# Sections 14.3, 15.1

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## Summary of the Sections

(1) A vector valued function is a function of the form $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ for $a \leq t \leq b$ is

$$s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt$$

(2) Arc length function : $s(t) = \int_a^t \|\mathbf{r}'(u)\| du$

(3) Speed is the derivative of distance with respect time:

$$v(t) = \frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

(4) We say that $\mathbf{r}(s)$ is an arc length parametrization if $\|\mathbf{r}'(s)\| = 1$ for all $s$.

(5) A level curve is the set of points given by an equation $f(x, y) = c$ with $c$ a fixed number.

(6) A contour map shows level curves $f(x, y) = c$ for equally spaced value of $c$.

(7) The closest two level curves are at a given point, the steepest the graph of $f$ is there.

(8) The direction of steepest ascent is always perpendicular to the level curve, and points towards higher altitudes.

## Problems

(1) Find the arc length for $1 \leq t \leq 3$:

(a) $\mathbf{r}(t) = \left\langle t, 4t^{\frac{3}{2}}, t^{\frac{3}{2}} \right\rangle$

(b) $\mathbf{r}(t) = \langle 2t+1, 8, 3t-4 \rangle$

(c) $\mathbf{r}(t) = \left\langle 4t^2+3, 6t^3, -3+t^3 \right\rangle$

(d) $\mathbf{r}(t) = \langle 5\sin(t)+2, t, 5\cos(t) \rangle$

(2) Find the speed of the curves at the time $t = 0$:

(a) $\mathbf{r}(t) = \langle \sin(3t),\ \sin(3t-\pi),\ t\cos(7t) \rangle$

(b) $\mathbf{r}(t) = \langle e^{t^2}+4t,\ ln(t^3+1),\ \sqrt{4t^4+1} \rangle$

(c) $\mathbf{r}(t) = \langle 3t^5 - 6e^t,\ \tan(3t),\ t-1 \rangle$

(3) The velocity of an airplane is given by $\mathbf{r}'(t) = \langle 3-4t^2, e^t-4, 5t^4 \rangle$. If the airplane starts at $t = 0$ at the point $P = (3, 6, 0)$

(a) Where is the airplane at time $T$?

(b) What is the total distance the airplane has flown by time $T$?

(4) Refer to the contour map drawn on the board:

(a) What is the average rate of change of altitude from point A to point B?

(b) What is the direction of steepest ascent in each of the points?

(c) Is the mountain steeper at point $A$ or at point $B$?

(5) If the atmospheric pressure at certain region is given by $P(x, y, z) = (x-4)^2 + \frac{25}{4}y^2 + 4(z-3)^2$, describe the isobars with pressures 3 and 0.