

PROBLEMS

(1) Compute the following integrals

(a) $\int_{-1}^1 \int_0^\pi x^2 \sin(y) \, dy \, dx$

(b) $\int_0^1 \int_0^2 (x + 4y^3) \, dx \, dy$

(2) Compute the following integrals using symmetry

(a) $\int \int_{\mathcal{R}} \sin(x) \, dA, \quad \mathcal{R} = [0, 2\pi] \times [0, 2\pi]$

(b) $\int \int_{\mathcal{R}} y^2 x^3 \, dA, \quad \mathcal{R} = [-4, 4] \times [8, 10]$

(3) Use Fubini's theorem to compute the following

(a) $\int_0^1 \int_0^\pi y \sqrt{1 + xy} \, dy \, dx$

(b) $\int_0^1 \int_0^\pi x e^{xy} \, dx \, dy$

(4) Compute the following integrals over the respective domains

(a) $f(x, y) = x$ over the domain $0 \leq x \leq 1, \quad 1 \leq y \leq e^{x^2}$

(b) $f(x, y) = \sin(x)$ over the domain bounded by $x = 0, \quad x = 1, \quad y = \cos(x)$

(5) Compute the integral of $f(x, y, z) = x$ in the region given by $x, y, z \geq 0$ above $z = y^2$ and below $z = 8 - 2x^2 - y^2$

(6) Find the volume of the region bounded by $z = 40 - 10y, \, z = 0, \, y = 0,$ and $y = 4 - x^2$

(7) You have a ceiling of a rectangular building given by $z = y^2 \sin(x)$ over the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1$. What is the average height of the ceiling?