

A REVIEW OF “A GRADUATE INTRODUCTION TO NUMERICAL METHODS” BY CORLESS AND FILLION

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I just finished the 1.7 kilogram book “*A Graduate Introduction to Numerical Methods*” by Corless and Fillion [4]. I read the book chapter-by-chapter, covering about a chapter a week, taking it with me as I traveled to three different countries.¹ Every instructor of a graduate or advanced undergraduate numerical analysis course should consider teaching from this book. It deservedly made the ACM Computing Reviews’ list of notable books and articles in 2013 [2].

What is numerical analysis? This is a difficult question and your answer is likely to be a personal interpretation of the fuzzy interplay between “numerics” and “analysis”. The book’s 869-page answer is a delight. It is an inclusive celebration of Maple’s symbolic manipulations and MATLAB’s numerical computations, Turing’s mathematical modeling [8] and Strang’s linear algebra [7], Higham’s backward error analysis [5] and Trefethen’s practical approximation theory [9]. All the introductory numerical analysis topics are included such as interpolation, floating point arithmetic, numerical linear algebra, and differential equations, as well as less common topics such as the barycentric formula, resultants, automatic differentiation, and the Lambert W function. The authors are experts on these special topics, see [1, 3]. The broad range of topics in this book shows that numerical analysis is an impressively rich and mature subject.

The book comes in four parts: Part I on error analysis, polynomials, and rootfinding, Part II on numerical linear algebra, Part III on interpolation, differentiation, and quadrature, and Part IV on differential equations. There are also helpful appendices on floating-point arithmetic, complex numbers, and introductory linear algebra. To summarize the book I made a word cloud of the text (see Figure 1), where the size of a word indicates the frequency of its occurrence. One can also use this word cloud as a visual blurb or as a possible answer to “What is numerical analysis?”.

It is a pleasure to see that the three large words are “problem”, “method”, and “compute”. The key schema of computational mathematics! It is the schema that is repeated hundreds of times in the book. The authors are often in investigative mode, using Maple or MATLAB or formulas to discover something interesting. A broad selection of numerical methods are described during the numerous journeys of discovery, in a manner suitable for a textbook and in a way that is very different to *Numerical Recipes* [6], which is better designed for quick reference. The unwavering andragogical style to Corless and Fillion’s writing is masterful.

Next in the word cloud I notice the common words “polynomial” and “interpolate”. This highlights one of the unique features of the book: the extensive use of polynomial interpolants and the barycentric formula, both for practical computations and theoretical derivations. For example, how many textbooks use the barycentric formula for Hermite interpolants to derive cubic splines? Or, to develop multistep methods? Or, how many start by interpreting the discrete Fourier transform as a

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¹Security control in the United States will search a backpack containing a hardback copy of Corless and Fillion’s book because it is 3 inches thick. Each time one must explain the unusual carry-on item.

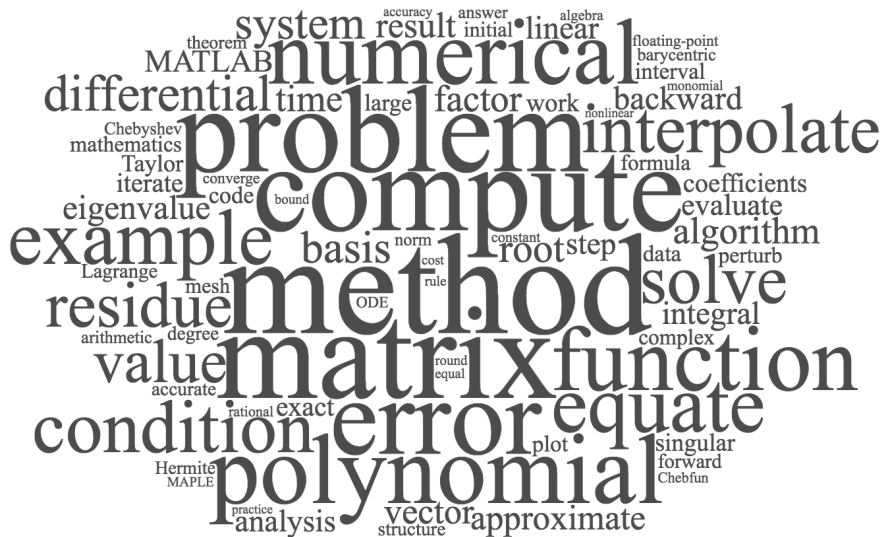


FIG. 1. A word cloud of the text and one possible answer to “What is numerical analysis?”

polynomial in the monomial basis evaluated at the roots-of-unity? The use of polynomial interpolants and the barycentric formula feels surprisingly fresh and modern.

Another common word is “example”, which appears on average about once per page. We know that numerical analysis is useful for a diverse range of applications, and the book considers examples on the orbit of three bodies under gravity, the spread of a measles epidemic, the flight path of a spinning golf ball, and others. The book’s example-oriented narrative is informal (see xkcd comic on p. 543 and footnotes) and well-illustrated with nearly 200 figures. This style makes the textbook engaging and friendly. An instructor could easily cover the examples given in the book during class; preferably, in a similar investigative mode.

Two other keywords that cannot be missed are “condition (number)” and “error”. The book uses backward error analysis and condition numbers throughout, not only for the numerical linear algebra chapters, but also for quadrature, rootfinding, and the numerical solution of differential equations. The error analysis is professionally satisfying and will be reassuring to conscientious learners. In the preface and afterwords, the authors present a thought-provoking discussion on the importance of backward error in numerical analysis, which is of independent interest and deserves attention.

