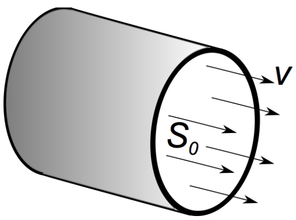
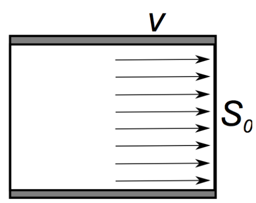
Math 1920 Workshop 1: Flux

This workshop is an application of the dot product: flux. Consider a fluid flowing through a region in space. Imagine a slice or a cross-section of the region. Then the volume of fluid that flows across the cross-section per unit time is called the flux.

Problem 1 – Flux through a pipe

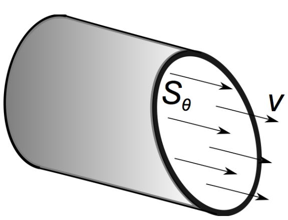
a) Fluid is flowing down a pipe at velocity $\vec{v} = 5$ m/sec. The cross sectional area S_0 of the pipe is 3 m^2. What is the volume of fluid flowing out of the pipe each second? This flow rate is called the volumetric flux of the fluid velocity.

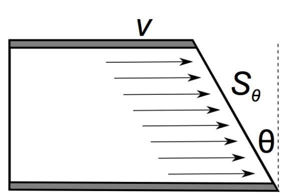




b) Suppose the pipe is cut on the diagonal as shown in the figure below. Now it has a new cross–sectional area S_θ at an angle θ. Note that $S_\theta = \frac{S}{\cos\theta}$. Briefly state why this makes sense.

Is the flux out of the pipe greater than, less than or equal to the one in part **a)**? Explain your answer.

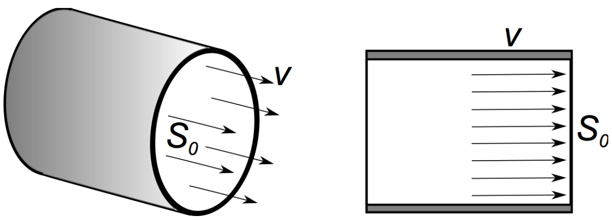


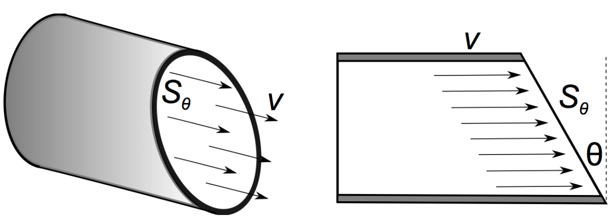


c) Let us now treat area as a vector. Consider a generic flat surface with constant area S and *unit vector* (a vector of length 1) $\vec{n}$, normal to the surface pointing 'outward'. We can define the area vector as $\vec{S} = S\vec{n}$. The magnitude of $\vec{S}$ is the area S, and its direction is normal to the area.

If we have a constant velocity vector defined over our surface, the flux is the dot product of area and velocity vectors, i.e. flux $= \vec{v} \cdot \vec{S}$.

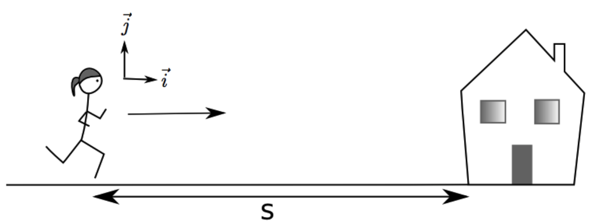
Use this definition of flux to redo part **a)** and **b)**. Draw the normals to S_0 and S_θ and calculate the flux as $\vec{v} \cdot \vec{S} = S(\vec{v} \cdot \vec{n})$.

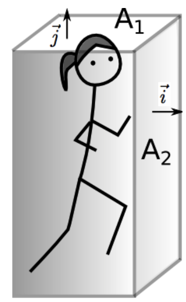




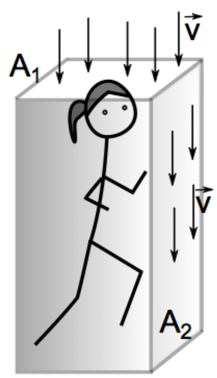
Problem 2 – A fun question to ponder

Imagine that you are on your way home when suddenly it rains - and you don't have an umbrella! Home is a distance s away when the rain starts falling. For simplicity let's assume that you are modeled as a box, as shown below.

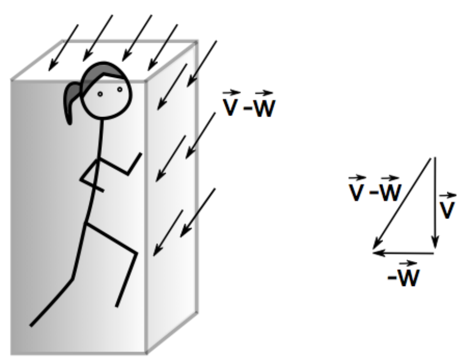




Let $\vec{i}$ be a unit vector perpendicular to area A_2 and $\vec{j}$ a unit vector perpendicular to area A_1. Figures (a) and (b) below show how the rain appears to fall when you are stationary and when you are moving forward with velocity $\vec{w}$. We assume the rain is evenly distributed and falling straight down with velocity $\vec{v}$. The velocity of the rain relative to your body is $\vec{v} - \vec{w}$.



(a) At rest



(b) In motion with velocity $\vec{w}$

a) Write down the area vectors for the surfaces A_1 and A_2 using $\vec{i}$ and $\vec{j}$ as direction vectors. Write $\vec{v}$ and $\vec{v} - \vec{w}$ in terms of $\vec{i}$ and $\vec{j}$. Recall that $\vec{v}$ has length $\|\vec{v}\|$ and the surfaces A_1 and A_2 have area $\|A_1\|$ and $\|A_2\|$ respectively.

b) The amount of water hitting a surface **per unit time** is represented by the flux through the surface. Calculate the amount of water hitting A_1 per unit time. Calculate the amount of water hitting A_2 per unit time.

c) How long will it take you to travel the distance s to your house when you are traveling with velocity $\vec{w}$? Write an equation that calculates how much water will hit you as you travel the distance s to your house.

d) If you want to minimize how wet you get, is it better to walk or run through the rain?

In the Future:
If the velocity vector is constant over the area, the flux is the dot product of area and velocity vectors: flux $= \vec{v} \cdot \vec{S}$.
If the area is curved in 3D, and/or the vector field is not constant over the area flux is a 'surface integral': $\int_S \vec{v} \cdot d\vec{S}$ of the vector field $\vec{v}$ through the surface S in space. It is the sum (integration) of the 'small fluxes' ($\vec{v} \cdot d\vec{S}$) through small area vectors $d\vec{S}$ on the surface S. We'll see surface integrals in the future.