

SUMMARY OF THE SECTIONS

(1) Double integral in *polar coordinates*:

$$\iint_{\mathcal{D}} f(x, y) dA = \int_{\theta_1}^{\theta_2} \int_{r=r_1(\theta)}^{r_2(\theta)} f(r \cos \theta, r \sin \theta) r dr d\theta.$$

(2) Triple integral in *cylindrical coordinates*:

$$\iiint_{\mathcal{R}} f(x, y, z) dV = \int_{\theta_1}^{\theta_2} \int_{r=r_1(\theta)}^{r_2(\theta)} \int_{z=z_1(r, \theta)}^{z_2(r, \theta)} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta.$$

(3) Triple integral in *spherical coordinates*:

$$\iiint_{\mathcal{R}} f(x, y, z) dV = \int_{\theta_1}^{\theta_2} \int_{\phi=\phi_1}^{\phi_2} \int_{\rho=\rho_1(\theta, \phi)}^{\rho_2(\theta, \phi)} f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta.$$

PROBLEMS

(1) Sketch the region indicated and integrate $f(x, y)$ over it using polar coordinates.

(a) $f(x, y) = y(x^2 + y^2)^3$; $y \geq 0, x^2 + y^2 \leq 1$. (b) $f(x, y) = e^{x^2+y^2}$; $x^2 + y^2 \leq R$.

(2) Sketch the region and evaluate the integral using polar coordinates.

(a) $\int_0^2 \int_x^{\sqrt{3}x} y \, dy \, dx$ (b) $\int_{-1}^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) \, dy \, dx$

(3) Use cylindrical coordinates to calculate $\iiint_{\mathcal{W}} f(x, y, z) dV$ for the given function and region.

(a) $f(x, y, z) = xz$; $x^2 + y^2 \leq 1, x \geq 0, 0 \leq z \leq 2$. (b) $f(x, y, z) = x^2 + y^2$; $x^2 + y^2 \leq 9, 0 \leq z \leq 5$.

(4) Use cylindrical coordinates to find the volume of the region bounded below by the plane $z = 1$ and above by the sphere $x^2 + y^2 + z^2 = 4$

(5) Use spherical coordinates to calculate the triple integral of $f(x, y, z)$ over the given region.

(a) $f(x, y, z) = y$; $x^2 + y^2 + z^2 \leq 1, x, y, z \geq 0$. (b) $f(x, y, z) = x^2 + y^2$; $\rho \leq 1$.