18th IMA Leslie Fox Prize in Numerical Analysis

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July 24, 2017

On Monday the 26th of June 2017, six of our young and bright numerical analysts presented their work at the University of Strathclyde for a chance to win the prestigious Leslie Fox prize. The topics of the 40-minute talks ranged from multipreconditioning to manifold-valued functions and randomized linear algebra. The six talks were of high quality.

David Griffiths started the day by describing Professor Leslie Fox himself. Leslie was a prominent British numerical analyst at the University of Oxford, who was particularly noted for his work on relaxation methods, finite difference methods, and numerical methods for partial differential equations. As David recalled, he was one of the last numerical analysts to use human computers! After Leslie's retirement in 1983, former students and colleagues set up the Leslie Fox Prize in recognition of his broad contributions to Numerical Analysis. The IMA subsequently took over the administration of the prize.

The audience were then treated to six delightful presentations. For the reader's convenience, we briefly describe the talks in the same order that the presentations were given on the day.

Aretha Teckentrup from the University of Edinburgh gave the first talk on "Posterior consistency for Gaussian process approximations of Bayesian posterior distributions" [7]. In many Bayesian inverse problems, sampling from a posterior distribution via Markov Chain Monte Carlo methods requires the evaluation of a model. For computational efficiency, the model can be replaced by a surrogate that is based on Gaussian emulators. Despite their widespread use, the error introduced by these emulators was not well understood. Aretha showed how to rigorously bound the distance between the true posterior distribution and different Gaussian emulator approximations. These results can then bound errors in other quantities of interest such as the error in expected values.

Nicole Spillane from École Polytechnique spoke about "An adaptive multipreconditioned conjugate gradient algorithm" [6]. Her work improves on the iterative method for symmetric positive definite linear systems introduced by Bridson and Greif [2] that chooses, at each iteration, an optimal combination from a set of preconditioners. Nicole introduced a test that automatically determines whether multipreconditioning is useful, and showed that the resulting adaptive algorithm can save considerable computation on a range of problems. Her ideas are ideally suited to balancing domain decomposition (BDD) preconditioners, since each local subdomain solve can be treated as a separate preconditioner. She demonstrated the method on several industrial applications involving heterogeneous materials, one of them being a composite weave pattern.

Robert Gower from École Normale Supérieure told us about "Randomized iterative methods for linear systems" [4]. He introduced a two-parameter family of sketch-and-project iterative solvers for $Ax = b$ that had six equivalent interpretations. Particular choices of the two parameters—an inner product and a random matrix—give the randomized Kaczmarz method, randomized Newton method, randomized coordinate descent method, and random Gaussian pursuit as special cases. Using this framework, Robert was able to develop new variants of classical algorithms and make links between existing methods. He presented a unified proof of linear convergence in a single theorem that recovered many of the known asymptotic convergence rates of existing methods.

After the morning talks we paused for a buffet lunch in the John Anderson building with some time to chat to all the candidates. Around 1:30pm we were ushered back into the lecture room for the final three talks.

Mario Berljafa from KU Leuven presented a talk titled "Generalized rational Krylov decompositions with an application to rational approximation" [1] in which he characterised the relationship between the rational Arnoldi matrix decomposition and rational Krylov spaces. His rational implicit Q theorem shows that the rational Arnoldi decomposition is essentially determined by a column vector and a set of poles. The

theorem provides a new perspective on rational Krylov spaces, allowing for better theoretical and algorithmic understanding. These ideas were then employed for rational least squares fitting in RKFIT, which is a toolbox for computations with rational functions.

Lise-Marie Imbert-Gérard spoke about "Interpolation properties of generalized plane waves" [5]. A new set of generalized plane waves were derived to obtain a numerical method for the scalar wave equation with smoothly varying coefficients. Lise-Marie employed a Trefftz finite element method in which shape functions approximately satisfy a homogeneous differential equation on each element. There was an ingenious construction of the underlying shape functions for variable coefficient problems that involved a hierarchy of linear subsystems. Their interpolation properties were also studied, and the order of convergence of the method determined.

Evan Gawlik gave the last talk on "Interpolation of manifold-valued functions via the polar decomposition" [3]. He showed us how the generalized polar decomposition could be used for the interpolation and extrapolation of manifold-valued data on symmetric spaces—smooth manifolds with inversion symmetry about every point. It was a general framework for constructing finite elements on Lorentzian metrics as well as computing the exponential map on the Grassmannian. Numerical experiments indicated that the interpolant operators have optimal approximation properties.

After the talks the committee deliberated for a short while to discuss the various merits of each presentation. During this time the audience were wagering small cash notes. The committee entered the room at exactly 4:30pm and the room hushed in anticipation. Andy Wathen announced that Nicole Spillane received the first-place prize for her multipreconditioning work, while the others were given second-place prizes. David Griffiths had the pleasure of officially presenting the awards to the prize winners. Everyone applauded the candidates and all the talks throughout the day. A photo was then taken with all the participants (see Figure 1).

Figure 1: Participants of the Leslie Fox prize meeting in 2017. Front row, left to right: Andy Wathen, Nicole Spillane, and David Griffiths; middle row: Aretha Teckentrup, Lise-Marie Imbert-Gérard, Robert Gower, Evan Gawlik, and Ben Leimkuhler; back row: Des Higham and Mario Berljafa.

This year was especially competitive. There were a total of 37 submissions to the Leslie Fox prize from 14 different countries: Australia, Belgium, Canada, China, England, France, Germany, Hong Kong, Hungary, Italy, Japan, Scotland, Sweden and the USA. We believe this is a record number of submissions, demonstrating that the prize is being widely advertised.

Many of the previous prize winners have gone on to have distinguished careers in numerical analysis and the authors believe the same will happen for this shortlist too. The day was a wonderful celebration of the emerging talent in numerical analysis.

If you are under 31 years old on the 1st of January 2019 and have authored a worthy paper in numerical analysis, then please consider submitting for the next Leslie Fox prize meeting. If you will be over 31 years

old, then please encourage your younger colleagues.

References

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