

PROBLEMS

- (1) Find critical points of the following functions. Determine whether they are local maxima, minima or saddle points.
 - (a) $xy^3 + 2y - x - 5$
 - (b) $e^x - xe^y$
 - (c) $x \ln(x+y)$
- (2) Find extremal points of the following functions with the given constraints:
 - (a) $f(x, y, z) = xy + 2z$ $x^2 + y^2 + z^2 = 36$
 - (b) $f(x, y, z) = x^2 - y - z$ $x^2 - y^2 + z = 0$
- (3) Consider the ellipsoid $3x^2 + y^2 - 2xy + 4z^2 = 9$. What is the maximal y coordinate in this ellipse?
- (4) Three positive numbers add up to 70. What is the biggest their product can be? When does this happen?
- (5) What is the point in the plane $2x - 6y + z = 7$ that is closest to $P = (1, 1, 1)$
- (6) Prove the following special case of the Cauchy-Schwarz inequality $\sqrt{3} \cdot \sqrt{a^2 + b^2 + c^2} \geq a + b + c$ using Lagrange multipliers.
- (7) Maximize $x^a y^b z^c$ on the unit sphere intersected with $x, y, z \geq 0$