HW # 5 DUE THURSDAY, OCTOBER 5
MATH 6670, FALL 2017

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Given a cone $C \leq \mathbb{R}^n$ defined by finitely many integral linear inequalities, we define $TV_C := \mathrm{Specm}\, \mathbb{C}[C \cap \mathbb{Z}^n]$ as its associated **toric variety**.

#1. Let $F \leq C$ be a face of C, i.e. where some of those inequalities are actually equalities. Show that $TV_F \hookrightarrow TV_C$ by identifying F's ring with the quotient of C's by an ideal.

#2. Let S be the blowup algebra, blowing up $\mathbb{C}^2$ at the origin.
Find a cone C whose associated algebra is S.
(It'll be 3-dimensional, because TV_C will be the affine cone over $\widetilde{\mathbb{C}^2}$, not $\widetilde{\mathbb{C}^2}$ itself.)

#3. Let S be the blowup algebra blowing up $\mathbb{C}^3$ along $xy = 0$. Find a cone C whose associated algebra is S.

#4. Determine the singular locus of $\mathrm{Specm}\, S$ and $\mathrm{Projm}\, S$ (from the question above).

#5. Let S, T be two finitely generated algebras over $\mathbb{C}$ (not necessarily coming from cones). Relate the spaces $\mathrm{Specm}\, S$, $\mathrm{Specm}\, T$, $\mathrm{Specm}\, (S \otimes_{\mathbb{C}} T)$, and also relate their singular loci.