

The reduced row-echelon  $2 \times 4$  matrices look like these:

$$\begin{aligned} & \begin{pmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \end{pmatrix} \begin{pmatrix} 1 & * & 0 & * \\ 0 & 0 & 1 & * \end{pmatrix} \begin{pmatrix} 1 & * & * & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{pmatrix} \begin{pmatrix} 0 & 1 & * & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ & \begin{pmatrix} 1 & * & * & * \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Over  $\mathbb{F}_p$ , there are  $p$  possibilities for each  $*$ . So the number of such matrices is

$$p^4 + p^3 + p^2 + p^2 + p + 1$$

plus

$$p^3 + p^2 + p + 1$$

plus

$$1.$$