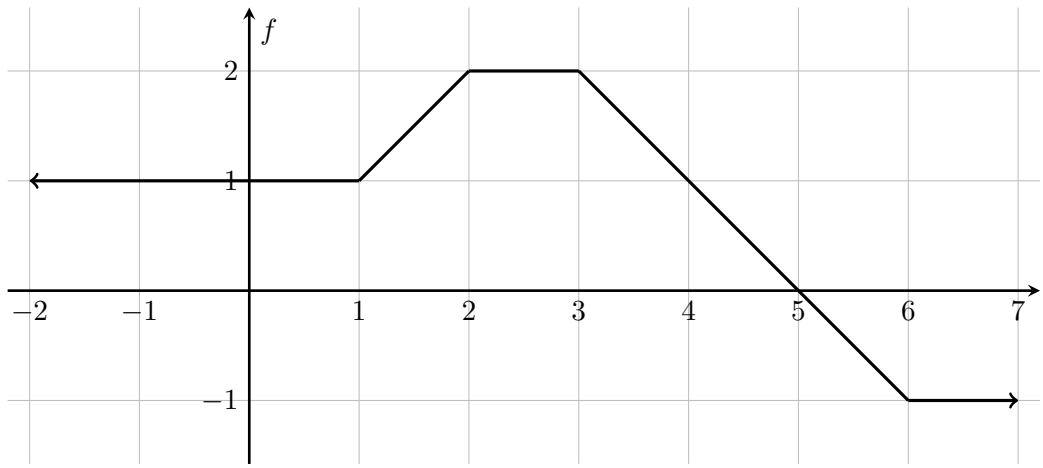
At this point, there are two general questions we would like to answer:

i) Does every continuous function have an antiderivative?

ii) Are the notions of integration and differentiation related and if so, how?
The answer to the first question will lead to to the answer of the second.

1. **Area Accumulation Functions**
Consider the function $f(x)$ whose graph is given below.



(a) Define $F(x) = \int_0^x f(t)\,dt$. What does this function represent?

(b) Evaluate the following:

$F(0) =$ $F(2) =$
$F(1) =$ $F(-1) =$

(c) Shade in and find the area represented by $F(3) - F(1)$.

(d) Find a formula for $F(x)$ between $x = 0$ and $x = 1$

(e) Give two values at which $F(x) = 0$. (Hint: assume the graph continues to the right.)

(f) Which is larger: $F(3)$ or $F(4)$? Explain.

(g) Which is larger: $F(5)$ or $F(6)$? Explain.

(h) Give the open intervals on which $F(x)$ is increasing and decreasing. Explain.

2. Let us consider the function $Z(x) = \int_0^x 2\,dt$.

(a) Sketch a graph of $f(t) = 2$ for $t \geq 0$.

(b) Use geometry to fill in the chart below and generalize it for an arbitrary value of x.

x	0	1	2	3	...	x
$Z(x)$						

(c) Using the values in your table, sketch a graph of Z. What is the slope? The y-intercept?

(d) How would we change the definition of $Z(x)$ given above so that the slope would be 3?

3. **Fundamental Theorem of Calculus (Part 1)**
Let $f(t)$ be continuous function on $[a, b]$. Let $F(x) = \int_a^x f(t)dt$.

(a) Sketch the quantities $F(x)$ and $F(x + h)$ where h is a small positive number. To make it simple, take a function $f(t) > 0$.

(b) Approximate $F(x + h) - F(x)$ by the area of a rectangle.

(c) This motivates the Fundamental Theorem of Calculus.

4. For the following questions, use the First Fundamental Theorem of Calculus to find $F'(x)$.

   (a) $F(x) = \int_1^x \sqrt[4]{t}\, dt$

   (b) $F(x) = \int_0^x \sec^2 t\, dt$

   (c) $F'(x)$ for $F(x) = \int_3^{5x^2+8} \cos(t)\, dt$ (Don't forget about the chain rule!)

   (d) $F'(x)$ for $F(x) = \int_{\sin(x)}^{x^2} \frac{1}{t^3}\, dt$ (Can you break this up somehow?)

5. A particle is moving along the x-axis. The position of the particle at time t seconds is given by $x(t) = (t-1)(t-3)^2$, $0 \le t \le 5$. Find the total distance (in meters) the particle travels in 5 seconds.