Sections 15.3, 15.4, 15.5

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Math 1920 - Andres Fernandez

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Summary of the Sections

1. For small small changes Δx, Δy we have:

$$f(a+\Delta x, b) \approx f(a,b) + f_x(a,b)\Delta x$$

$$f(a, b+\Delta y) \approx f(a,b) + f_y(a,b)\Delta y$$

2. Clairaut's theorem states that mixed partials are equal as long as all functions we are dealing with are continuous. Hence, we can take higher partial derivatives in any order we please.

3. The linearization of$f$ in two and three variables:

$$L(x,y) = f(a,b) + f_x(a,b)(x-a)f_y(y-b)$$

$$L(x,y,z) = f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$$

4. If f_x and f_y exist and are continuous in a disk containing (a,b), then f is differetiable at (a,b).

5. **Equation fo the tangent plane** to z= f(x,y) at (a,b).

$$z = f(a,b) + f_x(a,b)(x-a)f_y(y-b)$$

6. The **gradient** of a function f is fiven by $\nabla f = \left\langle \frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f, \frac{\partial}{\partial z} f \right\rangle$.

7. **Chain rule for paths**: $\frac{d}{dt} f(\mathbf{r}(t)) = \nabla f_{\mathbf{r}(t)} \cdot \mathbf{r}'(t)$

8. The directional derivative with respect to $\mathbf{u}$ a unite vector is given by $D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u}$

9. If the angle between $\mathbf{u}$ and ∇f is θ, then $D_{\mathbf{u}} f = \|\nabla f\| \|\mathbf{u}\| \cos(\theta)$

10. The equation of the tangent plane to the level surface $F(x,y,z) = k$ at point $P = (a,b,c)$ is

$$\nabla F_P \cdot \langle x-a, y-bz-c \rangle$$

Problems

1. Suppose that we have a cylinder of radius $r = 90$cm and height $h = 6$cm. Estimate the change in volume if we increase the radius by 2cm.

2. Find f such that:

   (a) $\frac{\partial}{\partial x} f = 6x^2 y$, and $\frac{\partial}{\partial y} f = 2x^3 - 3$

   (b) $\frac{\partial}{\partial x} f = e^x - y \sin(xy)$, and $\frac{\partial}{\partial y} f = -x \sin(xy) + 5y^4$

3. Find the tangent plane to the following graphs at the given point:

   (a) $f(x, y) = ln(4x^2 - y^2)$ at $(1, 1)$

   (b) $f(x, y) = e^{\frac{x}{y}}$ at $(2, 1)$

4. At which points is the vector $\mathbf{n} = \langle 2, 7, 2 \rangle$ normal to the tangent plane of $z = xy^3 + 8y^{-1}$?

5. Let $H(x, y) = 4x^{\frac{3}{2}} y^4$. Show that we have $\frac{\Delta H}{H} \approx \frac{3}{2} \frac{\Delta x}{x} + \frac{\Delta y}{y}$. If x is increased by 5% and y is decreased by 6%, what is the percentage of change of H? Does H increase or decrease?

6. Determine whether the derivative of the function along the path is zero, positive or negative in each of the points A, B, C (see blackboard).

7. Determine the derivative of the function along the path given:

   (a) $f(x, y, z) = yx^2 - e^{xy} + ln(x)$ $\mathbf{r}(t) = \langle t^2, ln(t), \sqrt{t} \rangle$

   (b) $f(x, y, z) = \sin(xyz)$ $\mathbf{r}(t) = \langle e^t, \cos(4t), -t \rangle$

8. Find the normal vector to the ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$ at $(9, 0, 0$. When is the tangent plane normal to the vector $\langle 0, 1, 2 \rangle$?

9. Find a function such that $\nabla f = \left\langle 2xe^y - z, y^3 + x^2 e^y, -x \right\rangle$