There are so many intriguing features of the book that can be seen directly from the word cloud. For example, the word “residue” appears nearly 800 times because complex analysis is used extensively (another unique feature of the book), “MATLAB” is two-times larger than “Maple”, and for every occurrence of “bound” there is a “round” (suggesting a delightful balance between error analysis and floating point arithmetic).

More than just common words, there are also amusing maxims scattered throughout the book. Here is a selection.

It’s what a good numerical analyst does anyway. The book excels at teaching someone how to be a “good” numerical analyst. A student will get into the habit of using a computer to conjure up a theorem, a theorem to certify numerical computations, and a diagram to illustrate a concept. Each chapter also has a welcomed

Notes and References section that details places in the literature to find out more.

Which method is the best method? It's like a game of Rock-Paper-Scissors. In the game of rock-paper-scissors neither rock, paper, nor scissors always trumps the opponent's gesture. An analogous situation occurs for many classes of numerical techniques. The book has an unbiased didactic tone that is particularly prominent when describing quadrature rules and the numerical methods for solving differential equations. This style will keep the material relevant for several decades.

A pupil from whom nothing is ever demanded which (s)he cannot do, never does all (s)he can. Each chapter has a good selection of problems that examine both the practice and the theory. One could easily set homework containing questions directly from the book, assuming the solutions are not already online. The book's charming and informative language carries over to the problems too. For example, Problem 13.22 is on the so-called Obsessive-Compulsive Disorder Euler method. Each chapter also has a section called *Investigations and Projects* that contains more involved and open-ended questions. These will challenge the more advanced students.

I don't care how quickly you give me the wrong answer. The book's central philosophy on numerical algorithms is: *reliability first, cost last*. This makes error analysis and conditioning a central activity, but not necessarily algorithmic complexity. I think this is a shame. Numerical analysts care about reliability *and* speed. As just one example, Chapter 6 on structured matrices spends the majority of time on structured backward error analysis and only briefly mentions fast matrix-vector products and matrix factorizations. Researchers usually exploit structure in matrices for computationally cost, storage, and scalability to real-world problems, not for an improved theoretical error bound. I did not enjoy the *cost last* philosophy of the book.

A problem is "stiff" if, in comparison, ode15s mops the floor with ode45 on it. This is one of the more memorable informal definitions of a *stiff* differential equation, and highlights again the conversational style of the book. Numerical methods for solving boundary value problems, delayed differential equations, and partial differential equations as well as Runge–Kutta and multistep methods are expertly described. There is even a detailed discussion on event handling and the role of numerical methods for chaotic systems.

One particularly illustrative example is the classic Hénon–Heiles equation, which is a nonlinear nonintegrable Hamiltonian system given by

$$\ddot{x}(t) = -\frac{\partial V}{\partial x}, \quad \ddot{y}(t) = -\frac{\partial V}{\partial y}, \quad V(x, y) = \frac{1}{2} \left(x^2 + y^2 + 2x^2y - \frac{2}{3}y^3 \right). \quad (0.1)$$

The energy $E(t) = V(x(t), y(t)) + \frac{1}{2}(\dot{x}(t))^2 + \dot{y}(t)^2$ is conserved during motion and serves as a way to gauge the numerical error in a method. The book gives inline MATLAB code to solve (0.1) for $0 < t \leq 10^5$ using `ode113`, as well as code to plot a Poincaré map of the phase space using event handling. I could not resist trying it out for myself. Figure 2 shows Poincaré maps for two different energies, and on the right we see a beautiful figure showing chaotic behavior with “islands” of nonchaotic phase space. The energy was conserved up to 8-digits. The inline codes in the book are an excellent starting point for students to begin further experimentation.

Corless and Fillion have clearly dedicated several years of their lives to this book. It has been worth it! The result is a carefully judged all-round numerical analysis

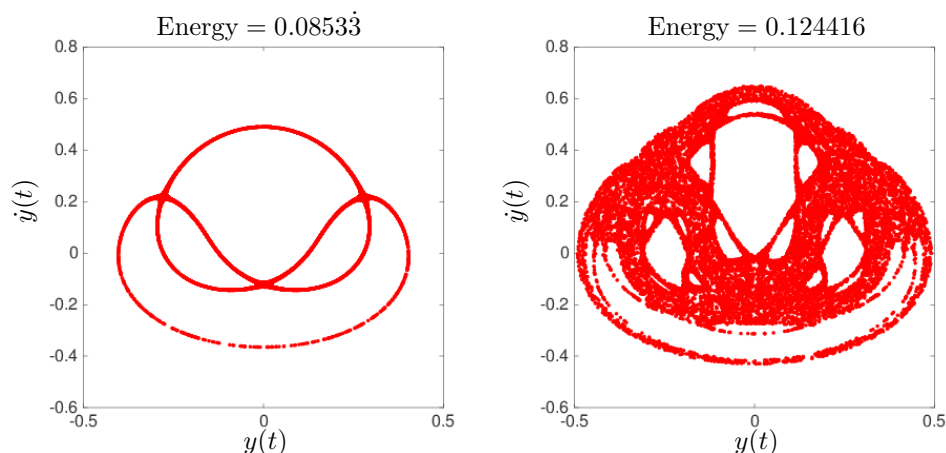


FIG. 2. Poincaré map of the Hénon–Heiles system for energy $32/375$ with $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0.2$ (left) and energy 0.124416 with $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0.24$ (right, see [4, Fig. 12.12]). A red dot is plotted in the phase space $(y(t), \dot{y}(t))$ whenever $x(t) = 0$ and $\dot{x}(t) > 0$. On the right one observes chaotic behavior with “islands” of nonchaotic phase space

textbook that expertly serves students and instructors. The book stands out in a crowded market and deserves worldwide acclaim. It is a textbook that numerical analysis can be very proud of.

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