1. Last time we talked about approximating the area under a curve using sums of rectangles. Is there a way to use this idea to find the actual area under a curve (and not just an approximation)?

   Ask this question to the students and help them get to the formula: $\lim_{\Delta \to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k$. *(p. 318 in the textbook)*
   Here we assume that Δ *is the length of the largest interval (it is thus positive).*
   Explain what each part of this formula ***means****:*

   - *the length of the intervals are going to zero and they DON'T have to have the same length,*
   - *we can take any value of the function within each interval,*
   - *the number of intervals is going to infinity.*

   Telate this to the pre-class activity: taking the limit (with intervals becoming infinitely small) enable us to "reconcile" the left-handed and right-handed rectangles approximations. This also explains how we can actually compute the area under a half-circle even though one rectangle is "always" flat.

2. Let us consider a function $f(x)$ on an interval $[a, b]$. **By definition**, the integral of f from a to b, denoted by

$$\int_a^b f(x)dx$$

   is ... *the area under the function between* a *and* b.

   Explain what the different parts of this notation mean: upper and lower limits, function, dummy variable.

3. Compute the following integrals. Drawing the graphs of these functions will probably help you.

   a) $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

   Underline here that we are computing the area under this curve and NOT looking for the antiderivative!

   b) $\displaystyle\int_{-2}^{2} (2 - |x|)\, dx$

4. Let us now look at one important property of integrals.

   a) Compute the integral $\int_{-3}^{3} -\sqrt{9-x^2}\,dx$. What do you conclude from this example?

   Integrals give the singed area.

   b) Then, what is $\int_{-1}^{1} x\,dx$

5. Let us see one more property of integrals.

   a) Compute the integrals $\int_{-3}^{0} \sqrt{9-x^2}\,dx$ and $\int_{0}^{3} \sqrt{9-x^2}\,dx$.

   b) How are they related to $\int_{-3}^{3} \sqrt{9-x^2}\,dx$? What property of integrals is this an example of?

   Here mention that $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$. *Also mention that* $\int_a^a f(x)dx = 0$.

   c) Compute $\int_{0}^{0} \sqrt{9-x^2}\,dx$.