

## PROBLEMS

- (1) Compute the integral  $\int \int_{\mathcal{S}} x^2 y \, dS$ , where  $\mathcal{S}$  is the surface given by  $z = \sqrt{3}x + y^2$ ,  $-1 \leq x \leq 1$ , and  $0 \leq y \leq 1$ .
- (2) Calculate  $\int \int_{\mathcal{S}} (x^2 + y^2) e^{-z} \, dS$ , where  $\mathcal{S}$  is the cylinder given by  $x^2 + y^2 = 9$  and  $0 \leq z \leq 10$ .
- (3) Compute flux of the vector field  $\mathbf{F} = \langle x, y, z \rangle$  through the part of the unit sphere with  $0 \leq z \leq \frac{\sqrt{3}}{2}$  with normal pointing outwards.
- (4) Calculate the flow rate of a fluid with velocity vector field  $\mathbf{v} = \langle 2x, y, xy \rangle$  across the part of the cylinder  $x^2 + y^2 = 9$  where  $x, y \geq 0$  and  $0 \leq z \leq 4$ .
- (5) Suppose that you submerge a closed surface (for example a sphere) into the ocean. What is the net flux of water through the surface?  
  
Now you can answer the following question without a single computation: find the flux of the constant vector field  $\langle 2, 3, 6 \rangle$  along the unit sphere centered at  $(3, 1, 5)$ .
- (6) Calculate the flow rate of a fluid with velocity vector field  $\mathbf{v} = \langle 2x, y, xy \rangle$  across the part of the cylinder  $x^2 + y^2 = 9$  where  $x, y \geq 0$  and  $0 \leq z \leq 4$ .
- (7) Determine the flow for the vector field  $\mathbf{v} = \langle x - y, z + y + 4, z^2 \rangle$  through the surface given by  $y = 1 - x^2 - z^2$  and  $y \geq 0$  oriented in the positive  $y$  direction.