

PROBLEMS

- (1) Calculate the flow rate of a fluid with velocity vector field $\mathbf{v} = \langle 2x, y, xy \rangle$ across the part of the cylinder $x^2 + y^2 = 9$ where $x, y \geq 0$ and $0 \leq z \leq 4$.
- (2) You are asked to compute the line integral of $\mathbf{F} = \langle 5y, 3 \sin(y) \rangle$ over the path enclosing $\frac{5}{7}$ of a circle (oriented counterclockwise), as shown on the board. Use Green's theorem (otherwise it's a pain).
- (3) Use Green's theorem to do the following computations
 - (a) The area of the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$
 - (b) The circulation of $\mathbf{F} = \langle y, 3x \rangle$ along the boundary of the of radius R centered at the point $(4, 6)$.
- (4) Suppose that you know the circulation of $\mathbf{F}$ along the boundary of the circle of radius 6 (oriented counterclockwise) is 20. What is the circulation along the circumference of radius 1 (oriented counterclockwise) if we know that $\frac{\partial}{\partial x} F_2 - \frac{\partial}{\partial y} F_1 = \cos(x^2 + y^2)$ on the annulus $1 \leq r \leq 6$?
- (5) What is the flux of the vector field $\mathbf{F} = \langle 4x^3, 4y^3 - y^2 \rangle$ across the boundary of the circle of radius 5 centered at the origin?