

List of publications and preprints.

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0. *On the nonexistence of certain morphisms from Grassmannian to Grassmannian in characteristic 0*. Ph.D Thesis. University of Chicago (2005).

This work proves some properties of the big Chern classes, which are then used to obtain results about the non-existence of certain morphisms between Grassmannians in characteristic 0. Moreover, it is proven that the big Chern classes of a vector bundle E over a smooth scheme X with an ample line bundle can be recovered from the components of the Chern character of E and the Atiyah class of the tangent bundle of X .

1. *On the nonexistence of certain morphisms from Grassmannian to Grassmannian in characteristic 0*. Submitted for publication.

This paper contains the main geometric results of my thesis.

2. *The big Chern classes and the Chern character*. International Journal of Mathematics **19** no. 6 (2008),699-746. Also available in the Arxiv at math.AG/0512104.

Let X be a smooth scheme over a field of characteristic 0. It is well known (from a paper of M. Kapranov) that the Atiyah class of the tangent bundle T_X of X equips $T_X[-1]$ with the structure of a Lie algebra object in the derived category $D^+(X)$ of bounded below complexes of $\mathcal{O}_X$ -modules with coherent co-homology. We realize this structure as the Lie bracket of an "almost free" Lie algebra $\mathcal{L}$ generated over $\mathcal{O}_X$ such that $\mathcal{L}$ is a bounded below complex of $\mathcal{O}_X$ -modules. We then give a theorem for $\mathcal{L}$ paralleling the computation of the differential of the inverse exponential map of a Lie algebra. We also show that the complex of poly-differential operators with Hochschild co-boundary is the universal enveloping algebra of $\mathcal{L}$, and hence $T_X[-1]$, in

$D^+(X)$. This enables us to interpret the Chern character of a vector bundle on X as the "character of a representation". The results obtained are then used to give an explicit formula for the big Chern classes of a vector bundle E on X in terms of the Chern character of E and the Atiyah class of T_X .

3. *The relative Riemann-Roch theorem from Hochschild homology.* Arxiv preprint math.AG/0603127. Submitted for publication.

This paper expands upon the core computations in a well known preprint of Markarian to compute Caldararu's Mukai pairing on Hochschild homology at the level of Hodge cohomology. It turns out that the Hochschild-Kostant-Rosenberg map twisted by the square-root of the Todd genus "almost preserves" the Mukai pairing, thus verifying a conjecture of Caldararu.

4. *Some notes on the Feigin-Losev-Shoikhet integral conjecture.* Journal of Noncommutative Geometry 2(2008), 405-448. Also available in the Arxiv at math.QA/0612298.

Given a vector bundle E on a compact complex manifold X of complex dimension n , B. Feigin, A. Losev and B. Shoikhet use "topological quantum mechanics" to construct a linear functional I_E on the completed Hochschild homology $\widehat{HH}_0(\mathcal{D}iff(E)) \simeq H^{2n}(X, \mathbb{C})$ of the sheaf $\mathcal{D}iff(E)$ of holomorphic differential operators on E (see their preprint math.QA/0401400). They conjecture that $I_E = \int_X$. This paper shows that $I_E = I_F$ for any two vector bundles E and F . Similar linear functionals can be constructed on the completed cyclic homology $\widehat{HC}_{2i}(\mathcal{D}iff(E))$ for each i . This yields linear functionals $I_{E,2i,2k}$ on $H^{2n-2k}(X, \mathbb{C})$ for $0 \leq k \leq i$. We show that $I_{E,2i,0} = I_E$ and that $I_{E,2i,2k}$ vanishes for $k > 0$.

5. *Integration over complex manifolds via Hochschild homology.* To appear in Journal of Noncommutative Geometry. Also available at arxiv:0707.4528.

Completes the proof of the Feigin-Losev-Shoikhet conjecture and generalizes it to noncompact complex manifolds with "bounded geometry" as well.

6. *The Mukai pairing and integral transforms in Hochschild homology.* Arxiv preprint arxiv:0805.1760. Submitted for publication.

The Hochschild homology $HH_\bullet(X)$ of a smooth proper scheme X over a field of characteristic 0 comes with two pairings: a "categorical" or "natural" pair-

ing constructed by D. Shklyarov, and the Mukai pairing of Caldararu. This paper proves a theorem computing the first pairing in terms of the second. Further, if X and Y are smooth and proper, an element $\Phi \in \text{perf}(X \times Y)$ gives rise to an integral transform from $\text{HH}_\bullet(X)$ to $\text{HH}_\bullet(Y)$ via two "a priori different" constructions: a natural construction of Shklyarov and a construction of Caldararu. This paper proves that these two constructions are equivalent. These results give rise to a Hirzebruch Riemann-Roch theorem for the sheafification of the Dennis trace map.

7. *A generalized Hirzebruch Riemann-Roch theorem.* Arxiv preprint arxiv:0808.3265. Submitted for publication.

This short note proves a generalization of the Hirzebruch Riemann- Roch theorem equivalent to the Cardy condition described by A. Caldararu. This is done using an earlier result in one of my papers that explicitly describes what the Mukai pairing on Hochschild homology descends to in Hodge cohomology via the Hochschild- Kostant-Rosenberg map twisted by the root Todd genus.