) $F'(x)$ for $F(x) = \int_{\sin(x)}^{x^2} \frac{1}{t^3}\, dt$ (Can you break this up somehow?)

5. A particle is moving along the x-axis. The position of the particle at time t seconds is given by $x(t) = (t-1)(t-3)^2$, $0 \le t \le 5$. Find the total distance (in meters) the particle travels in 5 seconds.

   Do this application of the FTC only if you have time.