

PROBLEMS

- (1) Show that the integral along any closed path of a conservative vector field is 0.
- (2) Find the integral of the following vector fields along the path given in the board, or explain why it cannot be done with the information given:

(a) $\langle z \sec^2(x), z, y + \tan(x) \rangle$

(b) $\langle y, e^{xy}, e^z \rangle$

(c) $\langle 2yx \cos(z) + 3x^2, x^2 \cos(z) - 3y^2 \sin(z), -x^2 y \sin(z) + 3z - y^3 \cos(z) \rangle$

(d) $\langle \frac{\tan(x) \cos(2y^2)}{x^2+y^2}, \frac{\tan(x)+3x^2-y}{x^2+y^2}, \frac{2y^2-5 \tan(y)+3zx^4}{x^2+y^2} \rangle$

- (3) Calculate the surface integral in each of the following cases:

(a)

$$\iint_{\mathcal{D}} x^2 dS \quad \text{where } \mathcal{D} \text{ is given by } x^2 + y^2 + z^2 = 9 \text{ and } x, y, z \geq 0$$

(b)

$$\iint_{\mathcal{D}} z dS \quad \text{where } \mathcal{D} \text{ is given by } x + y + z = 1 \text{ and } x, y, z \geq 0$$

(c)

$$\iint_{\mathcal{D}} e^{-z} dS \quad \text{where } \mathcal{D} \text{ is a vertical cylinder of radius 4 and bounded between } 0 \leq z \leq 4$$

- (4) Find the surface area of the part of the cone given by $x^2 + y^2 = z^2$ between the planes $z = 2$ and $z = 5$.