In calculus, we do a lot of adding. Since mathematicians tend to be lazy (or efficient, depending your point of view) there is a specific notation for sums, it is called the **sigma notation**.

To what sum does the following notation correspond to?

$$\sum_{k=1}^{1110} k$$

Here explain to the students the different "parts" of this notation: the dummy variable, the bounds.
Also, write down some "variations" of this sum (changing just one thing) to make sure everything is well understood

This notation is important because it is very often used in math!As it is often used in math. Our goal here is to be able to "go back and forth" between the notation with the dots and plus signs and the notation with the sigma. We will also compute some "simple" sums.

1. Rewrite the following sums using "+" signs and dots. Then compute these sums.

   (a) $\displaystyle\sum_{k=1}^{10}(k+1)$

   (b) $\displaystyle\sum_{k=7}^{10}(k+1)$ *Changing the lower bound*

   (c) $\displaystyle\sum_{k=1}^{5}(k+1)$ *Changing the upper bound*

   (d) $\displaystyle\sum_{n=10}^{10}(k+1)$ *Limit case where we don't have a sum anymore*

   Above are easy examples to have a soft start with this new notation.

2. Compute the following sums.

   (a) $\displaystyle\sum_{k=6}^{9}(3k+2)$

   (b) $\displaystyle\sum_{k=10}^{12}(k+1)^2$

   (c) $\displaystyle\sum_{k=1}^{5}k$

   (d) $\displaystyle\sum_{k=1}^{10}k$

   (e) $\displaystyle\sum_{k=1}^{100}k$ *Obviously, we want them to find the formula* $\frac{n(n+1)}{2}$

Space for exercises on p. 3 (if needed)

3. Let us consider the function $f(x) = \frac{1}{x}$. We want to estimate the area under this function between $x = 2$ and $x = 5$. Our goal is to make the connection between the sums below and various estimations of this area.
   Draw the graph of $f(x) = \frac{1}{x}$. Then for each of the following sums explain in words and pictures what they correspond to.

   (a) $\displaystyle\sum_{k=0}^{2} f(2+k) \cdot 1$

   (b) $\displaystyle\sum_{k=1}^{3} f(2+k) \cdot 1$

   (c) $\displaystyle\sum_{k=1}^{3} f\left(2 + k\frac{5-2}{3}\right) \cdot \left(\frac{5-2}{3}\right)$

   (d) $\displaystyle\sum_{k=1}^{3} f\left(1.5 + k\frac{5-2}{3}\right) \cdot \left(\frac{5-2}{3}\right)$

   This last exercise is an introduction to Rieman sums. It goes back to the last exercise of the previous worksheet.

(e) $\displaystyle\sum_{k=1}^{6} f\left(2 + k\frac{5-2}{6}\right) \cdot \left(\frac{5-2}{6}\right)$

(f) Write down the sum for an approximation of the area under $f(x) = 1/x$ between $x = 2$ and $x = 5$ using 10 right-handed rectangles.

4. Same question but for 1000 rectangles. Then, how can we write thus sum for "infinitely many" rectangles?

   This is how we are going to define the integral. But be careful, there are some subtleties here!

Challenge Exercises on Sums (only if you are **very** comfortable with the above material).

One can prove that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. Use this formula as well as the formula for the sum $\displaystyle\sum_{k=0}^{n} k$ found above to compute the following sums.

i) $\displaystyle\sum_{k=1}^{21} (3+k) =$

ii) $\displaystyle\sum_{k=14}^{74} k(k+1) =$