# Section 16.4

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Name: Solutions

October 18, 2018

## Summary of the Sections

(1) Double integral in *polar coordinates*:

$$\iint_{\mathcal{D}} f(x,y)\,dA = \int_{\theta_1}^{\theta_2} \int_{r=r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta.$$

(2) Triple integral in *cylindrical coordinates*:

$$\iiint_{\mathcal{R}} f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2} \int_{r=r_1(\theta)}^{r_2(\theta)} \int_{z=z_1(r,\theta)}^{z_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta.$$

(3) Triple integral in *spherical coordinates*:

$$\iiint_{\mathcal{R}} f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2} \int_{\phi=\phi_1}^{\phi_2} \int_{\rho=\rho_1(\theta,\phi)}^{\rho_2(\theta,\phi)} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\, \rho^2\sin\phi\, d\rho\, d\phi\, d\theta.$$

## Problems

(1) Sketch the region indicated and integrate $f(x,y)$ over it using polar coordinates.

(a) $f(x,y) = y(x^2+y^2)^3; \quad y \geq 0, x^2+y^2 \leq 1.$
SOLUTION: $0 \leq \theta \leq \pi, \quad 0 \leq r \leq 1$, the integral is $2/9$.

(b) $f(x,y) = e^{x^2+y^2}; \quad x^2+y^2 \leq R.$
SOLUTION: $0 \leq \theta \leq 2\pi, \quad 0 \leq r \leq \sqrt{R}$, the integral is $\pi(e^R - 1)$.

(2) Sketch the region and evaluate the integral using polar coordinates.

(a) $\int_0^2 \int_x^{\sqrt{3}x} y \, dy \, dx$
SOLUTION: $\frac{8}{3}$.

(b) $\int_{-1}^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2) \, dy \, dx$

SOLUTION: Divide the region in $\mathcal{D}_1 = \{2\pi/3 \leq \theta \leq \pi, 0 \leq r \leq -\sec\theta\}$ and $\mathcal{D}_2 = \{0 \leq \theta \leq 2\pi/3, 0 \leq r \leq 2\}$. The result is $\frac{\sqrt{3}}{2} + \frac{8\pi}{3}$.

(3) Use cylindrical coordinates to calculate $\iiint_{\mathcal{W}} f(x,y,z) dV$ for the given function and region.

(a) $f(x,y,z) = xz; \ x^2+y^2 \leq 1, x \geq 0, 0 \leq z \leq 2.$
SOLUTION: $\frac{4}{3}$.

(b) $f(x,y,z) = x^2+y^2; \ x^2+y^2 \leq 9, 0 \leq z \leq 5.$
SOLUTION: $\frac{5 \cdot 3^4 \pi}{2}$.

(4) Use cylindrical coordinates to find the volume of the region bounded below by the plane $z = 1$ and above by the sphere $x^2+y^2+z^2 = 4$

SOLUTION: $\frac{5\pi}{3}$.

(5) Use spherical coordinates to calculate the triple integral of $f(x,y,z)$ over the given region.

(a) $f(x,y,z) = y; \ x^2+y^2+z^2 \leq 1, \ x,y,z \leq 0.$
SOLUTION: $-\frac{\pi}{16}$.

(b) $f(x,y,z) = x^2+y^2; \ \rho \leq 1.$
SOLUTION: $\frac{8\pi}{15}$.