MATH 2230, FALL 2014

HOMEWORK #2, DUE THURSDAY SEP 11 AT BEGINNING OF CLASS

Exercises from the book [Hubbard and Hubbard, 4th edition]:

- 1.2.1b, 8, 13, 16, 17

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1. Let M be a "strictly upper triangular" $n \times n$ matrix, meaning, if $M_{ij} \neq 0$ then $i < j$.

a. Show that the first d diagonals above the middle of M^d vanish, i.e. $(M^d)_{ij} \neq 0$ implies $i + d \leq j$, for each $d > 0$.

b. Show that $M^n = 0$.

c. Use the infinite series $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots$ to guess a *finite* formula for the inverse matrix to $I + M$, where I is the $n \times n$ identity. Prove that your guess is indeed an inverse to $I + M$ (i.e. on both sides).

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2. Let M be a strictly upper triangular $n \times n$ matrix, with only entries $0, 1$, and such that $M_{ij} = M_{jk} = 1$ implies $M_{ik} = 1$, for all numbers $i < j < k$ in $[1, n]$. This is a way to picture a hierarchy like we did in class, where employee j reports to employee i if $M_{ij} = 1$ (and not 0).

a. Let $\vec{v}$ be a vector. Interpret the entries of $(I + M)\vec{v}$ in English.

b. Show that $I + M$ is invertible, and interpret its entries in English.