

PROBLEMS

(1) Use the chain rule to compute the partial derivatives:

(a) Find $\frac{\partial f}{\partial v}$, $\frac{\partial f}{\partial u}$ given $f(x, y) = \cos(x - y)$, $x = 3u - 5v$ and $y = -7u + 15v$.

(b) Find $\frac{\partial f}{\partial \theta}$ given $f(x, y, z) = xy - z^2$, $x = r \cos \theta$, $y = \cos^2 \theta$ and $z = r$.

(2) Compute $\frac{\partial z}{\partial y}$ where $e^{xy} + \sin(xz) + y = 0$.

(3) Suppose that f is a function of x and y , where $x = g(t, s)$ and $y = h(t, s)$. Show that f_{tt} is equal to

$$f_{xx} \left(\frac{\partial x}{\partial t} \right)^2 + 2f_{xy} \left(\frac{\partial x}{\partial t} \right) \left(\frac{\partial y}{\partial t} \right) + f_{yy} \left(\frac{\partial y}{\partial t} \right)^2 + f_x \left(\frac{\partial^2 x}{\partial^2 t} \right) + f_y \left(\frac{\partial^2 y}{\partial^2 t} \right)$$

(4) Find critical points of the following functions. Determine whether they are local maxima, minima or saddle points.

(a) $xy^3 + 2y - x - 5$

(b) $e^x - xe^y$

(c) $x \ln(x+y)$

(5) What is the point in the plane $2x - 6y + z = 7$ that is closest to $P = (1, 1, 1)$