Sections 13.7, 14.1, 14.2

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Summary of the Sections

(1) Conversion from rectangular coordinates to cylindrical/spherical coordinates:

(a) **Cylindrical:** $r = \sqrt{x^2 + y^2}$ $\quad \tan(\theta) = \frac{y}{x}$ $\quad z = z$

(b) **Spherical:** $\rho = \sqrt{x^2 + y^2 + z^2}$ $\quad \tan(\theta) = \frac{y}{x}$ $\quad \cos(\phi) = \frac{z}{\rho}$

With $0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq \pi$.

(2) Conversion from cylindrical/spherical to rectangular:

(a) **Cylindrical:** $x = r\cos(\theta)$ $\quad y = r\sin(\theta)$ $\quad z = z$

(b) **Spherical:** $x = \rho\cos(\theta)\sin(\phi)$ $\quad y = \rho\sin(\theta)\sin(\phi)$ $\quad z = \rho\cos(\phi)$

(3) A vector valued function is a function of the form

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

(4) The underlying curve C traced by $\mathbf{r}(t)$ is the set of points $(x(t), y(t), z(t))$ in space for t in the domain of $\mathbf{r}(t)$.

(5) Every curve can be parametrized in infinitely many ways.

(6) The projection of $\mathbf{r}(t)$ onto the xy-plane is the curve traced by $\langle x(t), y(t), 0 \rangle$. The projections onto the rest of the plane are found similarly.

(7) Limits, differentiation, and integration of vector-valued functions are performed componentwise.

(8) Differentiation rules:

(a) Sum rule: $(\mathbf{r_1}(t) + \mathbf{r_2}(t))' = \mathbf{r_1}'(t) + \mathbf{r_2}'(t)$

(b) Constant Multiple Rule: $(c\mathbf{r}(t))' = c\mathbf{r}'(t)$.

(c) Chain Rule: $\frac{d}{dt}\mathbf{r}(g(t)) = g'(t)\mathbf{r}'(g(t))$

(9) Product Rules:

(a) Scalar times a vector: $\frac{d}{dt}(f(t)\mathbf{r}(t)) = f'(t)\mathbf{r} + f(t)\mathbf{r}'(t)$

(b) Dot product: $\frac{d}{dt}(\mathbf{r_1}(t) \cdot \mathbf{r_2}(t)) = \mathbf{r_1}'(t) \cdot \mathbf{r_2}(t) + \mathbf{r_1}(t) \cdot \mathbf{r_2}'(t)$

(c) Cross Product: $\frac{d}{dt}(\mathbf{r_1}(t) \times \mathbf{r_2}(t)) = \mathbf{r_1}'(t) \times \mathbf{r_2}(t) + \mathbf{r_1}(t) \times \mathbf{r_2}'(t)$

(10) The tangent line to $\mathbf{r}(t)$ at $\mathbf{r}(t_0)$ is given by the vecotr parametrization:

$$\mathbf{L}(t) = \mathbf{r}(t_0) + t\mathbf{r}'(t_0)$$

(11) The general solution to $\mathbf{R}'(t) = \mathbf{r}(t)$ is given by $\mathbf{R}(t) = \int \mathbf{r}(t)dt + \mathbf{c}$, where integration is done componentwise.

Problems

(1) Express the following points in cylindrical and spherical coordinates:

   (a) $(3, 2, 0)$
   (b) $(-2, 1, 1)$

(2) Express the following constraints (regions in space) in spherical coordinates:

   (a) The (filled) unit ball
   (b) The space between two spheres of radii 4 and 5 respectively
   (c) The region given by $x^2 + y^2 + z^2 \geq 4$, $y = \sqrt{3}x$ and $y \leq 0$.

(3) Draw the region given in spherical coordinates by $0 \leq \theta \leq \frac{\pi}{3}$ and $\rho \leq 3$.

(4) How can you express the surface given by $6x^2 - 8z^{\frac{1}{3}} + 5y^2 - 15 = 0$ in cylindrical coordinates?

(5) Find the radius, center and plane containing the following circles:

   (a) $\mathbf{r}(t) = 7\mathbf{i} + (12\cos t)\mathbf{j} + (12\sin t)\mathbf{k}$.
   (b) $\langle \sin t, 0, 4 + \cos t\rangle$.

(6) Let $\mathbf{r_1} = \langle 3t^2 - 2,\ ln(5t^4 + 4),\ 4t + 5\rangle$ and $\mathbf{r_2} = \langle 5t + 3,\ e^{6t-7},\ 7t + 2\rangle$ be two vector valued functions. Do they collide? Do they intersect? If so find the points of collision/intersection.

(7) Let $\mathbf{r}_1(t) = \langle t^2, t^3, t\rangle$ and $\mathbf{r}_2(t) = \langle e^{3t}, e^{2t}, e^t\rangle$. Evaluate the derivative by using the appropriate product rule.

   (a) $\frac{d}{dt}(\mathbf{r}_1(t) \cdot \mathbf{r}_2(t))$.
   (b) $\frac{d}{dt}(\mathbf{r}_1(t) \times \mathbf{r}_2(t))$.

(8) Solve the following initial value problem:

$$\mathbf{r}''(t) = \left\langle 3 - t,\ sin(9\pi t),\ \frac{-7}{x^2} \right\rangle, \qquad \mathbf{r}'(1) = \langle 2, 0, 1\rangle, \qquad \mathbf{r}(1) = \langle 4, 0, 2\rangle$$

(9) A fighter plane, which can shoot a laser beam straight ahead, travels along the path

$$\mathbf{r}(t) = \langle t - t^3, 12 - t^2, 3 - t\rangle.$$

Show that the pilot cannot hit any target on the x-axis.